Free surface stabilisation in Elmer/Ice

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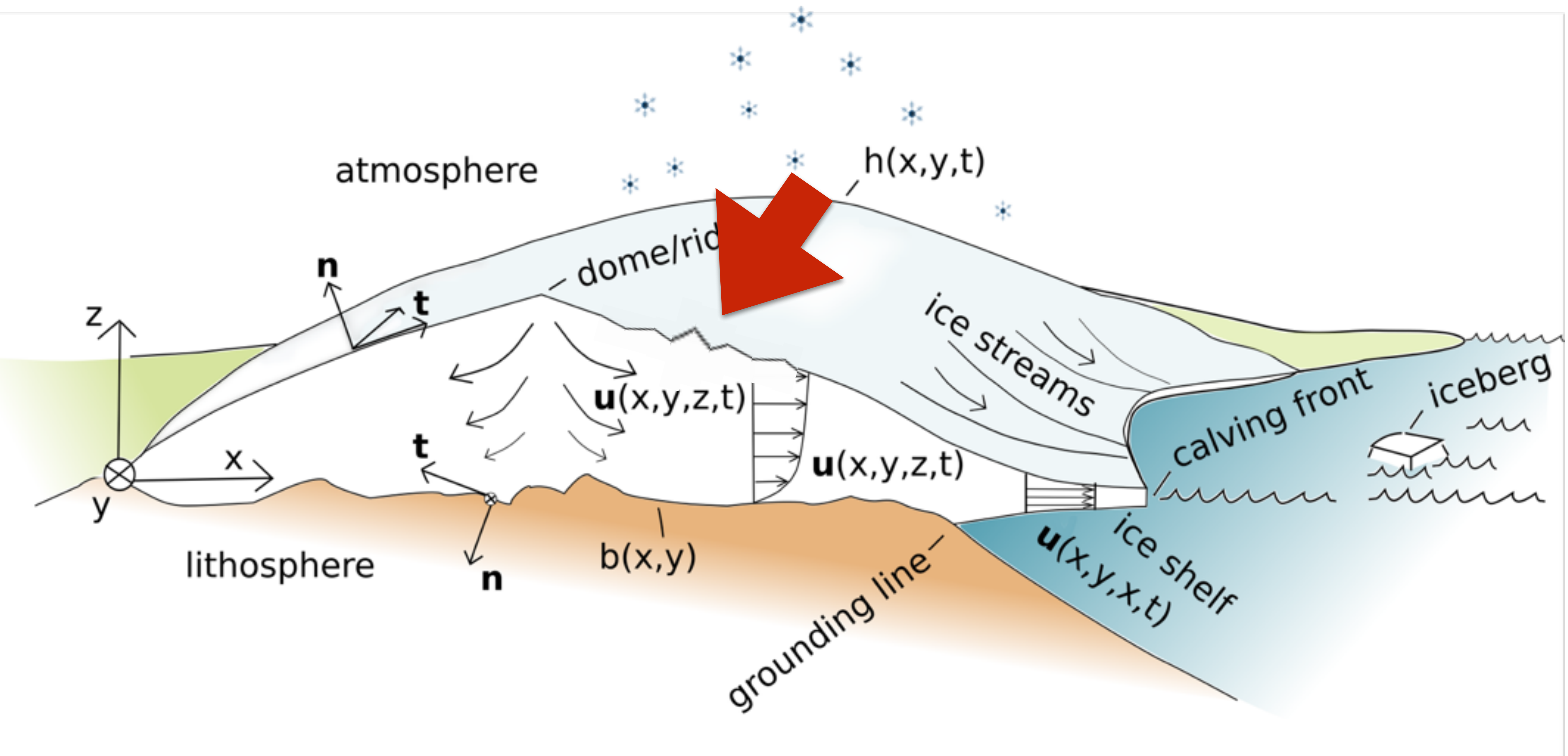
EGU General Assembly 2023



atmosphere
h(x,y,t)
dome/rid
n
t
z
x
y
u(x,y,z,t)
u(x,y,z,t)
t
n
ice streams
calving front
iceberg
lithosphere
b(x,y)
grounding line
ice shelf
u(x,y,x,t)

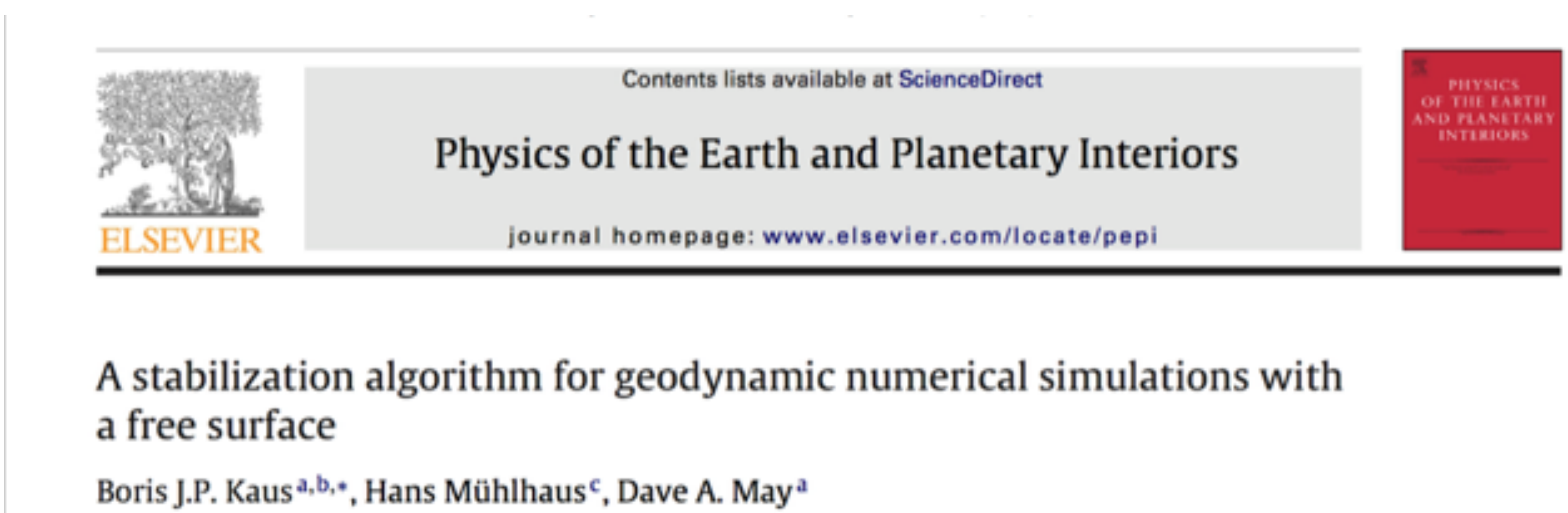
Mimicking a fully implicit time-stepping scheme

Weak form of Stokes equations

$$(\mathbf{S}(\mathbf{Du}), \mathbf{Dv})_{\Omega(t)} - (p, \nabla \cdot \mathbf{v})_{\Omega(t)} + \textbf{boundary terms} = (\rho \mathbf{g}, \mathbf{v})_{\Omega(t)}$$

$$\int_{\Omega(t)} \rho \mathbf{g} \cdot \mathbf{v} dV$$

$$\int_{\Omega(t+\Delta t)} \rho \mathbf{g} \cdot \mathbf{v} dV \approx \int_{\Omega(t)} \rho \mathbf{g} \cdot \mathbf{v} dV + \theta \Delta t \int_{\partial\Omega(t)} (\mathbf{u} \cdot \mathbf{n} + a_s)(\rho \mathbf{g} \cdot \mathbf{v}) dV$$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ELSEVIER

A stabilization algorithm for geodynamic numerical simulations with a free surface

Boris J.P. Kaus[a,b,*], Hans Mühlhaus[c], Dave A. May[a]

Adaptation and testing for simple ice sheet simulations


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Increasing stable time-step sizes of the free-surface problem arising in ice-sheet simulations

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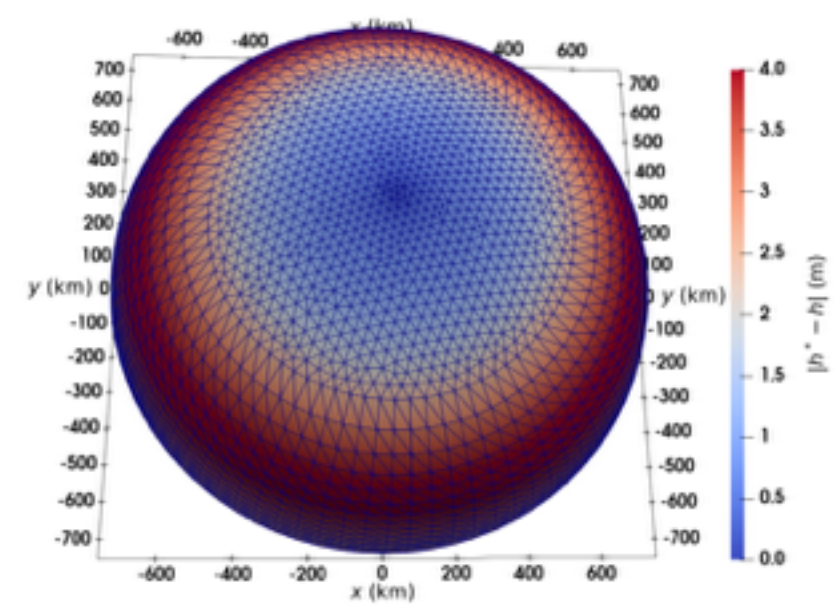
$$\int_{\Omega(t+\Delta t)} \rho \mathbf{g} \cdot \mathbf{v} dV \approx \int_{\Omega(t)} \rho \mathbf{g} \cdot \mathbf{v} dV + \Delta t \int_{\partial\Omega(t)} (\mathbf{u} \cdot \mathbf{n} + a_s)(\rho \mathbf{g} \cdot \mathbf{v}) dV$$

15-30 times bigger time steps



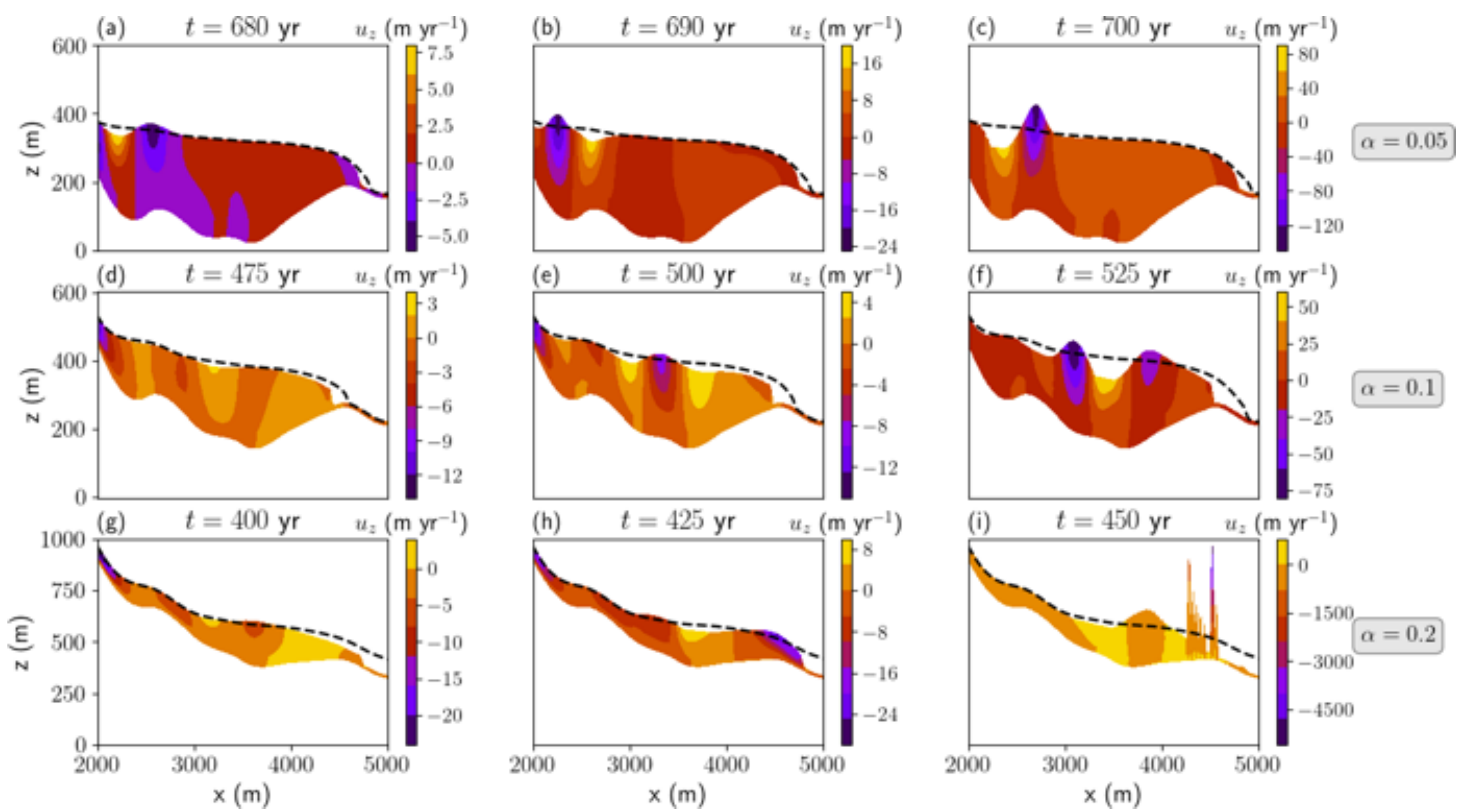
Implementation in Elmer/Ice

implemented in/fem/src/modules/IncompressibleNSVec.f90

```
DO p=1,nd
  DO q=1,nd
    DO i=1,dim
      DO j=1,dim
        STIFF( (p-1)*c+j,(q-1)*c+i ) = &
          STIFF( (p-1)*c+j,(q-1)*c+i )  &
           - s * FSSAtheta * dt * LoadVec(j) * Basis(q) * Basis(p) * Normal(i)
      END DO
    END DO
  END DO
END DO
```

Validation on synthetic case

vs.

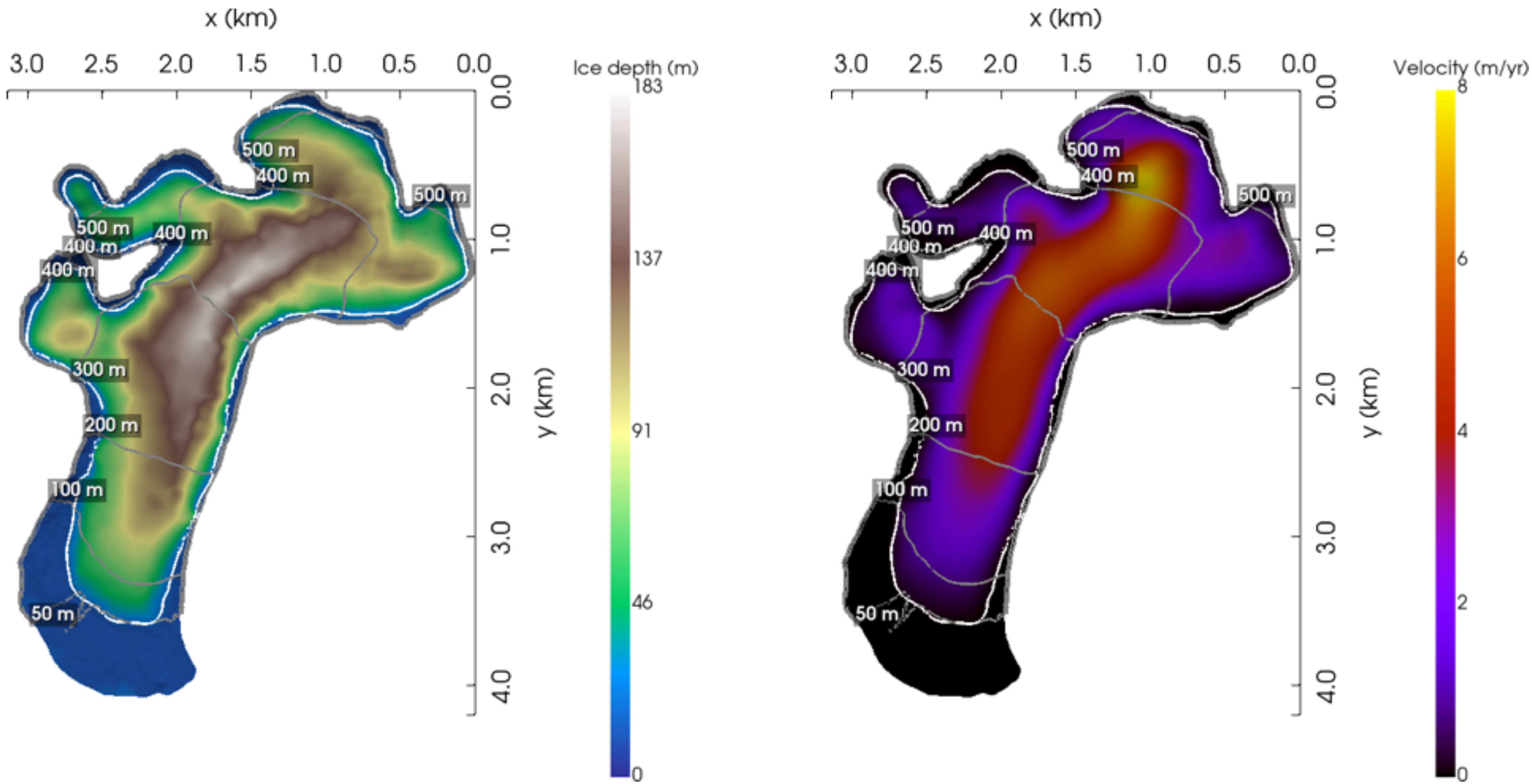


What to specify in sif-file

```
Boundary Condition 3
  Name = "surface"
  Top Surface = Equals "Zs"
  Target Boundaries = 2
  Body ID = 2
  FSSA Theta = Real 1.0
  FSSA Flag = String “full”  !normal
  Zs Lower Limit = Equals RefZs
  FSSA Accumulation = Variable Coordinate 1
  Real Procedure "accum" "accum"
End
```

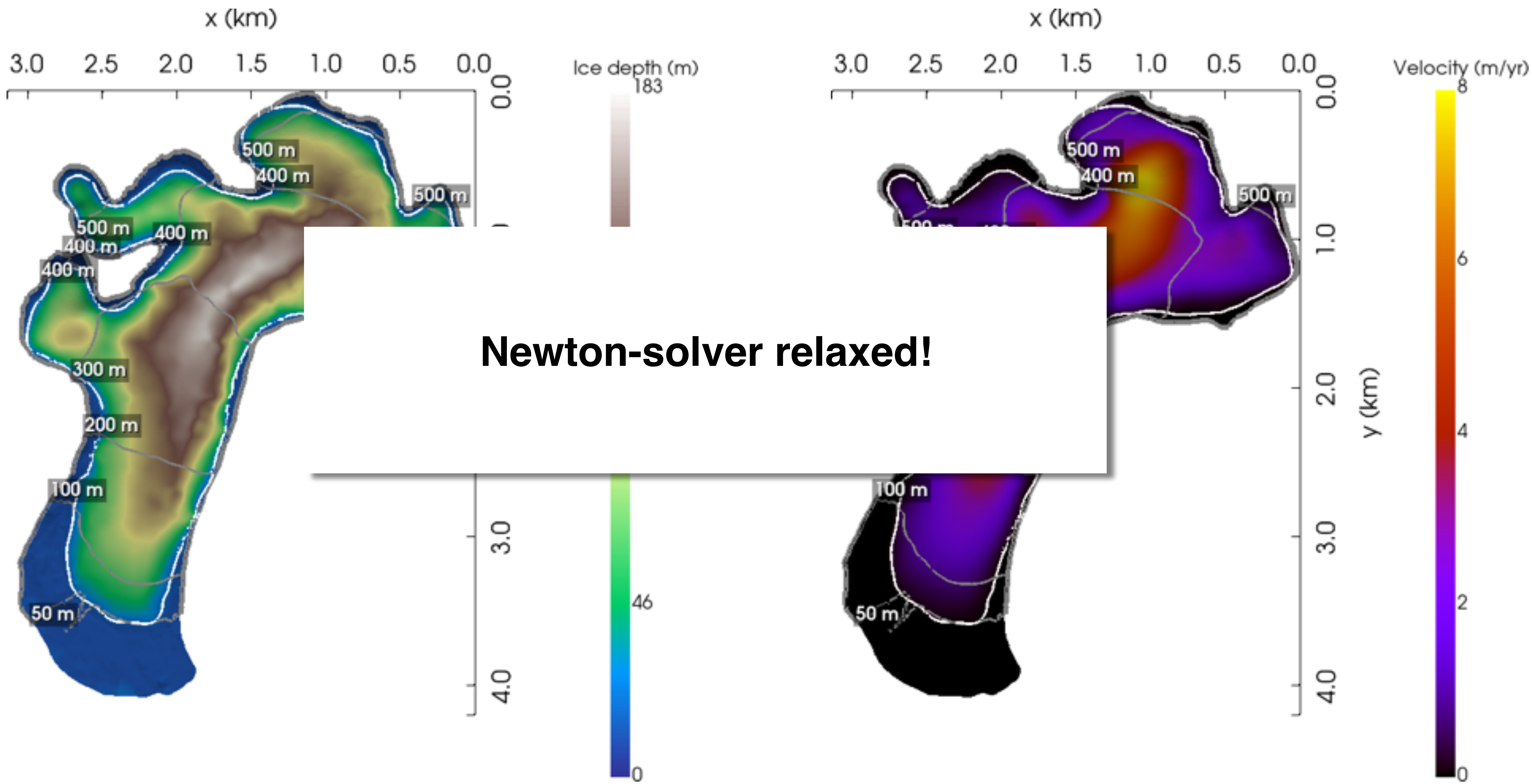
branch: fssa-merge

Midtre Lovénbreen



Setup (e.g. SMB) follows form Välisuo et al (2007), DEM:s from Jack Kohler

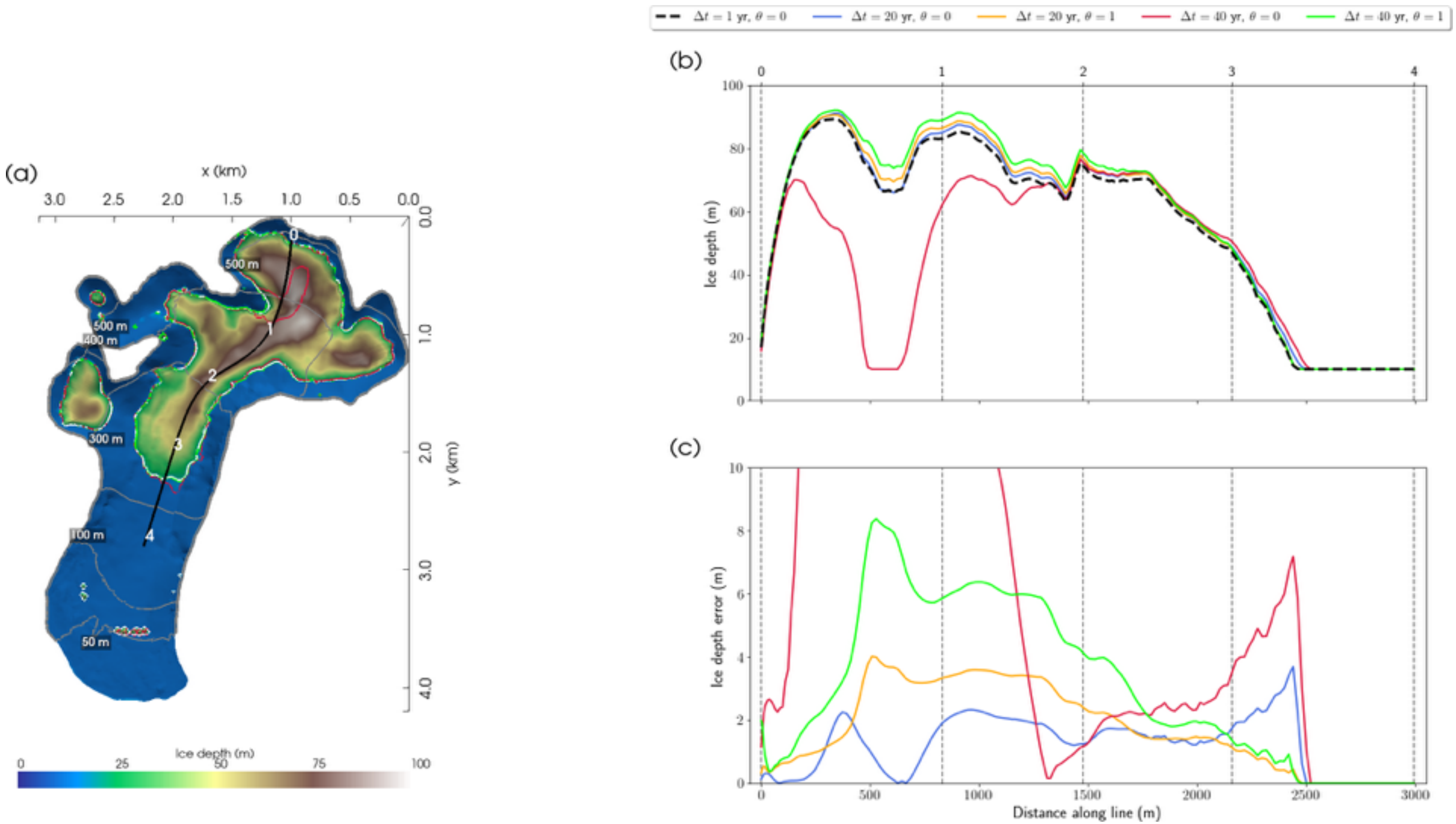
Midtre Lovénbreen



Newton-solver relaxed!

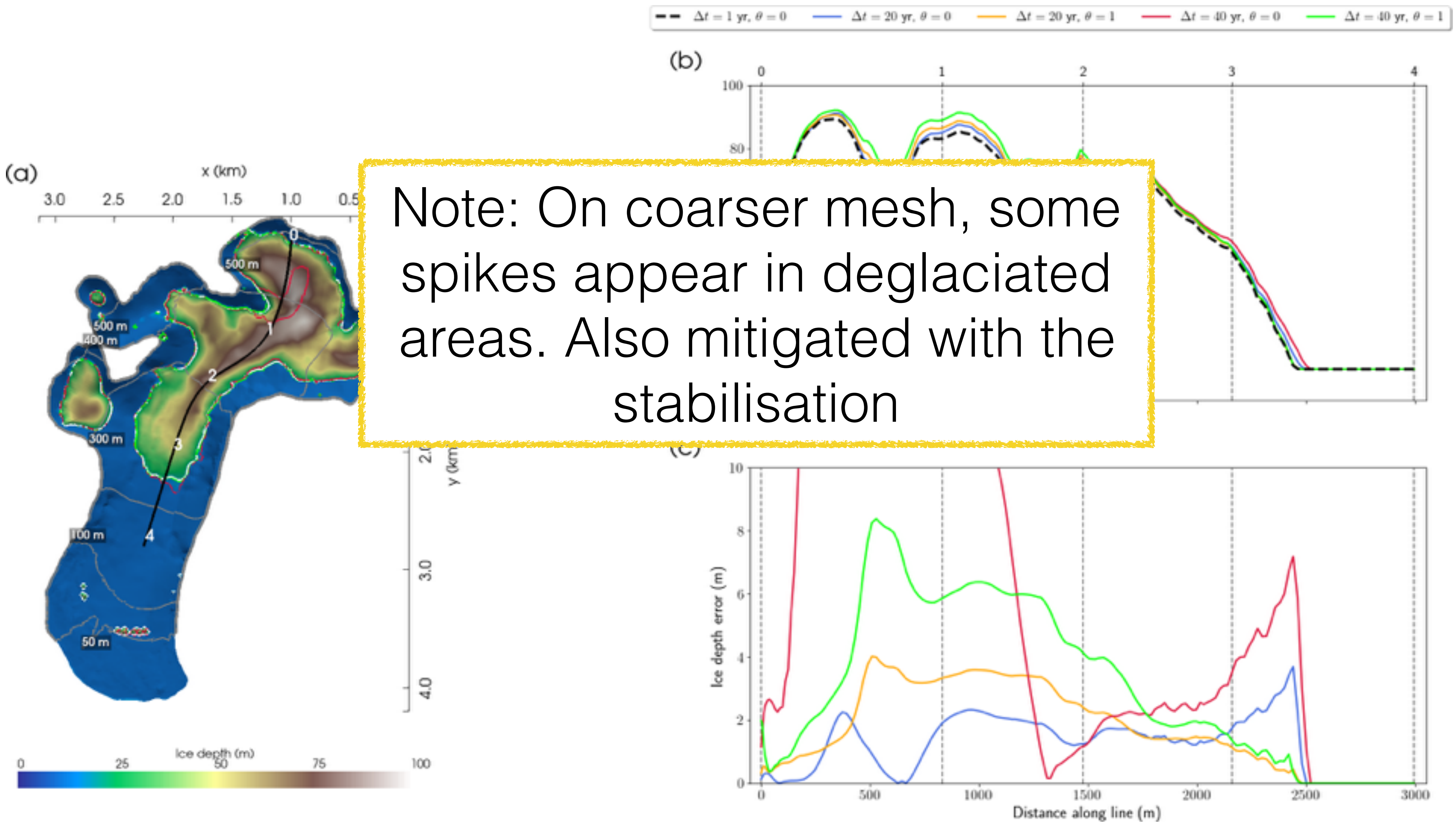
Setup (e.g. SMB) follows form Välisuo et al (2007), DEM:s from Jack Kohler

After 200 years



A. Löfgren, T. Zwinger, Peter Råback, C. Helanow, J.Ahlkrona, Increasing Numerical Stability of mountain Valley Glacier Simulations - Implementation and Testing of Free-Surface Stabilisation in Elmer/Ice (manuscript in preparation)

After 200 years



Note: On coarser mesh, some spikes appear in deglaciated areas. Also mitigated with the stabilisation

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Other Elmer-related work

How does the **eigenvalues of a ParStokes-like preconditioner** depend on the critical shear rate and Glen flow law parameter n?

(i) *For* $\tilde{S} = M$: ← *Not viscosity-scaled*

$$c_0^2 \varepsilon^{2-\mathfrak{p}} \leq \lambda \leq \frac{(\varepsilon^2 + \|\mathbf{D}\mathbf{u}_h^k\|_\infty^2)^{\frac{2-\mathfrak{p}}{2}}}{\nu_0(1+\gamma(\mathfrak{p}-2))}$$

(ii) *For* $\tilde{S} = M_\nu$: ← *viscosity-scaled*

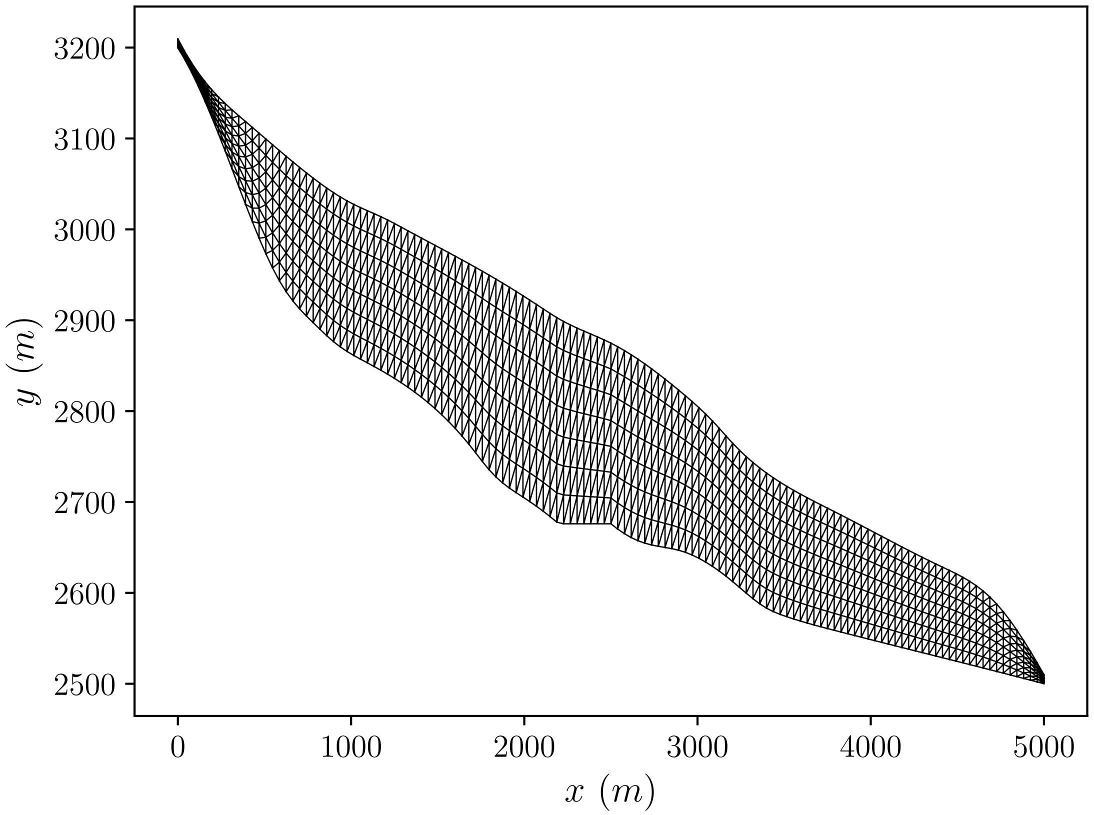
$$c_\nu^2 \leq \lambda \leq \frac{d}{1+\gamma(\mathfrak{p}-2)}$$

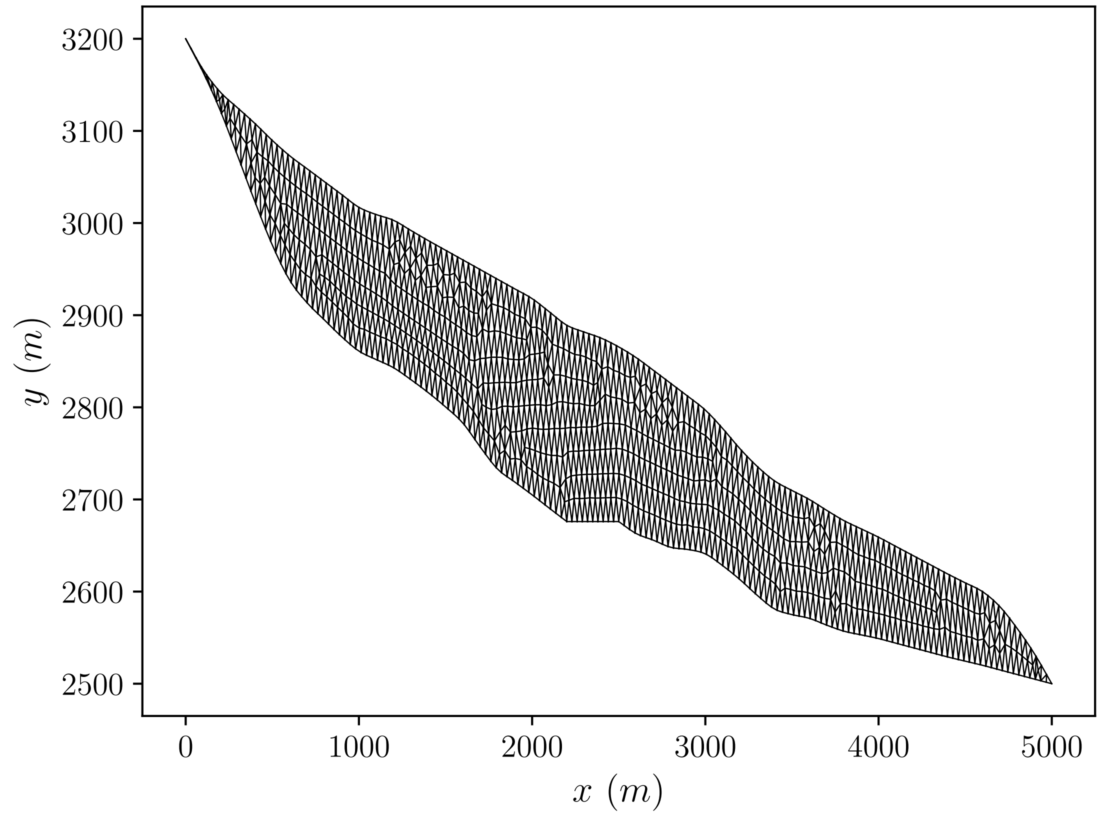
$\gamma = 1$ Newton

$\gamma = 0$ Picard

$$\mathfrak{p} = \frac{n+1}{n}$$

C. Helanow, J. Ahlkrona, *Preconditioning of singular power-law fluids describing shear-thinning flow: application to ice-sheet modeling (manuscript in preparation)*





(ii) *For* $\tilde{S} = M_\nu$*:*

$$c_\nu^2 \leq \lambda \leq \frac{d}{1+\gamma(\mathfrak{p}-2)}$$

$\gamma = 1$ Newton

$\gamma = 0$ Picard

$$\mathfrak{p} = \frac{n+1}{n}$$

Practical conclusion: For MINI-elements: Low minimum eigenvalues if low quality meshes. P2P1 not so sensitive to mesh quality.

C. Helanow, J. Ahlkrona, *Preconditioning of singular power-law fluids describing shear-thinning flow: application to ice-sheet modeling (manuscript in preparation)*

Elmer/Ice related outlook

- Try stabilisation on an ice sheet?
- Increased stability - worth awakening ISCAL (Igor Tominec, Clara Henry)