Elmer/Ice Glaciological Modelling

Glaciological Modeling with the Finite Element Package Elmer/Ice

Thomas Zwinger and ...

...

- CSC: Juha Ruokolainen, Peter Råback, Mikko Lyly (now ABB) and Mika Malinen
- LGGE, UJF, Grenoble: Olivier Gagliardini, Fabien Gillet-Chaulet, Gaël Durand, Lionel Favier, Jean Krug
- ILTS, Univ. of Hokkaido, Sapporo: Hakime Seddik, Ralf Greve
- BAS: Carlos Martìn
- Arctic Centre, Rovaniemi: Martina Schäfer, Rupert Gladstone
- Univ. Swansea: Sue Cook, Ian Rutt
- Univ. Uppsala: Jonas Thies, Josefin Alkrona

CSC

Projects and Partners

ice2sea

SVALI

norden

Top-level Research Initiative

LGGE

Laboratoire de Glaciologie et Géophysique de l'Environnement

ILTS

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School of Science
and Technology

北京師範大學
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GCESS
全球变化与地球系统科学研究院
College of Global Change and Earth System Science

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Contents

- General overview on ice-flow modeling
  - Basic equations (Stokes equations)
  - And their approximations (SIA,SSA)
- Introduction of Elmer/Ice
  - Ice sheet modelling with Elmer/Ice
- Some numerical concepts
  - Saddle point problem
  - Parallel runs

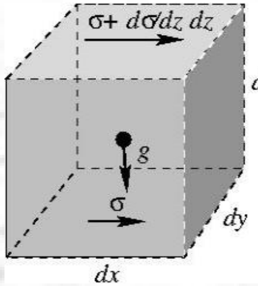
The Big Picture

- Until recently no ice sheet model (ISM) capable of correctly dealing with the **marine ice sheet** problem; tightly linked: **calving**
- Currently no deeper understanding of **basal sliding** processes
- Influence of **anisotropy** in ice flow
- ISM integration in **Earth System Models** (ESM)
  - **Glaciers** in warming climates
  - Local interaction of weather with ice surfaces

Ice Flow Dynamics

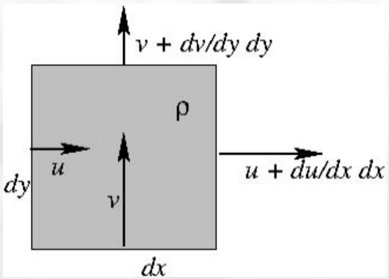
- Small Reynolds/Froude number limit → Stokes

$$\underbrace{\nabla \cdot \boldsymbol{\tau} - \nabla p}_{\text{sum of all surface forces}} + \underbrace{\varrho \mathbf{g}}_{\text{gravity}} = \mathbf{0},$$



- Incompressible fluid (if firn is neglected)

$$\underbrace{\nabla \cdot \mathbf{u}}_{\text{sum of all volume fluxes}} = 0$$



  – Definition strain rate tensor $\dot{\boldsymbol{\epsilon}} = \frac{1}{2}\left(\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}\right)$

  – And invariants $I_{\dot{\epsilon}} = \mathrm{tr}(\dot{\boldsymbol{\epsilon}}) = \nabla \cdot \mathbf{u}, \quad II_{\dot{\epsilon}} = \mathrm{tr}(\dot{\boldsymbol{\epsilon}}^2)$

Ice Flow Dynamics

- Shear thinning fluid → Norton-Hoff law with $n=3$ (Glen's flow law)

$$\boldsymbol{\tau} = \eta(II_{\dot{\epsilon}})\,\dot{\boldsymbol{\epsilon}} = \left(\frac{A^{-1/n}(T,p)}{2}(\sqrt{II_{\dot{\epsilon}}})^{(1-n)/n}\right)\dot{\boldsymbol{\epsilon}}$$

- Arrhenius law for rate factor

→ thermo-mechanical coupling

$$A(T,p) = 2E\exp\left(-Q/RT'\right)$$

Ice Flow Dynamics

Evolution of the free surface/flow height:

net accumulation

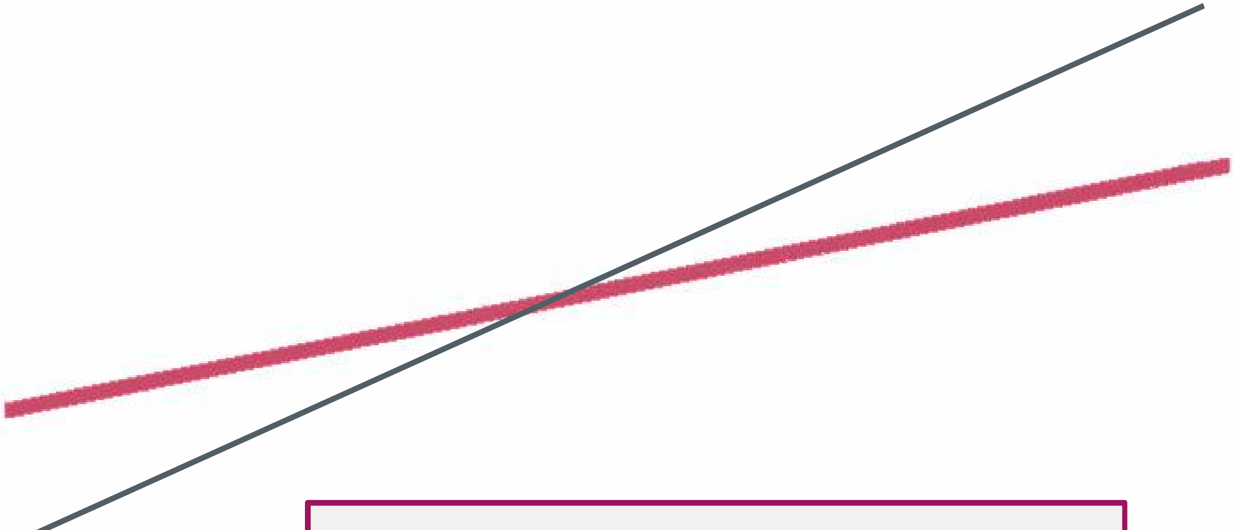
$$\frac{\partial h}{\partial t} + \underbrace{u_x \frac{\partial h}{\partial x} + u_y \frac{\partial h}{\partial y} - u_z}_{\text{convective transport}} = a$$

local change

coupling to climate

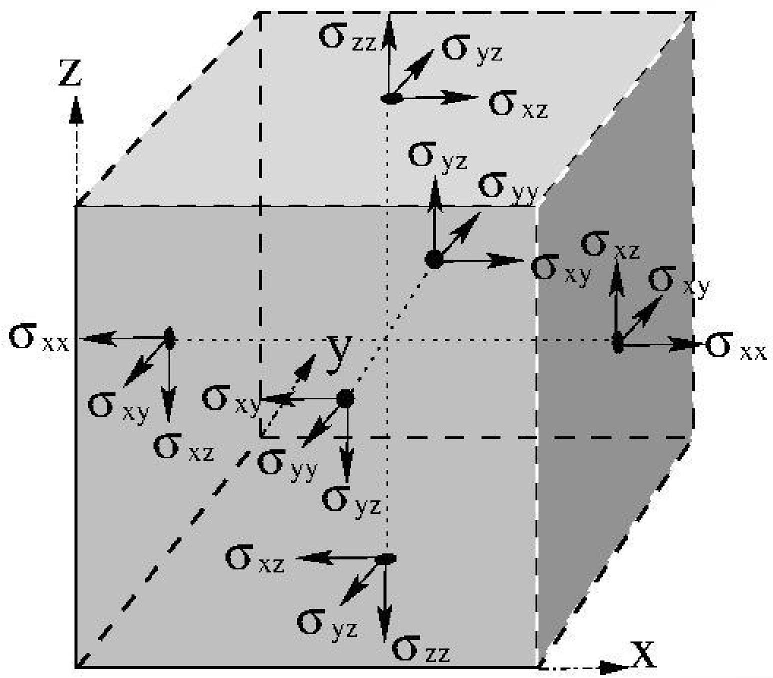
Ice flow modeling/dynamics

Ice Flow Dynamics



$$\frac{\partial h}{\partial t} + \underbrace{u_x \frac{\partial h}{\partial x} + u_y \frac{\partial h}{\partial y} - u_z}_{} = a$$

Ice Flow Dynamics



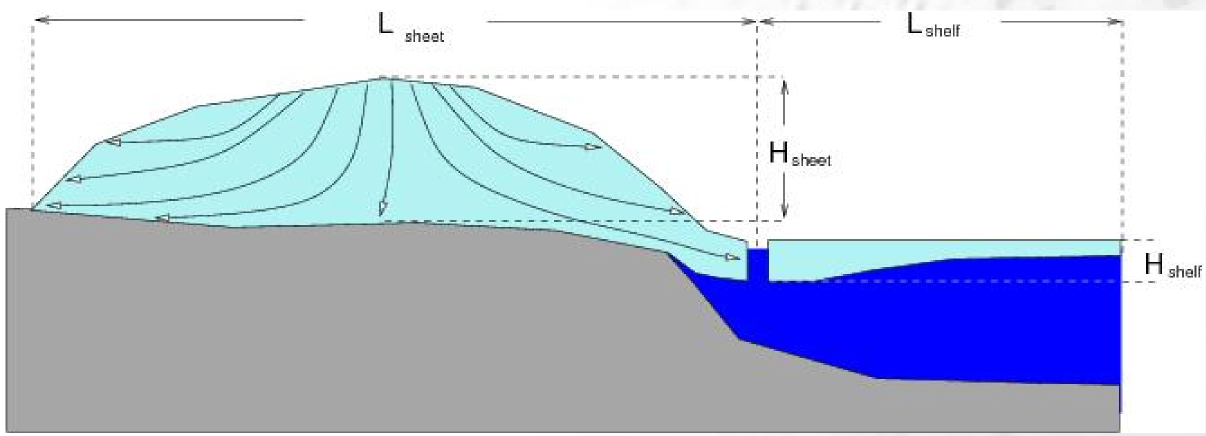
- Accounting for all components of the stress tensor = Full Stokes (FS) model
- “Cheating” by neglecting certain components:
  - Approximations, usually based on flatness assumption

Approximations to FS

- Shallow Ice Approximation (SIA)

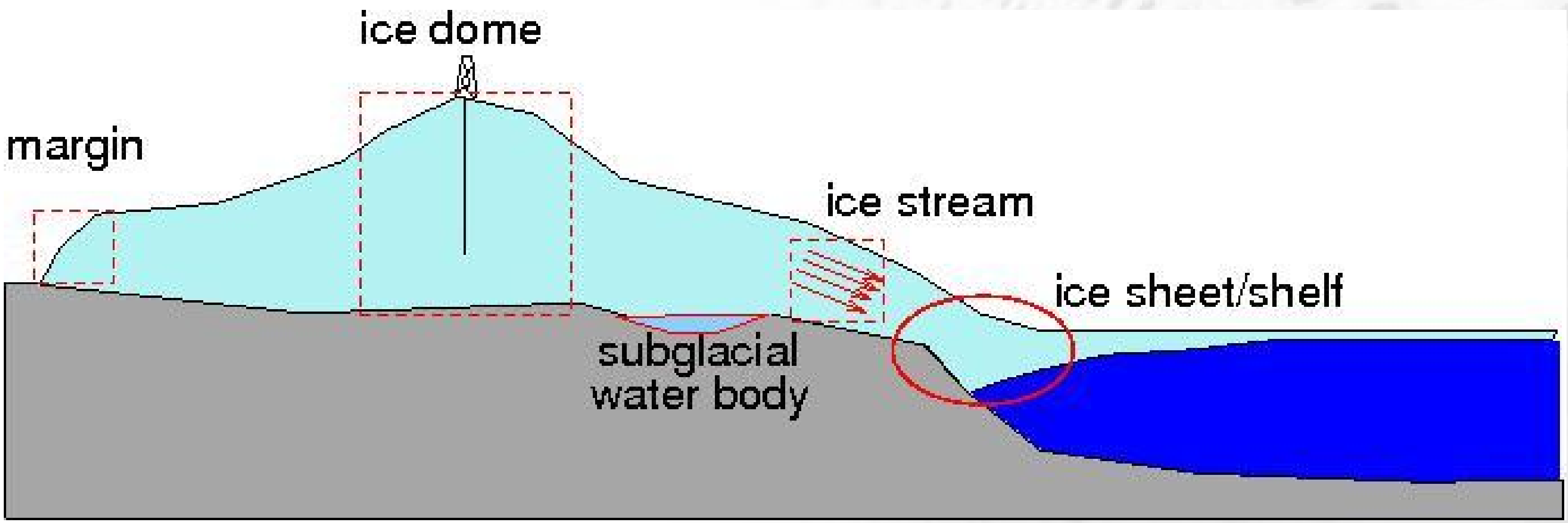
$$\epsilon = \frac{H}{L} \rightarrow 0$$

- Only vertical shear stresses
- Continuity equation → elliptic
- Momentum balance → simple relation (PDE)



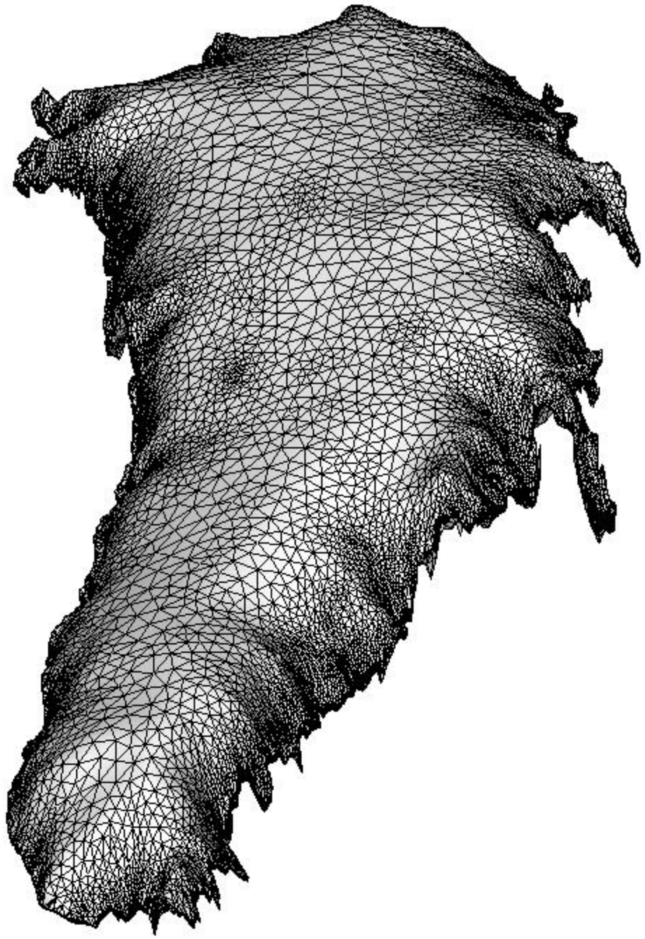
Approximations to FS

- Ice sheets and even glaciers are much longer than thick; generally SIA holds
- Limits of SIA and SSA:
  - Everywhere where $\varepsilon \rightarrow 0$ does not hold



Full Stokes?

- Stokes flow in 3D: 4 DOFs (u,v,w,p)
- Expensive!
- Greenland ice sheet (GIS): 1.7×10^6 km^2
- 500m resolution → 6×10^6 horizontal cells
- 15 vertical layers → 1×10^8 bulk elements/nodes



Elmer/Ice

- Elmer (= multi-physics package) with additional routines for Glaciology
- Maintained and supported by CSC
- Open Source (GPL2 or later)
  - Transparence (you co-own the code)
  - Sustainability (no license fees)
  - Viral effect of GPL (new code also GPL)
- Large international user community
  - Knowhow of well-established institutions
- Good level of support/documentation
  http://elmerice.elmerfem.org

Elmer/Ice

- Finite element method (FEM)
  - Able to produce adaptive meshes
  - Using linear elements and standard Galerkin with Stabilized Finite Elements
- Flow law → viscosity changes by order of magnitudes → bad conditioned system:
  - Direct parallel Solver
  - Special pre-conditioning
- (Massive) parallel computing
  - MPI
  - Multi-threading under development

Elmer/Ice

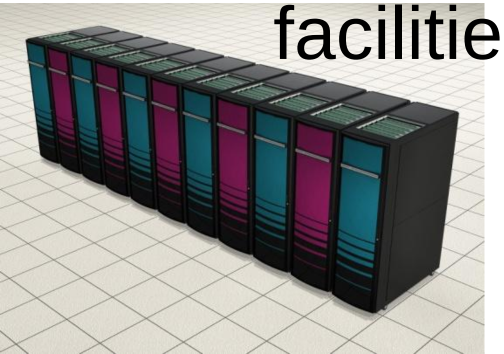
- Implemented Glaciological models:
  - Unstructured meshing
  - Deforming and moving meshes
  - Rheology: Glen, anisotropy, firn densification
  - Special sliding laws
  - NetCDF-readers (for geometry as well as coupling to climate)
  - Simple SMB (PDD)
  - Simple calving model
  - Inverse methods for data assimilation
  - Methods for tracer transport/dating

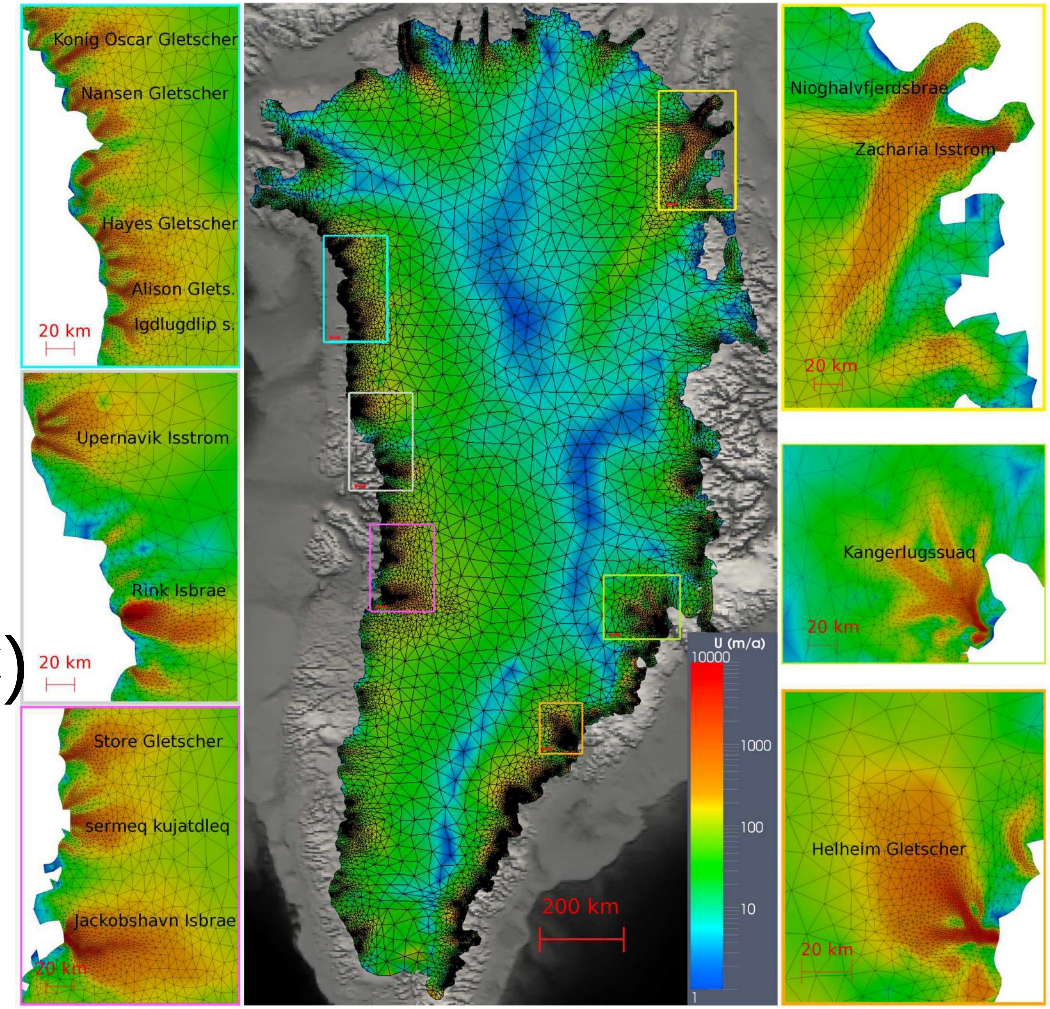
Elmer/Ice

- Currently developed/planned Glaciological models:
  - Coupling of SIA and FS
  - Hydrological model for soft-bedded (till) glaciers
  - Hydrological model for hard-bedded glacier
  - Enhanced calving laws (including issues with mesh updates)
  - Coupling to ocean model
  - Till deformation model

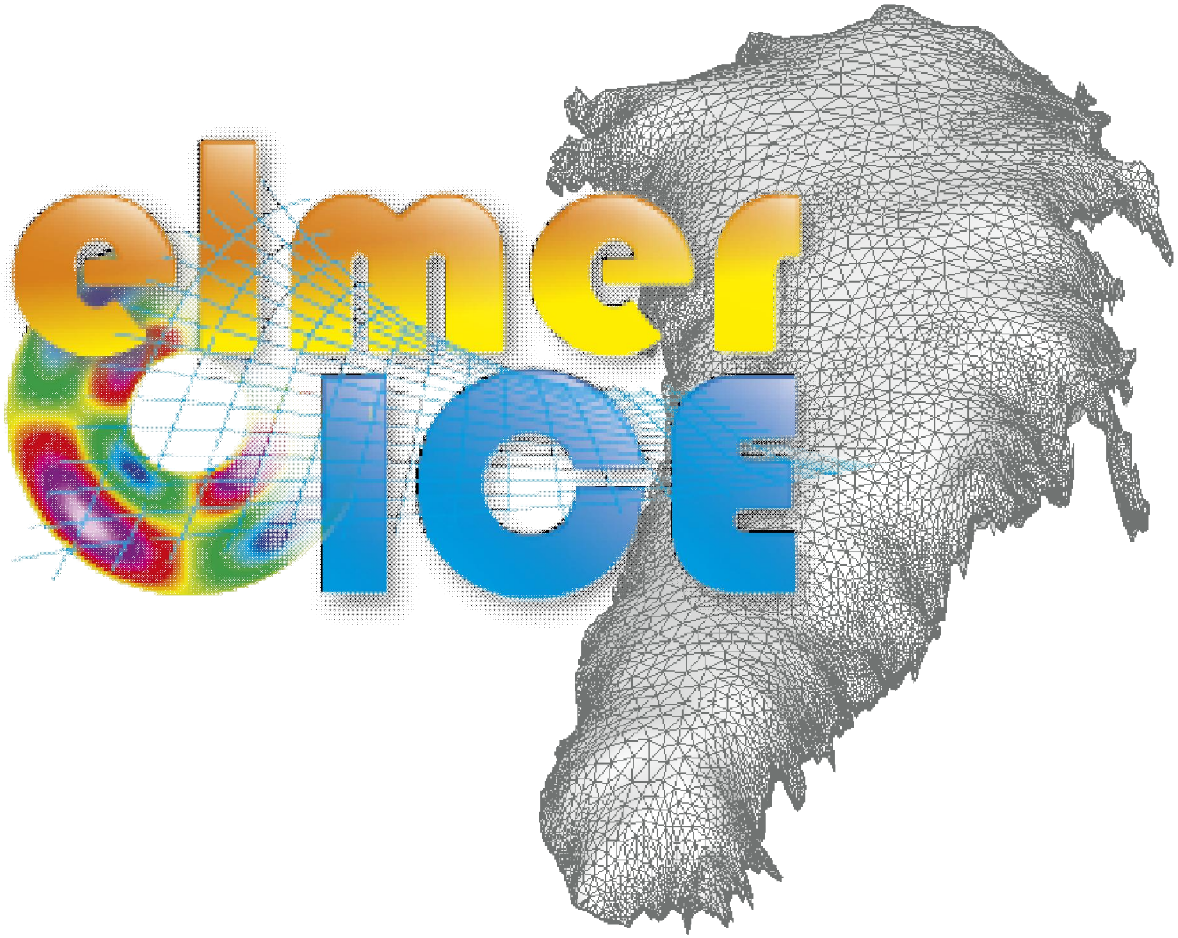
Ice-sheets

- Challenging by size
- Centuries rather than millennia
- Needs high performance computing (HPC) facilities





Different Applications of Elmer/Ice
Full Stokes modelling



Sliding

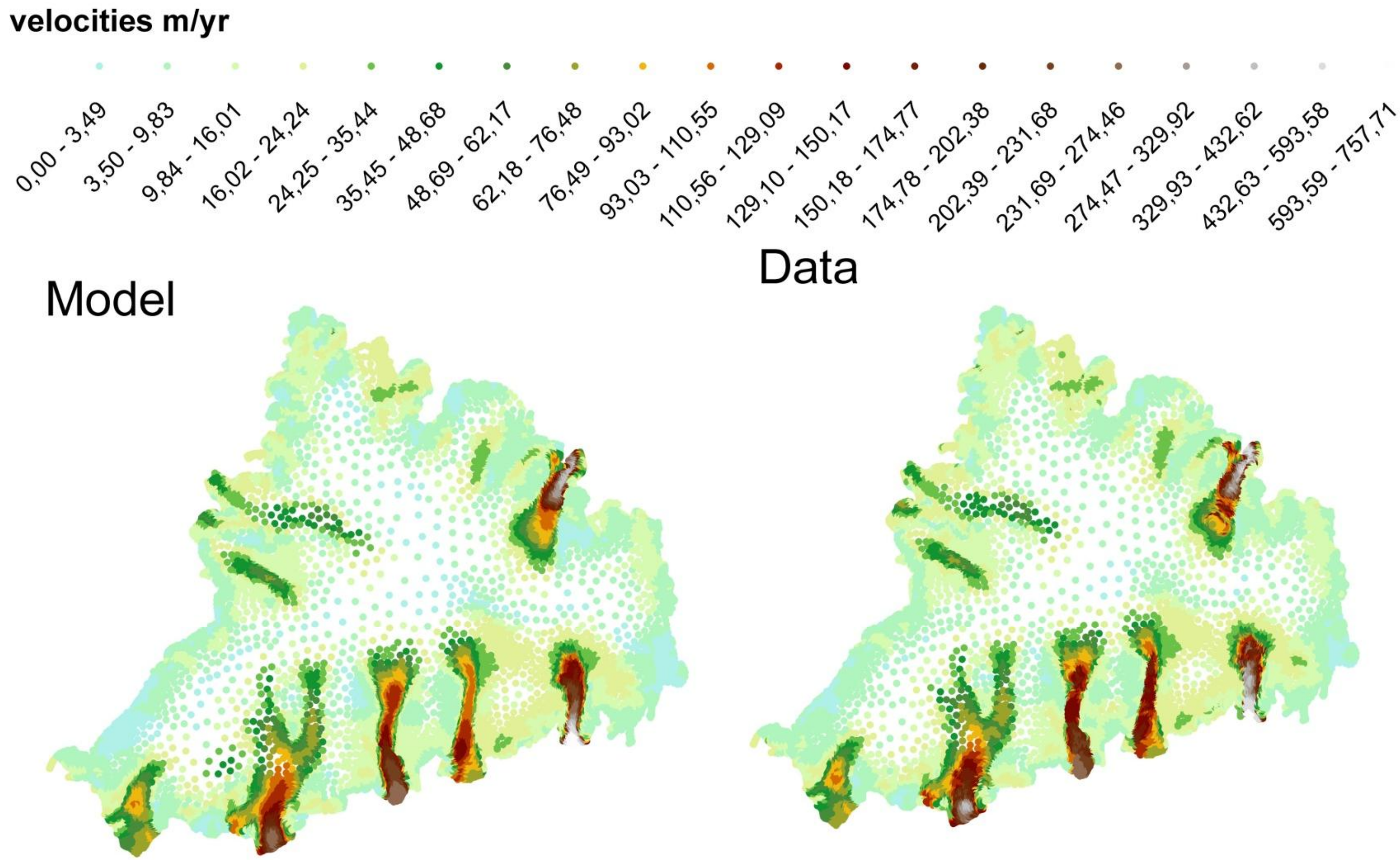
- Different types of sliding surfaces
  - Till (deforming sediment)
  - Hard rock
- Strong involvement of bed hydrology
- Bedrock processes not really understood and hard to determine from observations
- **Inverse Problem**: Determine sliding conditions from surface velocities

Sliding

- Solving iteratively Dirichlet (prescribed surface velocities) and Neuman (vanishing surface stress) problem
- Optimization with respect to the bed friction coefficient β by Gâteaux derivative $d_\beta J$ of cost function:

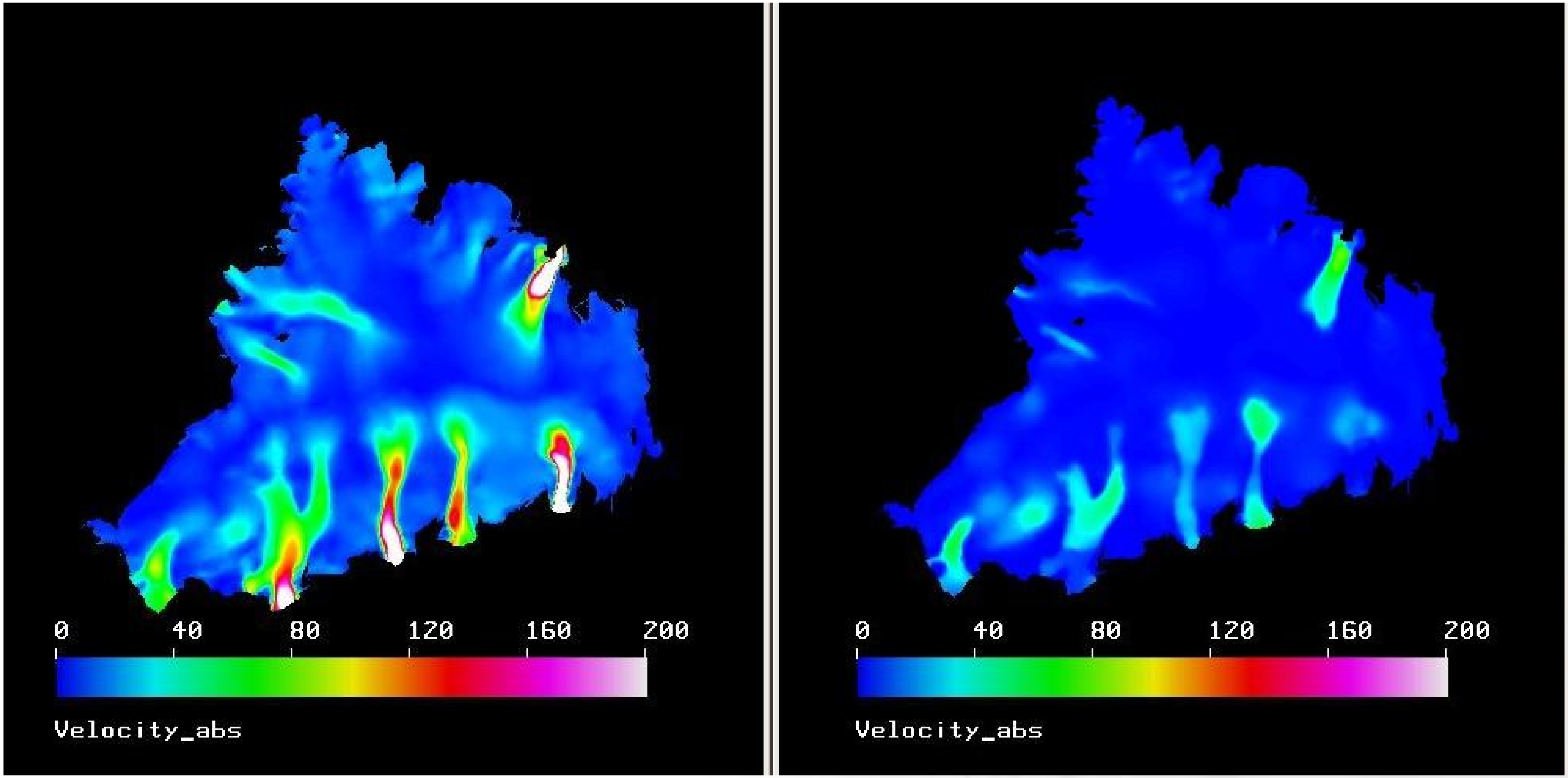
$$J = \int_A (\vec{u}_N - \vec{u}_D) \cdot (\sigma_N - \sigma_D) \cdot \vec{n} \, dA$$

Sliding



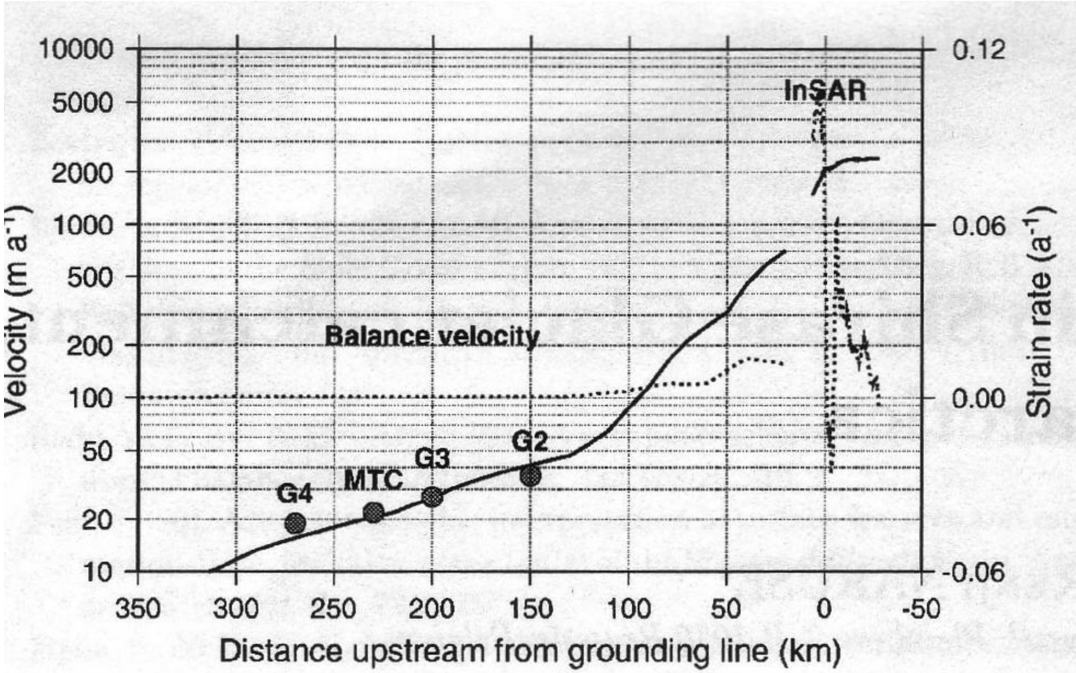
velocities m/yr
0,00 - 3,49
3,50 - 9,83
9,84 - 16,01
16,02 - 24,24
24,25 - 35,44
35,45 - 48,68
48,69 - 62,17
62,18 - 76,48
76,49 - 93,02
93,03 - 110,55
110,56 - 129,09
129,10 - 150,17
150,18 - 174,77
174,78 - 202,38
202,39 - 231,68
231,69 - 274,46
274,47 - 329,92
329,93 - 432,62
432,63 - 593,58
593,59 - 757,71
Model
Data

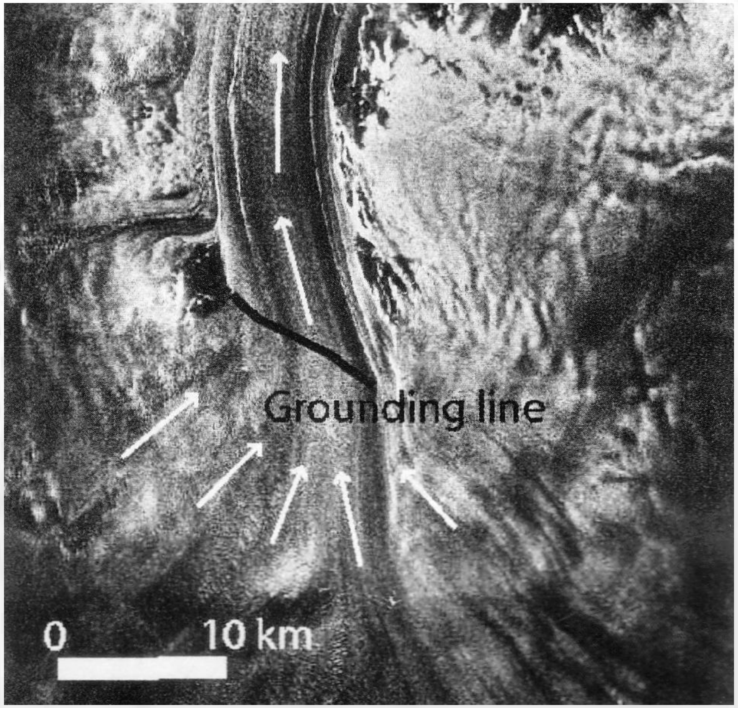
Sliding


0 40 80 120 160 200
Velocity_abs
0 40 80 120 160 200
Velocity_abs

Marine Ice Sheets

- Ice sheet that terminates at sea
- Ice that passes the grounding line (GL) contributes to sea level rise (SLR)

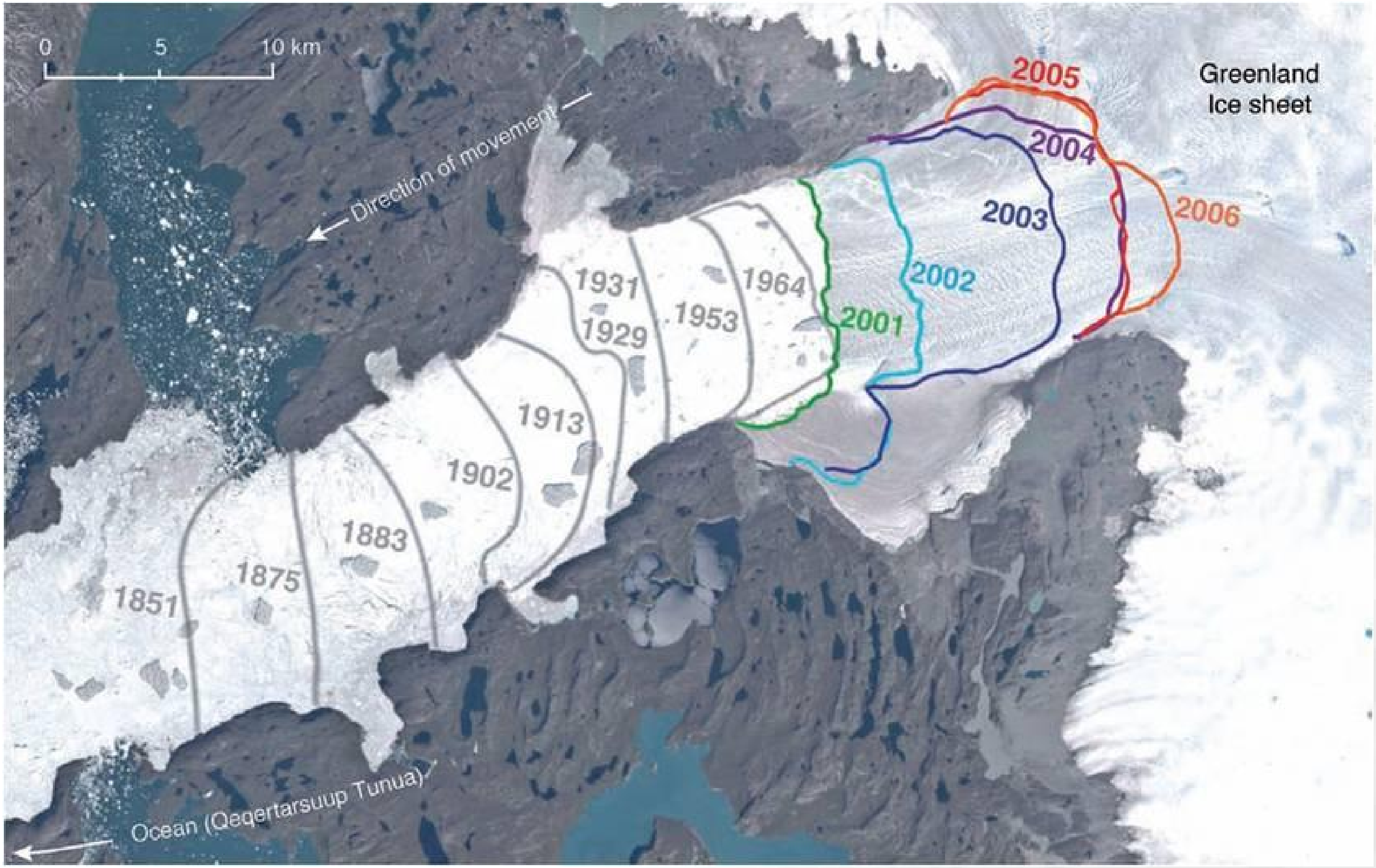




Pattyn & Naruse (2003) J. Glacio., 49, 166

Marine Ice Sheets

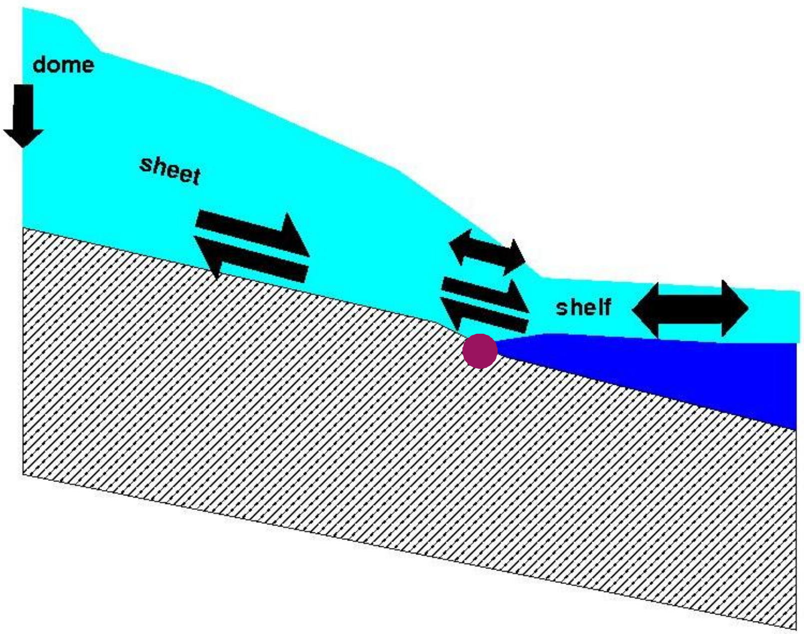
- Vast increase of outlet and retreat → SLR

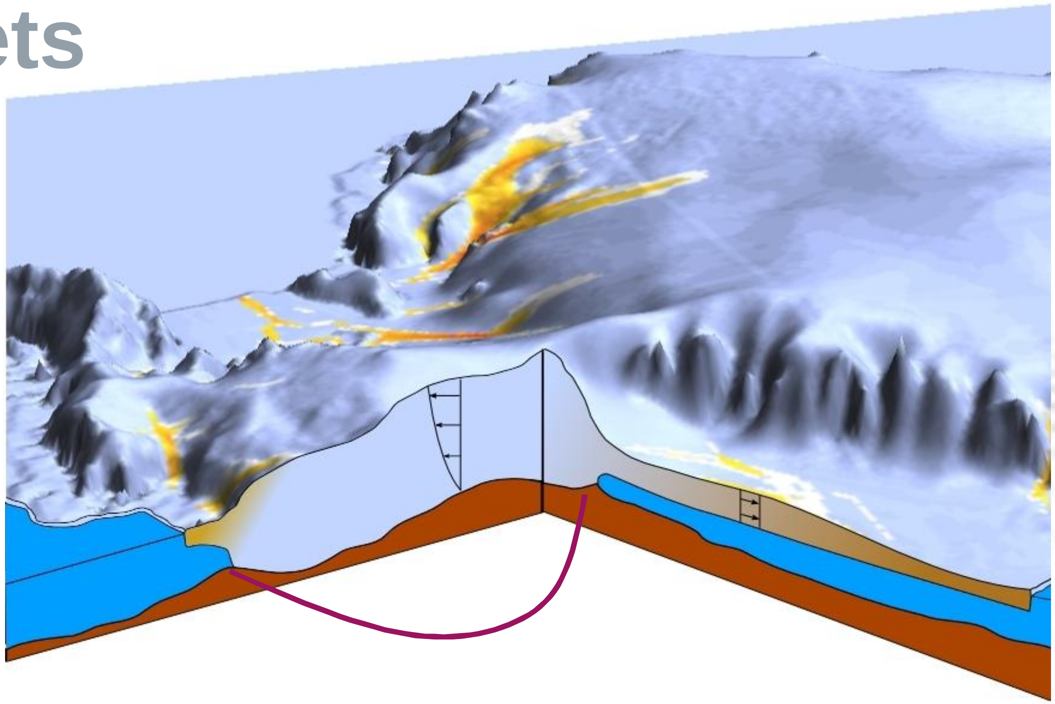


Howat et al. (2007)

Marine Ice Sheets

- Sudden change of stress field at GL



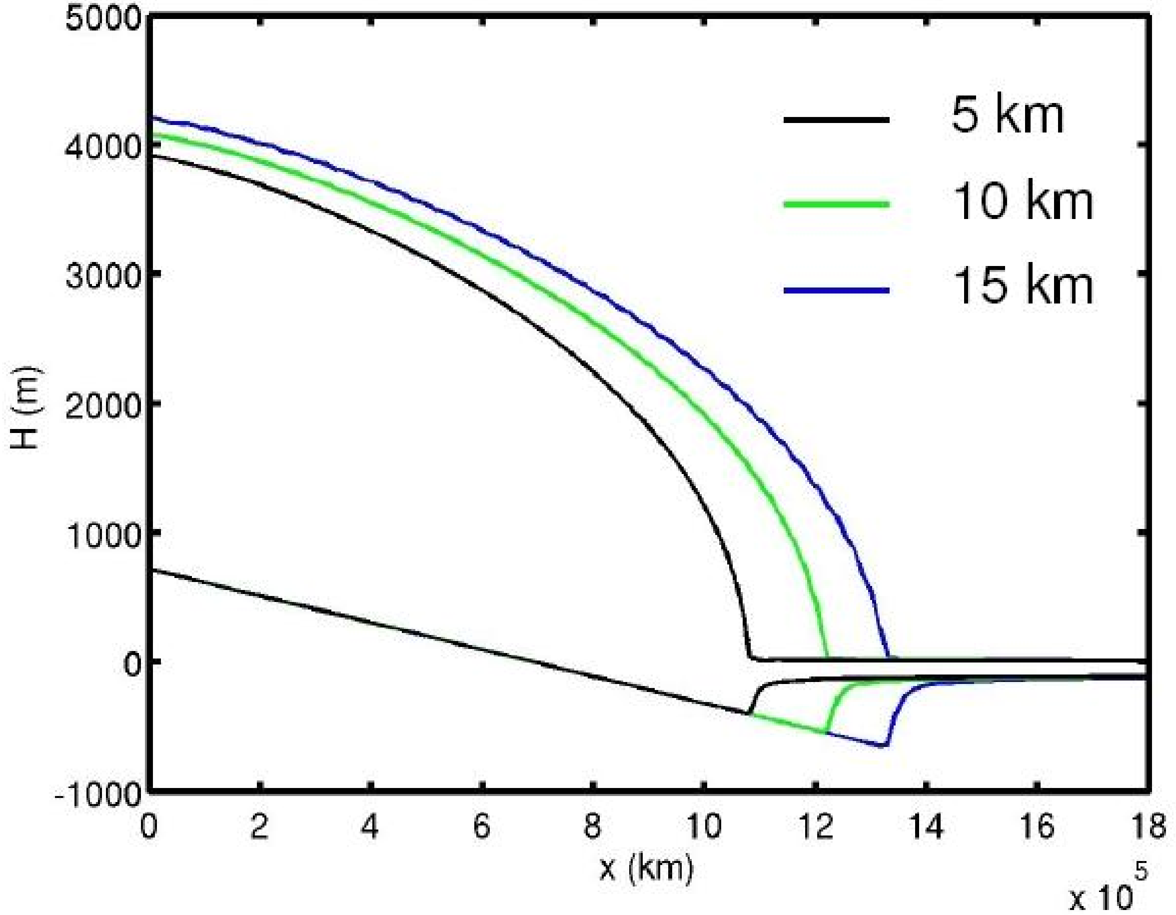


- Characteristics of a boundary layer

Marine Ice Sheets

- (Mesh) Size matters
- Different steady states depending on resolution
- Consistent for

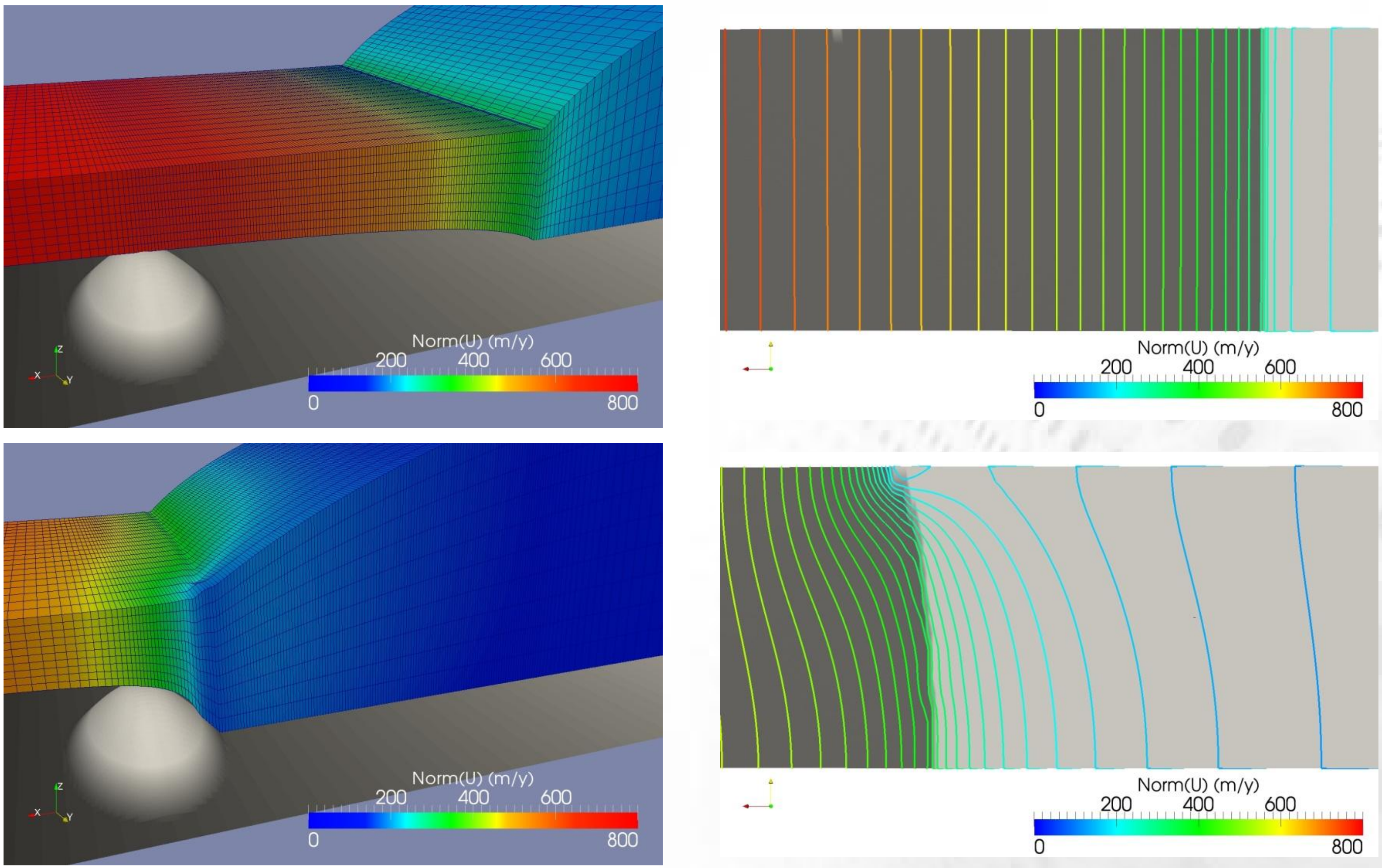
$$\Delta x < 200\,\mathrm{m}$$



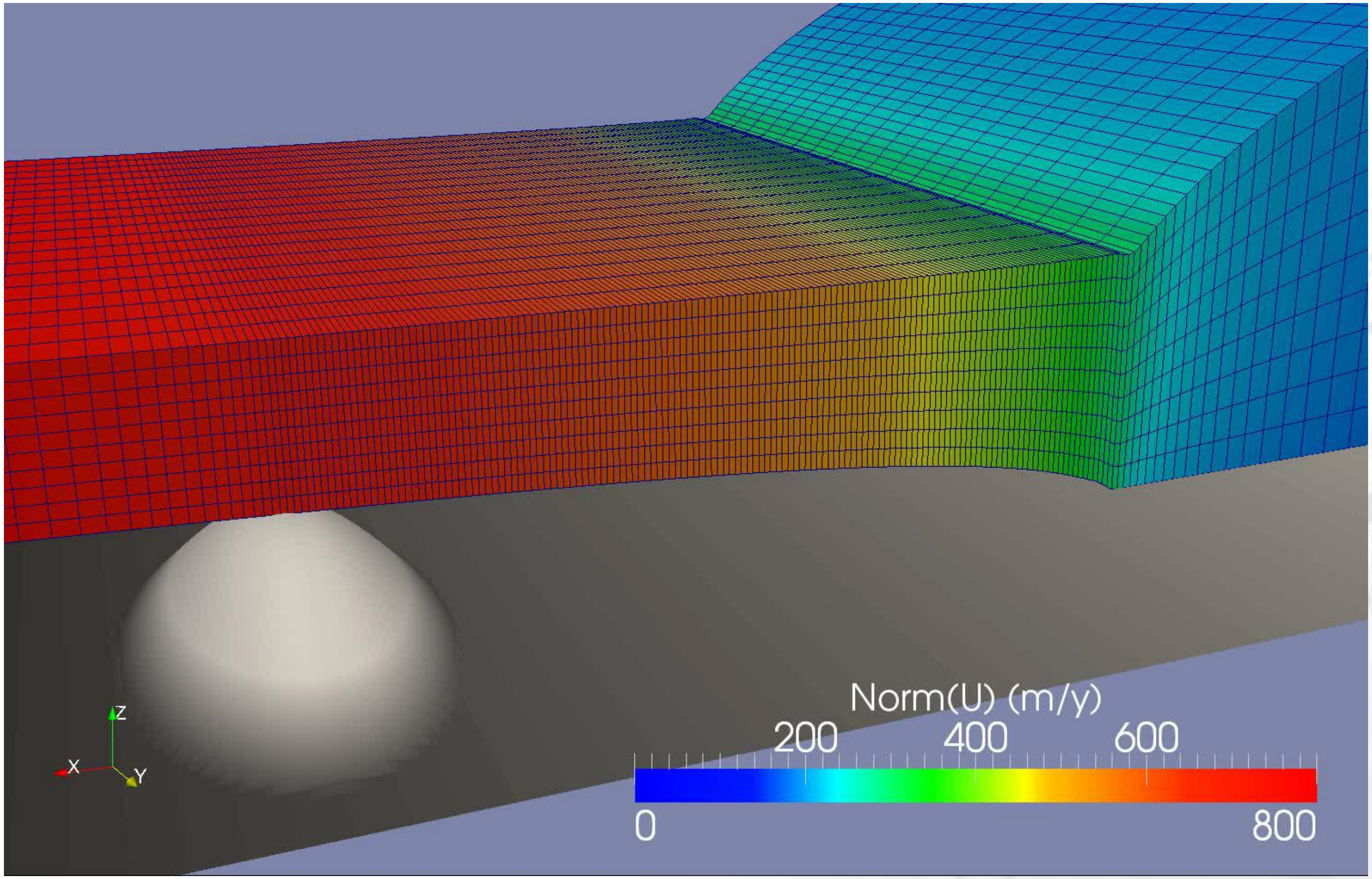
G. Durand, O. Gagliardini, T. Zwinger, E. Le Meur, and R. Hindmarsh (2009) *Full Stokes modeling of marine ice sheets: influence of the grid size*, Annals Glaciol., 52, 109-114

Marine Ice Sheets

- Moving to 3D – pinning point



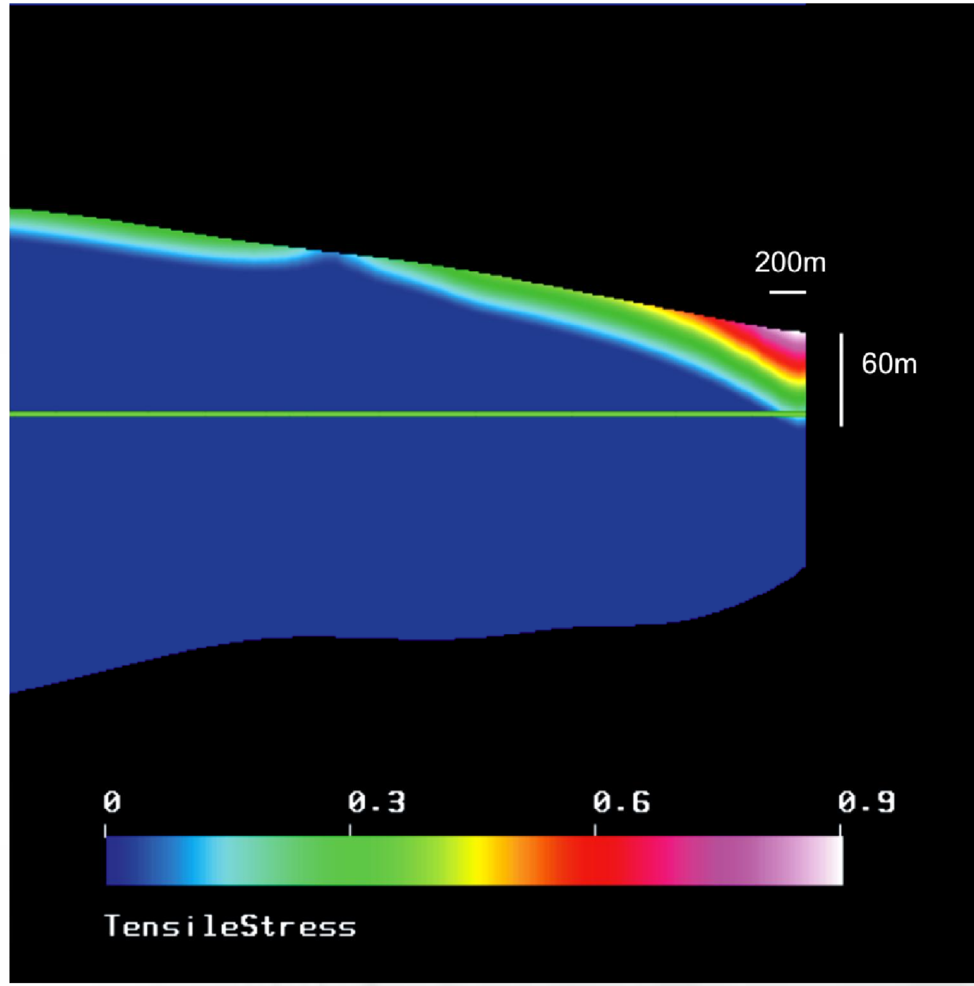
Marine Ice Sheets



L. Favier, O. Gagliardini, G. Durand, and T. *Zwinger A three-dimensional full Stokes model of the grounding line dynamics: effect of a pinning point beneath the ice shelf*
The Cryosphere Discuss., 5, 1995-2033, 2011

Calving

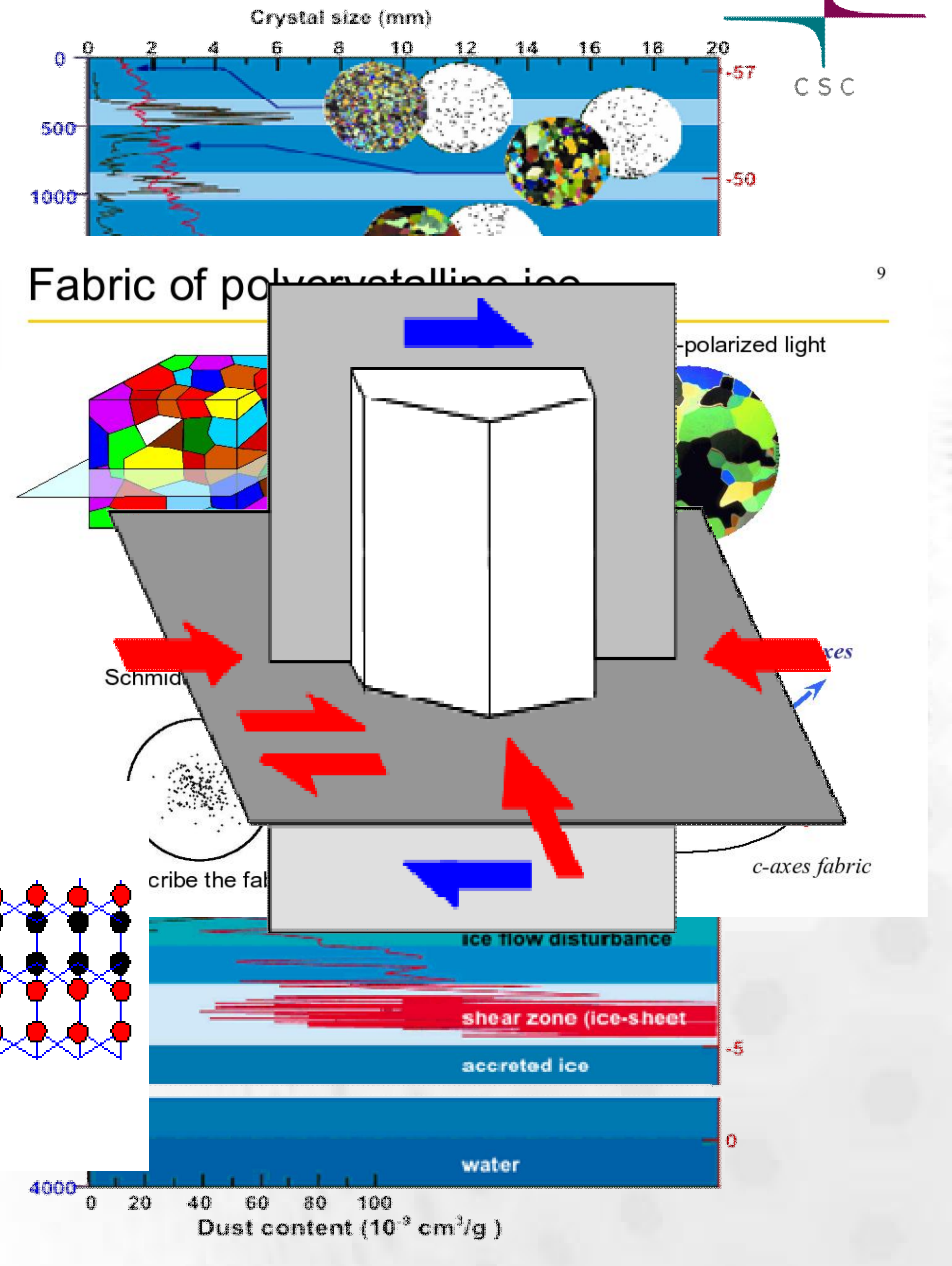
- Columbia glacier, Alaska, USA
- Nye formulation
- Including water level inside crevasses
  - Shift of terminus by 200 m – well within the range of observations

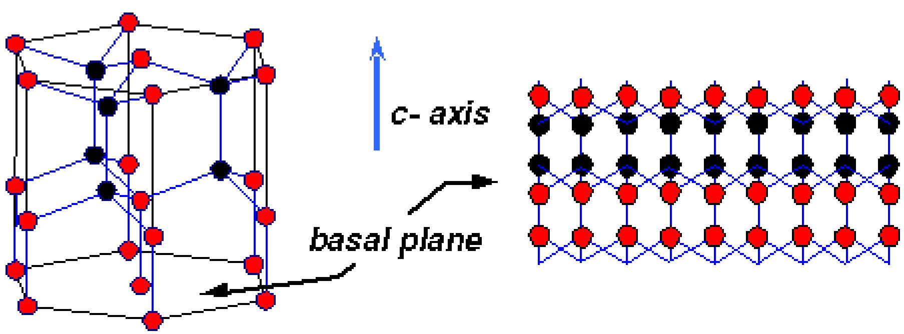


S. Cook, T. Zwinger, I.C. Rutt, S.O'Neel and T. Murray (2011) *Testing the effect of water in crevasses on a physically based calving model*, Annals Glac. 53, accepted

Anisotropy

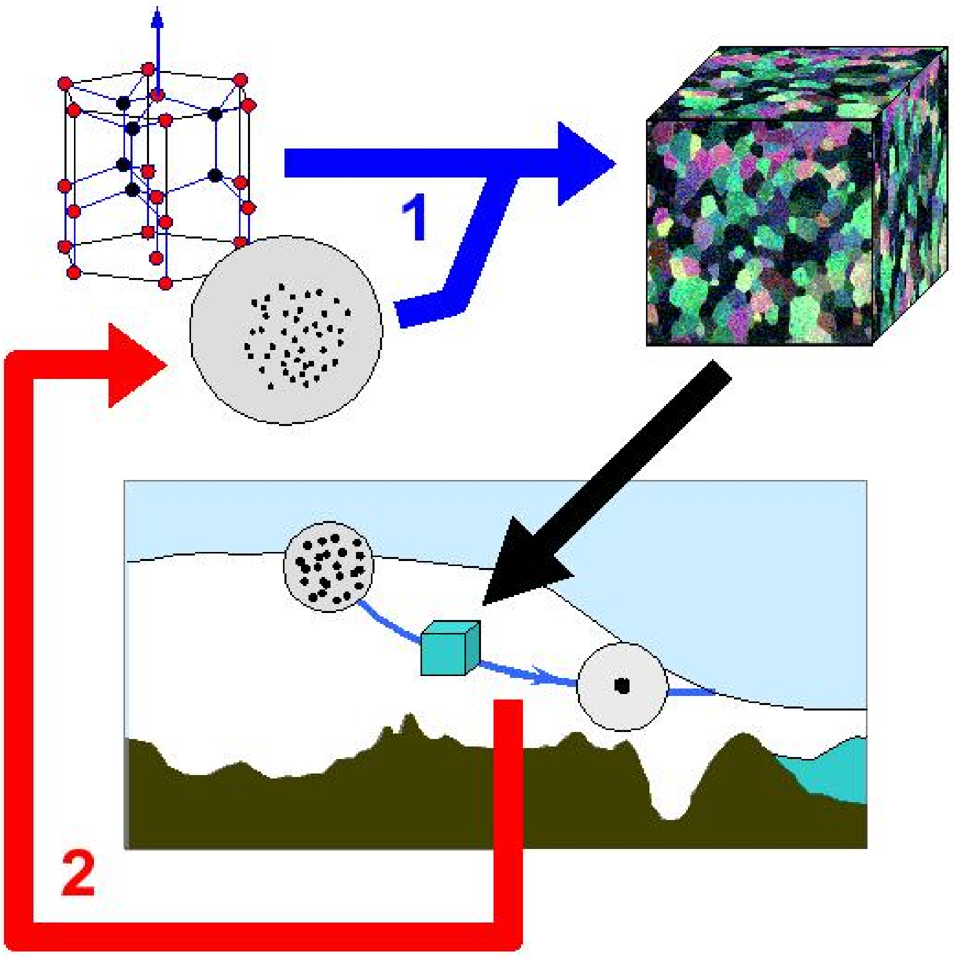
- Mono-crystalline ice is extremely **anisotropic**
- Polar ice is a **polycrystalline** material



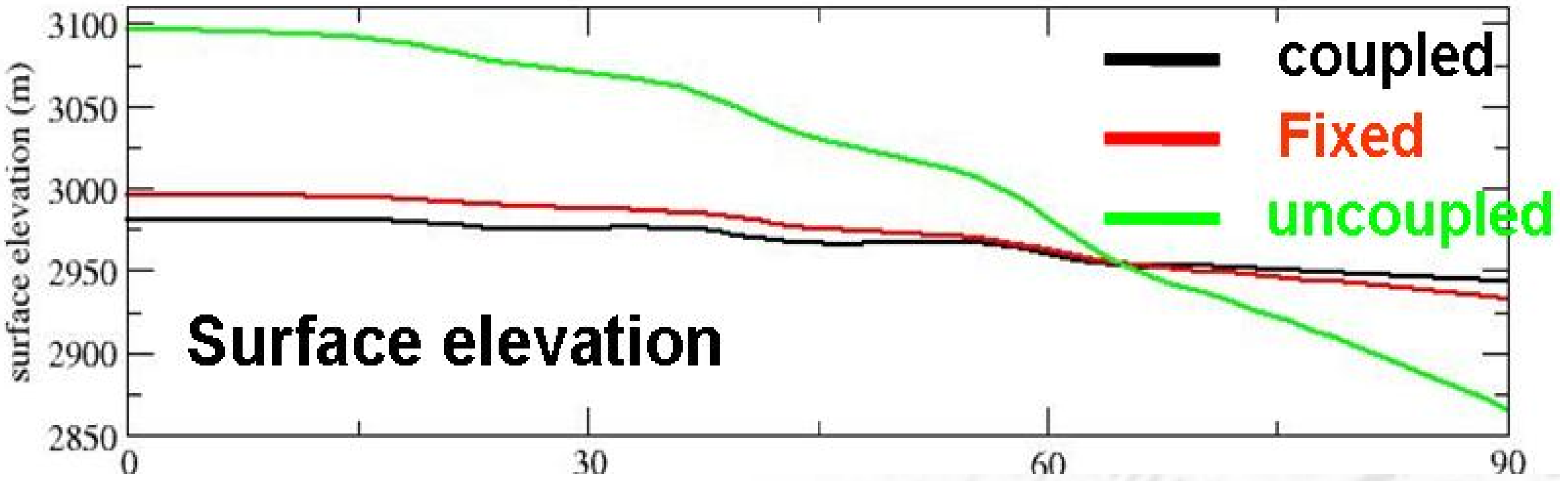


Anisotropy

- Fabric is influenced by shear history = dynamics
- Feedback via rheology



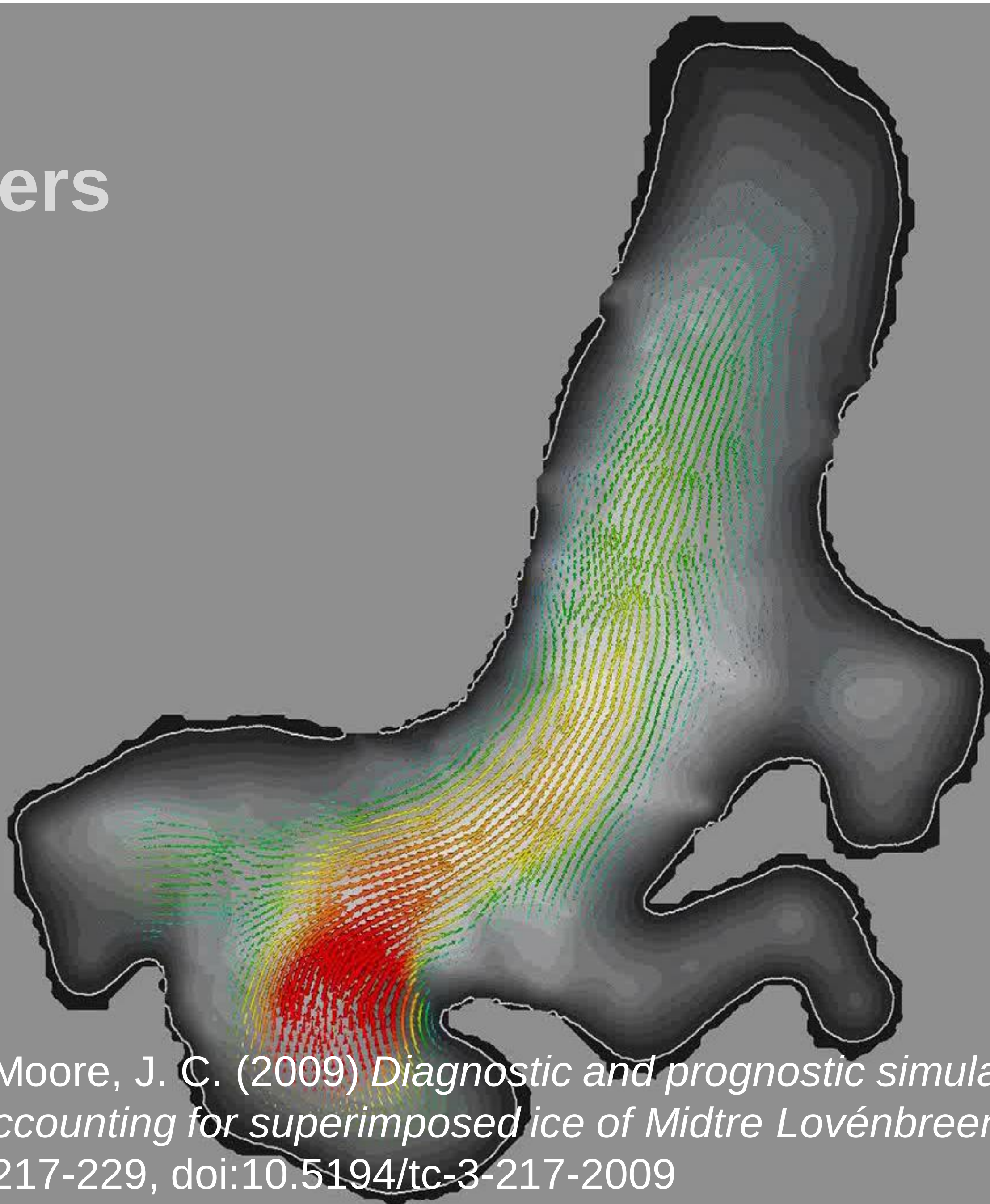
Material ice



- Influence of anisotropy

F. Gillet-Chaulet, O. Gagliardini, J. Meyssonnier, T. Zwinger, J. Ruokolainen (2006) *Flow-induced anisotropy in polar ice and related ice-sheet flow modelling*, J. Non-Newtonian Fluid Mech. 134, p. 33-43.

Glaciers



T. Zwinger and Moore, J. C. (2009) *Diagnostic and prognostic simulations with a full Stokes model accounting for superimposed ice of Midtre Lovénbreen, Svalbard*, The Cryosphere, 3, 217-229, doi:10.5194/tc-3-217-2009

Glaciers

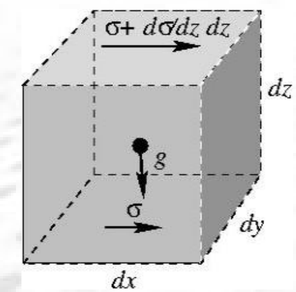
- Katabatic windfront impinging on blue ice area at Scharffenberg-bottnen, DML, EAAIS
- Elmer, VMS turbulentce model

Simulation by T. Zwinger and T. Malm

FEM Formulation

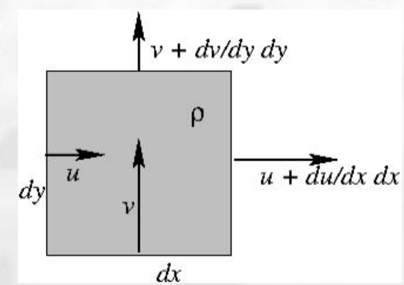
- Conservation of linear momentum

$$\underbrace{\nabla\cdot\boldsymbol{\tau}-\nabla p}_{\text{sum of all surface forces}}+\underbrace{\varrho\mathbf{g}}_{\text{gravity}}=\mathbf{0},$$



- Conservation of mass

$$\underbrace{\nabla\cdot\mathbf{u}}_{\text{sum of all volume fluxes}}=0$$



FEM formulation

- Weak formulation:

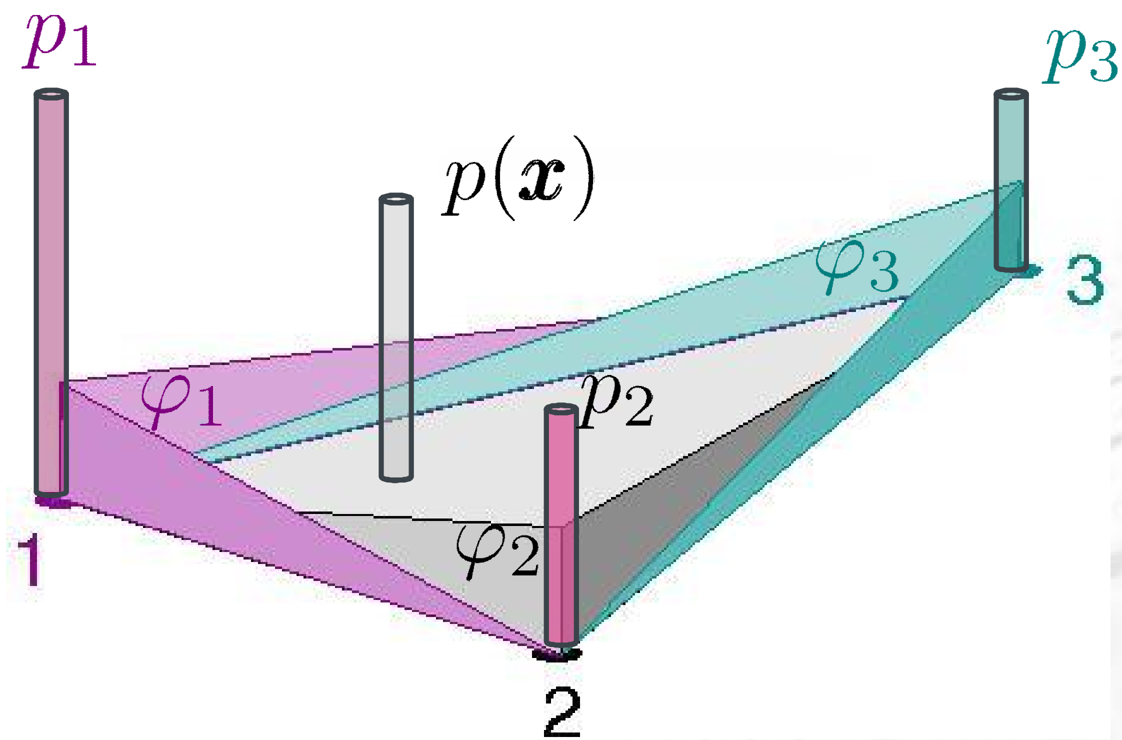
$$-\int_\Omega (\boldsymbol{\tau} - \mathbf{1}p) \cdot \nabla \varphi_\alpha d\Omega \quad + \quad \oint_\Gamma \boldsymbol{t} \varphi_\alpha d\Gamma$$

$$= \int_\Omega \varrho \boldsymbol{g} \varphi_\alpha d\Omega$$

- Linearization of deviatoric stress:

$$\boldsymbol{\tau}\big|_{(n)} = \eta(\dot{\boldsymbol{\epsilon}}\big|_{(n-1)})\dot{\boldsymbol{\epsilon}}\big|_{(n)}$$

FEM formulation

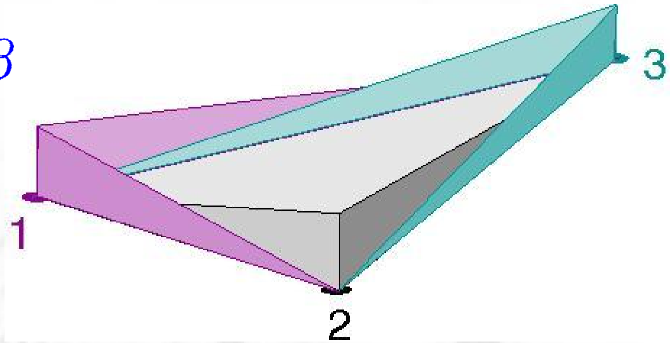


$$p(\boldsymbol{x}) = p_1\varphi_1|_{\boldsymbol{x}} + p_2\varphi_2|_{\boldsymbol{x}} + p_3\varphi_3|_{\boldsymbol{x}}$$

FEM formulation

- Discretization by weighting functions:

$$\boldsymbol{u} = \sum_\beta \boldsymbol{u}_\beta \psi_\beta, \qquad p = \sum_\beta p_\beta \chi_\beta$$



$$p_\beta \int_\Omega \chi_\beta \nabla \varphi_\alpha d\Omega \quad - \quad \boldsymbol{u}_\beta \int_\Omega \eta \dot{\epsilon}(\nabla \psi_\beta) \nabla \varphi_\alpha d\Omega$$

$$+ \oint_\Gamma \boldsymbol{t} \varphi_\alpha d\Gamma \quad = \quad \int_\Omega \varrho \boldsymbol{g} \varphi_\alpha d\Omega$$

FEM formulation

- Standard Galerkin: $\psi_\beta = \chi_\beta = \varphi_\beta$

$$p_\beta \int_\Omega \varphi_\beta \nabla \varphi_\alpha d\Omega - u_\beta \int_\Omega \eta \boldsymbol{D}(\nabla \varphi_\beta) \nabla \varphi_\alpha d\Omega$$

$$+ \oint_\Gamma \boldsymbol{t} \varphi_\alpha d\Gamma = \int_\Omega \varrho \boldsymbol{g} \varphi_\alpha d\Omega$$

$$S_{\alpha\beta} \chi_\beta = f_\alpha$$

FEM formulation

- Saddle point problem: $\boldsymbol{S} = \begin{bmatrix} -\eta D & \nabla \\ \nabla^T & 0 \end{bmatrix} = \begin{bmatrix} C^{-1} & A \\ A^T & 0 \end{bmatrix}$

$$\begin{bmatrix} C^{-1} & A \\ A^T & 0 \end{bmatrix} \cdot \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

- $\boldsymbol{S}$ is not positive definite
  - $\leftrightarrow$ no minimization problem
  - $\leftrightarrow$ $\exists$ non-trivial null-space for p (checkerboard)
  - $\leftrightarrow$ Babuska-Brezzi (aka. inf-sup) condition

FEM formulation

- Condition for **stability**:

$$\inf_p \sup_u \frac{p^T A u}{\left(u^T C^{-1} u\right)\left(p^T p\right)} \geq C$$

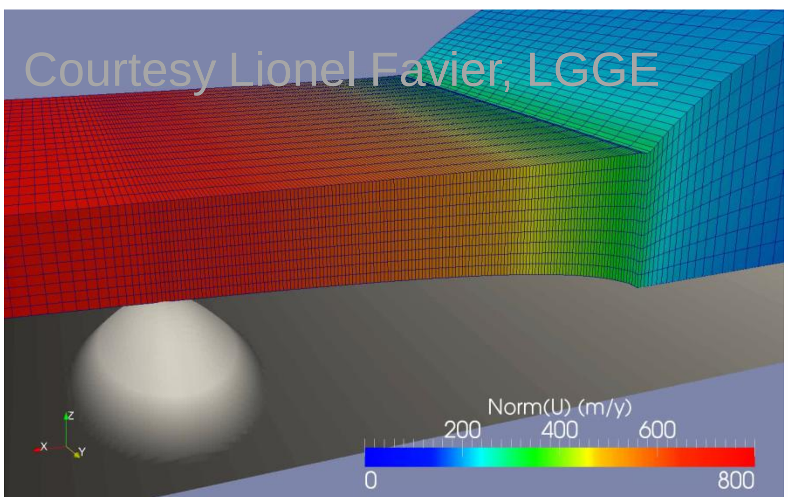
  – Depends on the space of the test-functions
  – **Stabilization**. Methods in Elmer:
    1. Residual square methods
    2. Residual free bubbles
    3. Taylor-Hood
    4. VMS

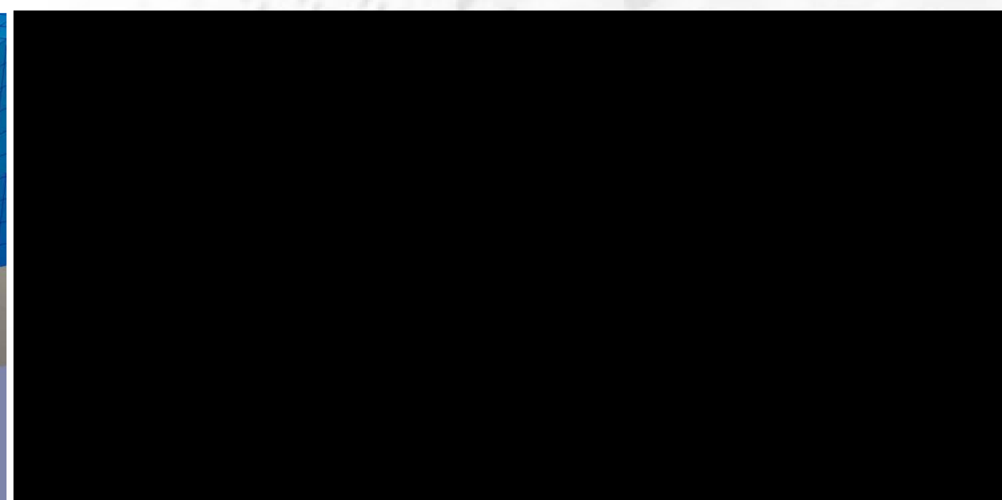
$$\begin{bmatrix} C^{-1} & A \\ A^T & \boldsymbol{B} \end{bmatrix} \cdot \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

Contact Problem

- No-penetration condition at bedrock
- Solving: $$\frac{\partial h}{\partial t} + u_x \frac{\partial h}{\partial x} + u_y \frac{\partial h}{\partial y} - u_z = a_\perp ,$$

  in combination with: $$h > b + \Delta\, h_{\mathrm{min}}$$
- Variational formulation → **Variational inequality**



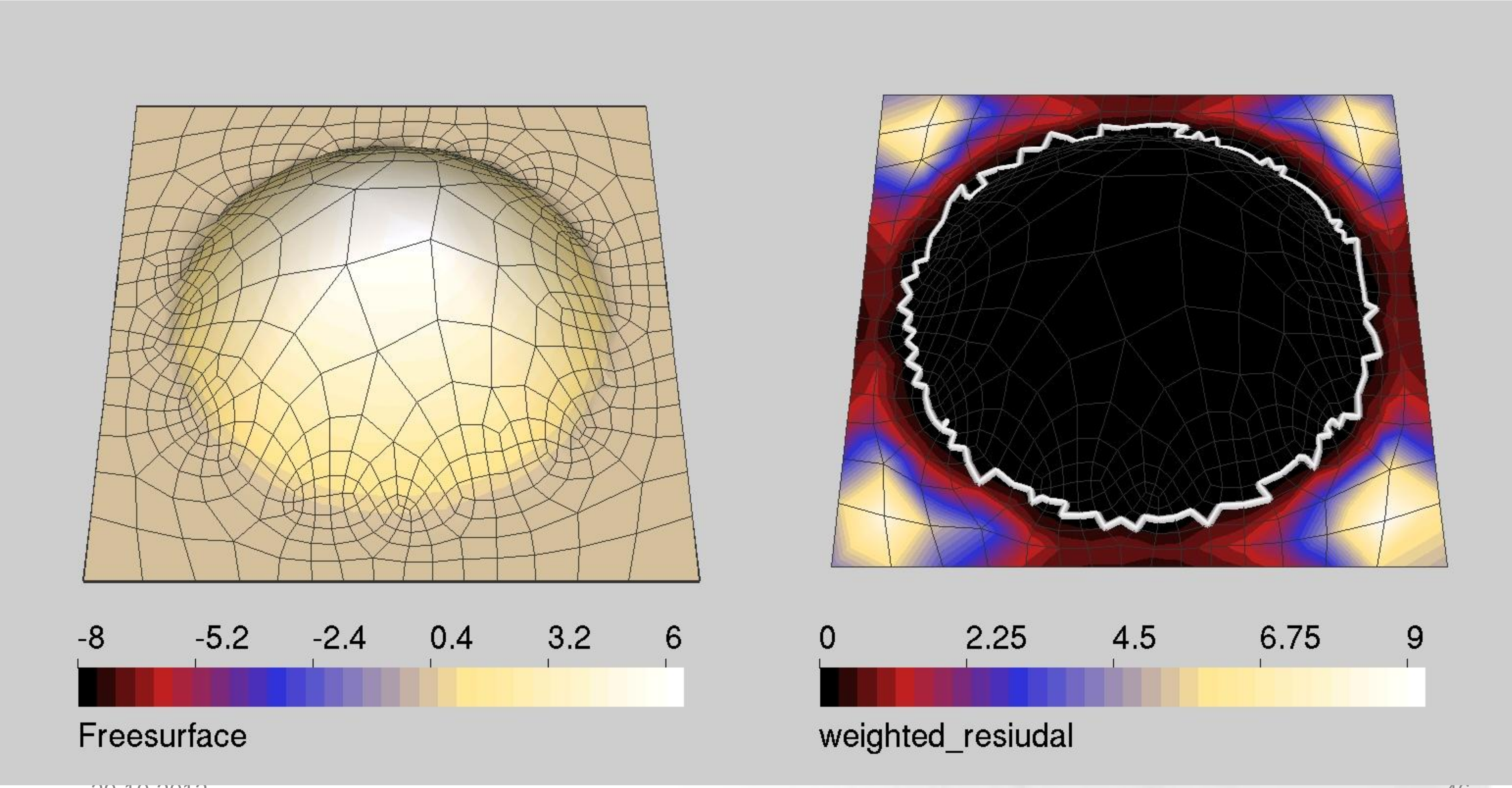


Contact Problem

- Solving system: $S_{ij}\, h_j = a_i, \quad h_j > h_{j,\min}$
- Flag, f_j, for active nodes initialized with 0
- Marking nodes with: $h_j < h_{j,\min} \Rightarrow f_j = 1$
- Manipulaltions: $\forall f_j = 1 : S_{ij} \rightarrow S'_{ij} = \delta_{ij},\, a_i = h_{j,\min}$
- Solving modified system: $S'_{ij}\, h_j = a'_i$
- Residual of unaltered system: $R_i = S_{ij} h_j - a_i$
- Flag turned back to 0: $h_j > h_{j,\min} \wedge R_i < 0$

Consistent, fast and robust algorithm

Contact Problem



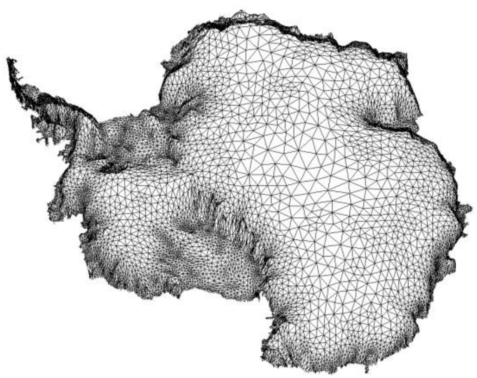
FEM Formulation

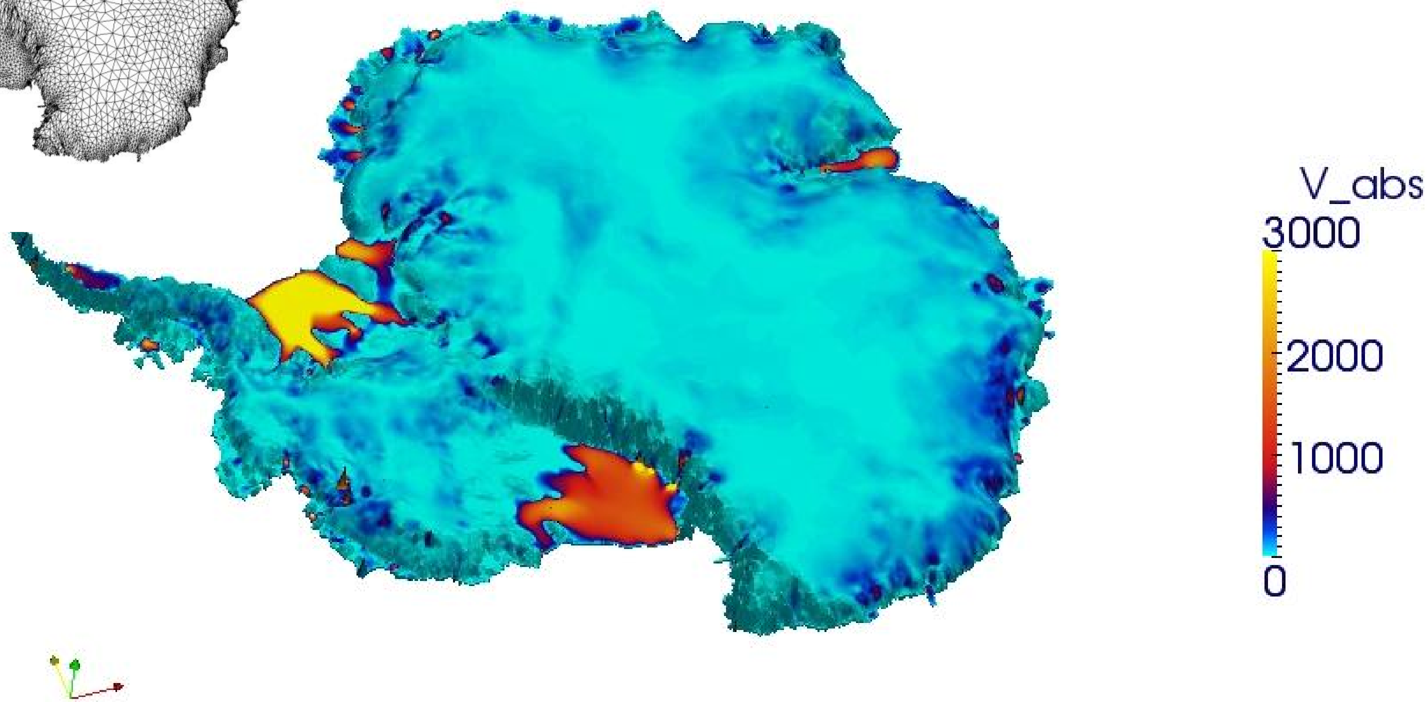
- Different types of elements:
  - Standard Lagrangian
  - Second order (edge + face centered)
  - P-elements (highest order: 9)
  - Mixed type elements (Taylor-Hood, Mini-element)
  - Discontinuous Galerkin
- BEM solver (e.g., Accoustics)
- Lagrangian particle tracker (alternative to DG)

Linear Algebra

- Internal CRS (compact row storage) scheme
- Different solution methods in Elmer:
  1. Direct solution: Unsymmetrical Multi-Frontal method UMFPack (serial) and MUMPS (parallel)
  2. Iterative = Krylov subspace: GMRES, CG, BiCGStab, GCR
  3. Multi-Grid: AMG (built-in), Boomer AMG
- Pre-conditioner:
  - Diagonal, ILU(N/T)
  - Hypre: Parasails, BoomerAMG
  - Block pre-conditioner

Block pre-conditioning



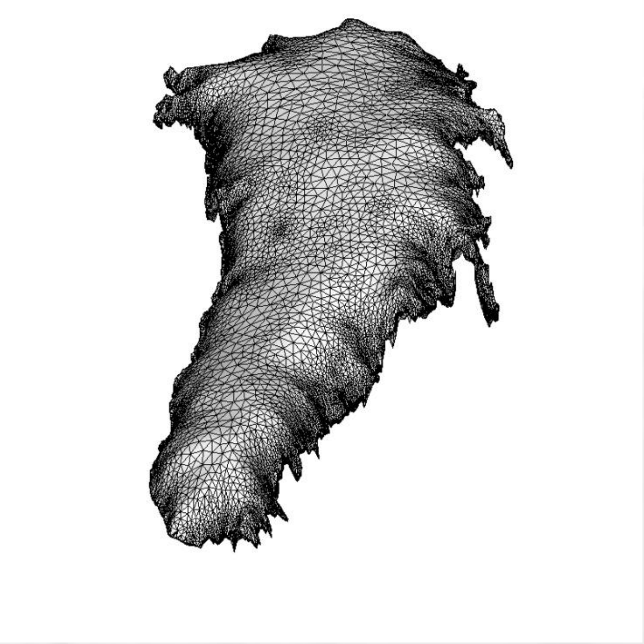


Linear Algebra

- Not re-inventing the wheel → interfacing with comming linear algebra packages:
  - LAPack
  - UMFPACK
  - MUMPS (ScaLAPack)
  - Hypre (pre-conditioners and AMG)
  - Pardiso (multi-threaded library)
  - SparsIter (Cholmod)
  - Trilinos (J. Thies)
  - To be continued

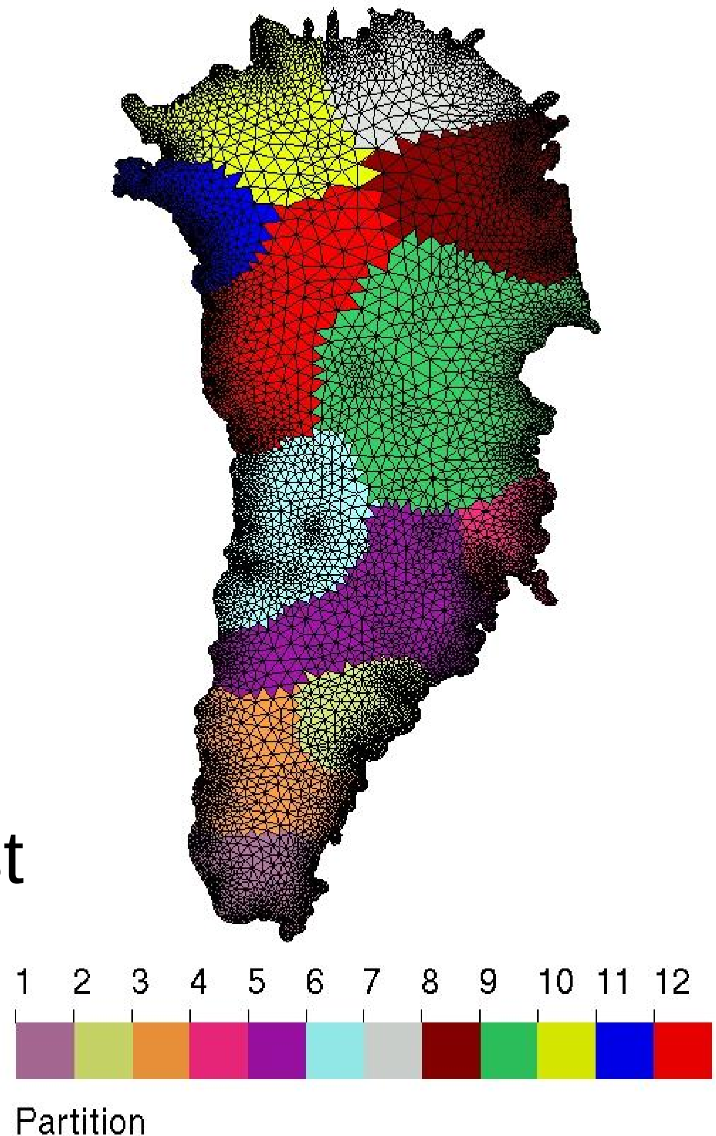
Parallel Concept of Elmer

- Modern HPC CPU’s already have 6+ cores
  - – Core frequency stagnating
  - – Need to go parallel for improvements
- Full Stokes is expensive
  - – 4 DOFs (3 x momentum, pressure)
- Greenland Ice Sheet:
  - – $1.7x10^6$ km^2
  - – 500 m resolution -> $6x10^6$ horizontal cells

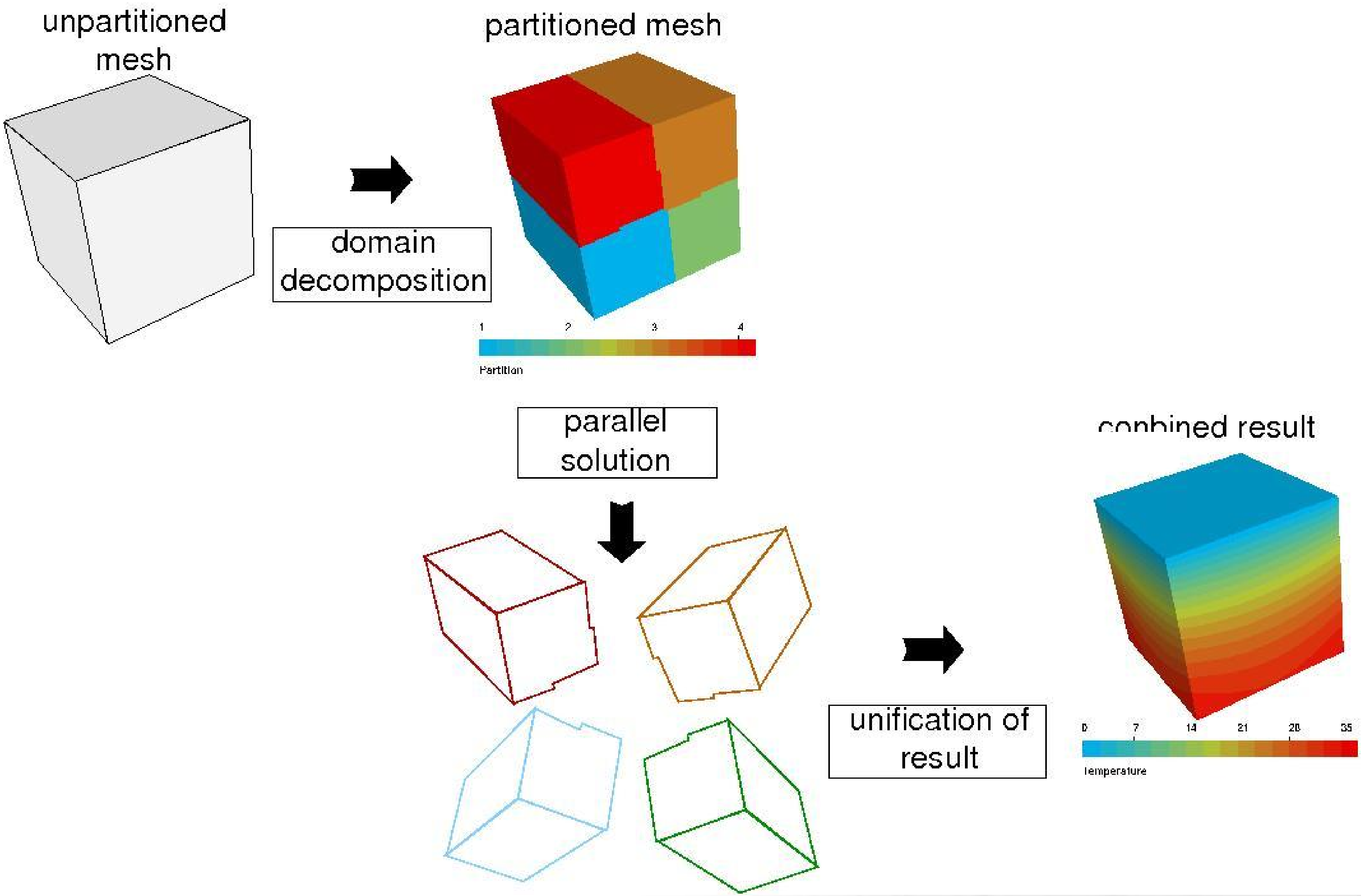


Parallel Concept of Elmer

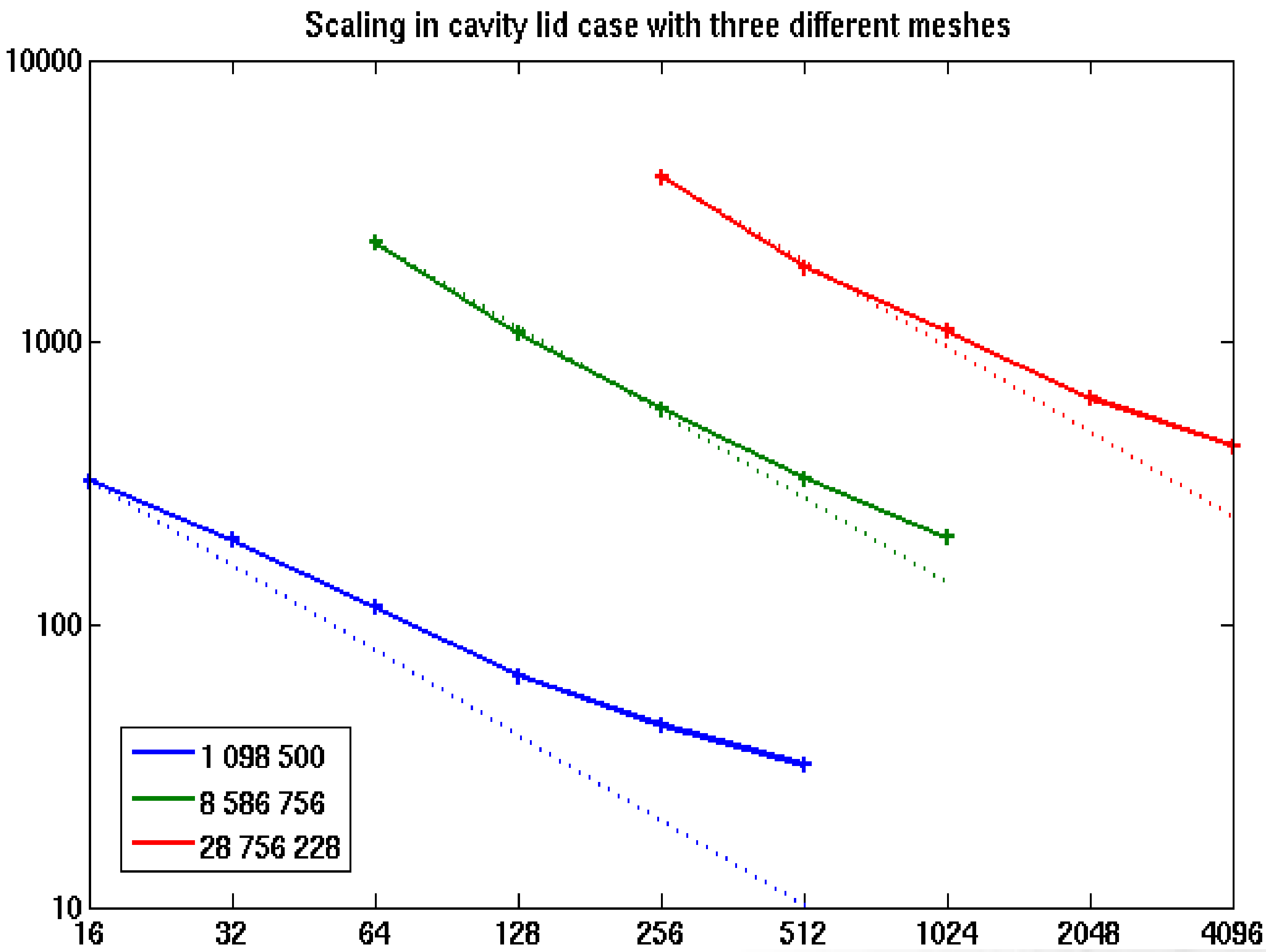
- Domain decomposition
- Additional pre-processing step (splitting)
- Every domain is running its ”own” ElmerSolver
- Parallel process communication: Message Passing Interface (MPI)
- Re-combination of ElmerPost output

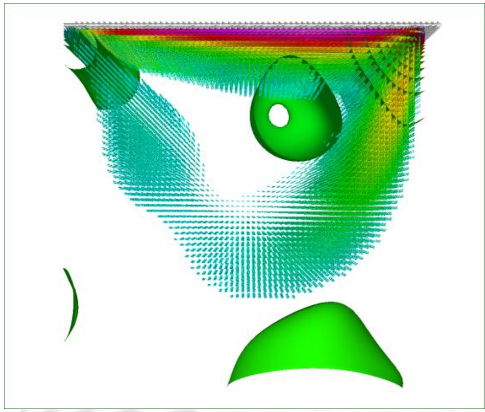


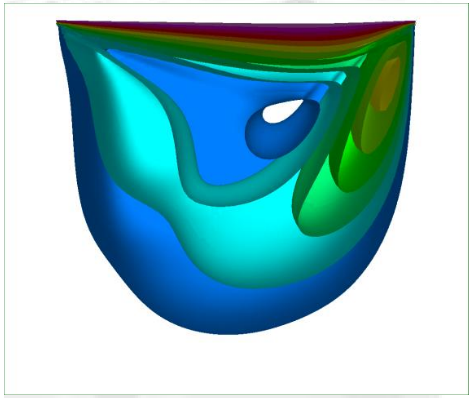
Parallel Concept of Elmer



Parallel Concept of Elmer



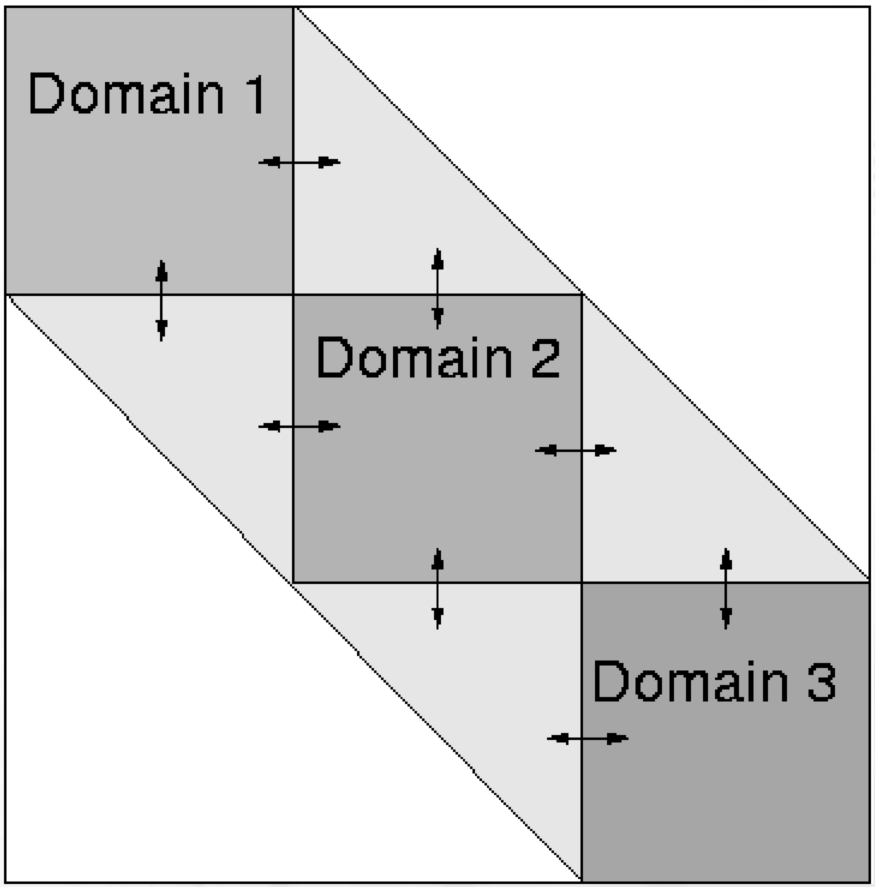




Scaling of wall clock time with dofs in the cavity lid case using GMRES+ILU0. Simulation Juha Ruokolainen, CSC, visualization Matti Gröhn, CSC .

Parallel Concept of Elmer

- Altered numerics in parallel
- ”Missing” parts of global system matrix, e.g., in ILU
- Communication expensive
- Direct parallel solver (MUMPS)
  - Memory hog
  - Does not scale at all



Thank you!