Elmer/Ice advanced Workshop
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Marine ice-sheets and the Grounding line problem

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Grounding Line

✓ The Physics
  - Dynamics of ice-sheets
  - The transition zone
  - Results from grounding line models

✓ Equations to be solved
  - The Schoof equation
  - Solution of a contact problem

✓ Implementation in Elmer/Ice
  - The basal boundary
  - How to evaluate the contact?
  - Mesh size issue
  - Interpolation of the friction

✓ Example
  - MISMIP test
Importance of ice-stream

Mass balance velocity

(Reprinted from Testut, 2000)
Better understanding of the processes controlling ice-streams dynamics:
- grounding line dynamics
- stress transmission across grounding line
Ice Discharge

What will be the future contribution of Ice Discharge for the next centuries?

Need accurate description of the Grounding Line dynamics

[Rignot et al., 2011]
EISMINT Results

- **No consensus** on the results
- **No consensus** on how the GL should be modelled
- It is **unclear** whether these results are indicative of neutral equilibrium

[Huybrechts, 1998]

Influence of the horizontal grid size

[Vieli and Payne, 2005]

Poor ability of the model to capture the GL dynamic until recently
Dynamical processes related to ice flow not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea level rise. Understanding of these processes is limited and there is no consensus on their magnitude. Current global model studies project that the Antarctic Ice Sheet will remain too cold for widespread surface melting and is expected to gain in mass due to increased snow fall. However, net loss of ice mass could occur if dynamical ice discharge dominates the ice sheet mass balance.

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Example
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Notation / Concept

Marine Ice-sheet
Ice stream

How the grounding line evolves for different scenarii?
Schoof's solution (2007) – MISI in 2D

\[ a x_g = q_B(x_g) = \left( \frac{\bar{A}(\rho_i g)^{n+1}(1 - \rho_i/\rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}} \]

- Confirms that there is no stable position of the GL on an upsloping bed
- For a given surface mass balance, gives the steady GL position (in 2D)
MISI in 3D

Marine ice sheets are not unconditionally unstable in two horizontal dimensions

[Gudmundsson et al., 2012]
Equations to be solved

Ice Flow
\[
\begin{aligned}
D &= A \tau_{e}^{n-1} S \\
\text{div } \sigma + \rho g &= 0 \\
\text{div } u &= 0
\end{aligned}
\]

Top free surface
\[
\frac{\partial z_s}{\partial t} + u_x \frac{\partial z_s}{\partial x} + u_y \frac{\partial z_s}{\partial y} - u_z = a_s
\]

Bottom free surface
\[
\frac{\partial z_b}{\partial t} + u_x \frac{\partial z_b}{\partial x} + u_y \frac{\partial z_b}{\partial y} - u_z = a_b
\]
with \( z_b \geq b \)

Ice - Bed contact
\[
z_b = b \text{ and } -\sigma_{nn} > p_w \quad \Rightarrow \quad u \cdot n = 0 \\
\begin{aligned}
\text{or } z_b &> b \\
\end{aligned}
\]

Ice - Sea
\[
z_b = b \text{ and } -\sigma_{nn} \leq p_w \quad \Rightarrow \quad \text{Buoyancy condition}
\]

\[ u_t = f_t(\sigma_{nt}) \]
Buoyancy BC

BC Stokes: if $z_b(x, t) > b(x)$

$$
\sigma_{nn}(x) = \rho_w g (l(t) - z_b(x, t)) \quad \text{and} \quad \sigma_{nt}(x) = 0
$$

$$
z_b(x, t) = z_b(x, t - dt) + u_n \sqrt{1 + (dz_b/\,dx)^2} \, dt
$$

$$
\sigma_{nn}(x) = \rho_w g (l(t) - z_b(x, t - dt)) - \rho_w g \sqrt{1 + (dz_b/\,dx)^2} \, dt \cdot u_n
$$
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The basal boundary

Condition applied on the basal boundary depend if
- the ice is in contact with the bedrock
- or the ice is in contact with the sea

The limit between grounded and floating parts (the GL) is unknown and solution of the contact problem

Add a $\text{Mask}$ variable (only on the basal surface) which tells if grounded, floating or at the GL

$\text{Mask} = 1$ if grounded
$\text{Mask} = -1$ if floating
$\text{Mask} = 0$ if at the GL
The basal boundary

In Elmer, the use of a **conditional Dirichlet** condition allows to deal with this evolving limit.

Example in the SIF:

```
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable Mask
   Real MATC "tx + 0.5"
```

- Mask = -1  →  the Dirichlet BC is not applied
- Mask = 1 or 0  →  the Dirichlet BC $u_n = 0$ is applied
The contact problem

\[ z_b = b \quad \text{and} \quad -\sigma_{nn} > p_w \quad \Rightarrow \quad \text{Ice - Bed condition} \]

\[ z_b = b \quad \text{and} \quad -\sigma_{nn} \leq p_w \]

or \[ z_b > b \]

\[ \Rightarrow \quad \text{Ice - Sea condition} \]

\[ \text{Stokes Solver} \]

Evaluate the condition \(-\sigma_{nn} > p_w\)

after the non-linear system has started converged

\[ \text{Upper Free Surface} \]

\[ \text{Bed Free Surface} \]

\[ \text{Grounded Mask Solver} \]

Evaluate the condition \(z_b > b\)
The contact problem

The condition $-\sigma_{nn} > p_w$ is in fact evaluated using nodal force (and not stress)

- the force exerted by the ice on the bed is given by the residual of the Stoles solution

In the Stokes solver

  Exported Variable 1 = Flow Solution Loads[Stress Vector:2 CEQ Residual:1]
  Calculate Loads = Logical True

- the nodal water force is the integrated water pressure with respect to the surface element

  add a new solver to integrate the water pressure
SUBROUTINE GetHydrostaticLoads( Model, Solver, dt, TransientSimulation )

VariableValues = 0.0_dp

DO t = 1, Solver % NumberOfActiveElements
   Element => GetActiveElement(t)
   IF ( ParEnv % myPe .NE. Element % partIndex ) CYCLE
   n = GetElementNOFNodes()

   BC => GetBC( Element )
   pwt(1:n) = -1.0 * ListGetReal(BC, 'External Pressure', n, &
                              Element % NodeIndexes, GotIt)

   CALL GetElementNodes( Nodes )
   IP = GaussPoints( Element )
   DO p = 1, IP % n

      stat = ElementInfo( Element, Nodes, IP % U(p), IP % V(p), &
                          IP % W(p), detJ, Basis, dBasisdx, ddBasisddx, .FALSE.)
      s = detJ * IP % S(p)

      Normal = NormalVector( Element, Nodes, IP % U(p), IP % V(p), .TRUE.)
      pwi = SUM(pwt(1:n)*Basis(1:n))
      PwVector(1:DIM) = pwi * Normal(1:DIM)

      DO i = 1, n
         Nn = Permutation(Element % NodeIndexes(i))
         DO j = 1, DIM
            VariableValues(DIM*(Nn-1)+j) = VariableValues(DIM*(Nn-1)+j) + PwVector(j) *
            s * Basis(i)
         END DO
      END DO
   END DO

   IF ( ParEnv % PEs>1 ) CALL ParallelSumVector( Solver % Matrix, VariableValues )
!------------------------------------------------------------------------------
END SUBROUTINE GetHydrostaticLoads
!------------------------------------------------------------------------------
The bed boundary condition

Boundary Condition 1
Target Boundaries = 1
Body Id = 3

Normal-Tangential Velocity = Logical True
Flow Force BC = Logical True
!
! Bedrock conditions
!
Slip Coefficient 2 = Variable Coordinate 1
Real Procedure "ElmerIceUSF" "SlidCoef_Contact"
Sliding Law = String "Weertman"
Weertman Friction Coefficient = Real $C
Weertman Exponent = Real $(1.0/n)
Weertman Linear Velocity = Real 1.0

Grounding line Definition = String “Discontinuous”
!
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable GroundedMask
Real MATC "tx + 0.5"
!
! Shelf conditions
!
External Pressure = Variable Coordinate 2
Real Procedure "ElmerIceUSF" "SeaPressure"

Slip Coefficient 1 = Variable Coordinate 2
Real Procedure "ElmerIceUSF" "SeaSpring"

The variable GroundedMask is updated in this User Function SlidCoef_Contact

Will only apply if the Dirichlet condition Velocity 1 = 0 is not applied

See note after
The user function SlidCoef.Contact

Test the contact condition:

\[
\text{Normal} = \text{NormalValues}(\text{DIM} \times (\text{NormalPerm}(jj)-1)+1 : \text{DIM} \times \text{NormalPerm}(jj))
\]
\[
\text{Fwater} = \text{Hydro}(\text{DIM} \times (\text{HydroPerm}(jj)-1)+1 : \text{DIM} \times \text{HydroPerm}(jj))
\]
\[
\text{Fbase} = \text{ResidValues}((\text{DIM}+1) \times (\text{ResidPerm}(jj)-1)+1 : (\text{DIM}+1) \times \text{ResidPerm}(jj)-1)
\]

\[
\text{comp} = \text{ABS}(\text{SUM}(\text{Fwater} \times \text{Normal})) - \text{ABS}(\text{SUM}(\text{Fbase} \times \text{Normal}))
\]

IF (comp \geq 0.0_{dp}) GroundedMask(Nn) = -1.0_{dp}

and return the sliding coefficient:
- appropriate if grounded
- 0 if floating

\[
\text{cond} = \text{GroundedMask}(\text{GroundedMaskPerm(nodenumber)})
\]

IF (cond > -0.5_{dp}) THEN
\[
\text{Bdrag} = \text{Sliding\_weertman(Model, nodenumber, y)}
\]
ELSE
\[
\text{Bdrag} = 0.0_{dp}
\]
END IF
Sensitivity to the grid size

Steady for different horizontal grid sizes

Problem: CPU cost!
Interpolation of the friction

Use Discontinuous!
Interpolation of the friction

Ny = 20

Ny = 40

Ny = 80
PIG example (Favier et al., 2014)
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Example GL_MISMIP

MISMIP Experiment 1a
Step 4
Start from Schoof geometry


[ELMER_TRUNK]/elmerice/Tests/GL_MISMIP
References


