ElmerIce Workshop

Study of a moving front using a level set method

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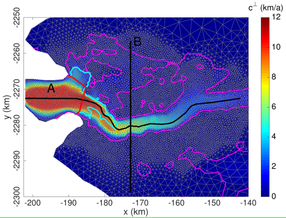
Objective

To study numerically, using ElmerIce moving calving front:

- 2D plan view (SSA Approx.).
- Using a level-set function.



Level set function

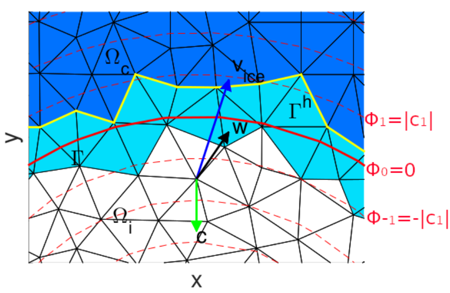


Figure: Bondzio et al., (2016)

We're interested in the front displacement, we define a level set function ϕ as follows

- It is zero at the border.
- It defines the distance to the front.
- It is signed as:

$$\phi \begin{cases} < 0, & \text{if } \vec{x} \in \Omega_i \\ = 0, & \text{if } \vec{x} \in \delta\Omega_i \\ > 0, & \text{otherwise} \end{cases}$$

- It allows us to define a mask for active and passive

Evolution equation of the level set function

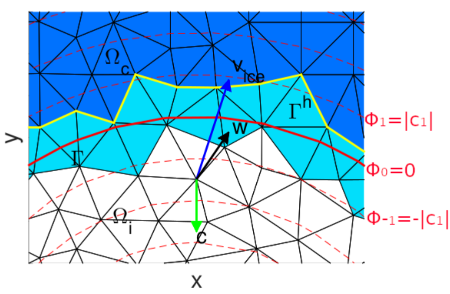


Figure: Bondzio et al., (2016)

In order to compute the temporal evolution of the front we compute the material derivative of the distance ϕ and neglecting the flow in the front, we obtain:

$$\frac{\partial \phi}{\partial t} + \left(v - a^{\perp}\right) \hat{n} \cdot \nabla \phi = 0.$$

with $w = v - a^{\perp}$ the ice velocity normal to the front, $\hat{n} = \frac{\nabla \phi}{|\nabla \phi|}$, and $a^{\perp}$ the ablation rate normal to the front.

Evolution equation of the level set function

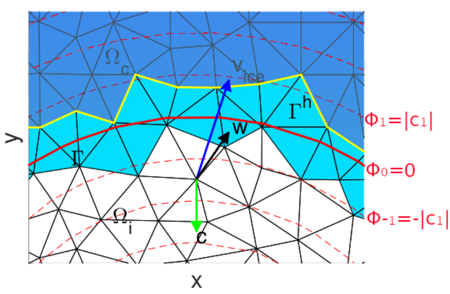


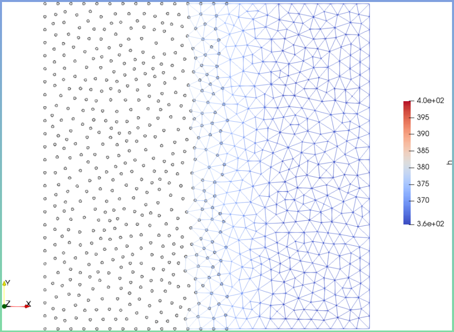
Figure: Bondzio et al., (2016)

We do not know the values of the level-set speed on the masked domain, so we export the values of the velocity at the front towards the passive domain by solving:

$$\hat{n} \cdot \nabla S = 0.$$

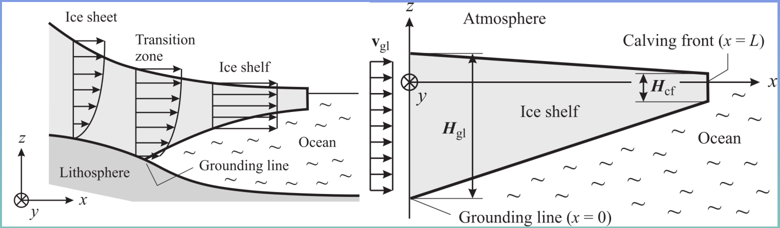
with $\hat{n} = \frac{\nabla \phi}{|\nabla \phi|}$ and S the value to export to the PASSIVE domain.

Test case: halogreedy elements



- Dirichlet boundary conditions on proc divisions require hallogreedy elements.

Test case: idealized ice shelf



- Analytical solution based on [Greve and Blatter (2009)].
- SSA Approximation.
- Free-slip lateral boundary condition.
- Constant incoming flow.
- The calving front position does not modify the steady-state configuration.

Test case: idealized ice shelf

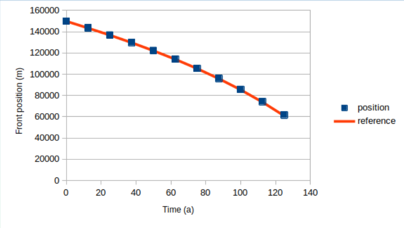
To test the level set solver solution, I solve the equations for the steady state ice-shelf. Then I compute the fronts position solving the previously shown eq.

$$\frac{\partial \phi}{\partial t} + \left(v - a^{\perp}\right) \hat{n} \cdot \nabla \phi = 0.$$

setting the ablation rate normal to the front, $a^{\perp}$ to an arbitrary constant value, and the ice velocity at the front, v, to be the normal SSA Velocity value. We made three experiments, departing from the same position, x_c:

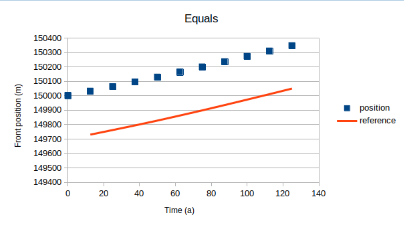
$$a^{\perp} \begin{cases} > v, & \text{then } v_{front} < 0, \text{ retreats} \\ = v, & \text{then } v_{front} = 0, \text{ does not move} \\ < v, & \text{then } v_{front} > 0, \text{ advances} \end{cases}$$

Front displacement



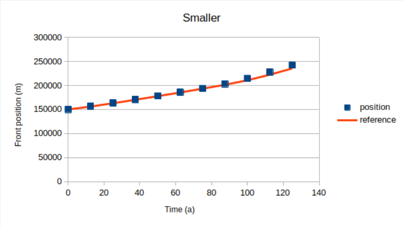
- Ablation larger than the SSA velocity, differs 1.4% with theorical value.

Front displacement



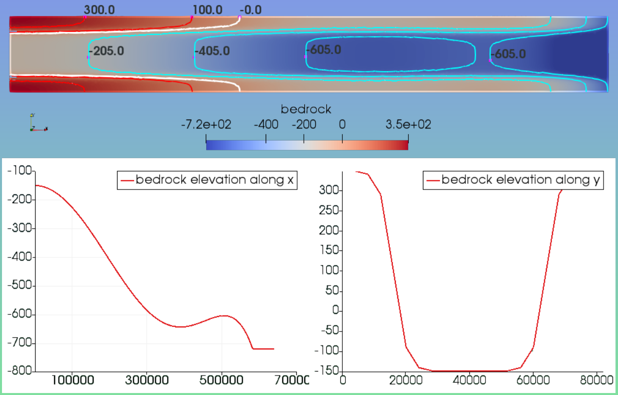
- Ablation almost equal to the SSA velocity, differs 0.2% with theorical value.

Front displacement



- Ablation smaller than the SSA velocity, differs 3% with theorical value.

Test case: Bedrock MISMIP



A more realistic test, using the idealized configuration MISMIP+. It's bedrock elevation is shown in this figure.

Ice Sheet MISMIP

To test the level set solver solution, we will only solve the level set equation. We impose the speed v_{front} as follows:

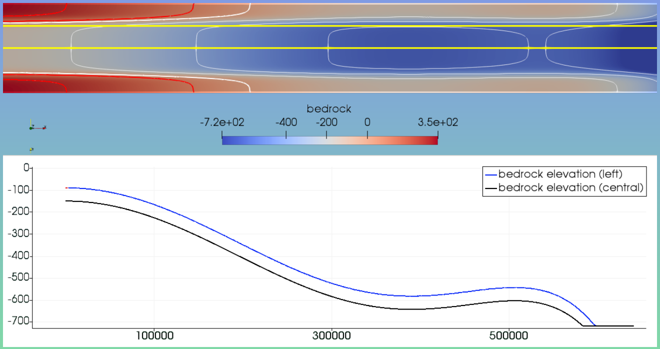
$$v_{front} = \begin{cases} cte, & \text{if } z_{bedrock} < 0, \text{ with } cte \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

where $z_{bedrock}$ is the bedrock elevation. The front will move at a velocity $\vec{v}_{front} = v_{front}\hat{n}$, where $\hat{n}$ is the unitary normal vector to the surface which points outwards to the ice.

Front displacement: ready for realistic case, test with different reinitialization frequencies

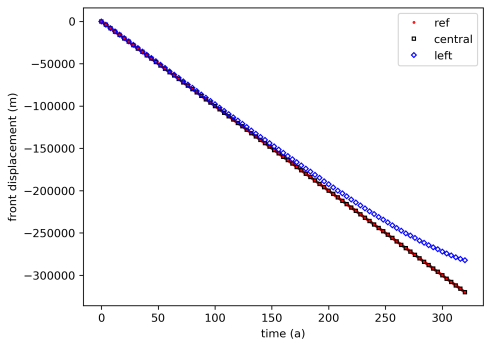
Figure: Comparison of different front speeds 1000 m/a yellow, 500 m/a red, and 250 m/a in green. All having a reinitialization of the level set for a distance of 4 km corresponding to the grid size.

Bedrock MISMIP - Following levelset evolution



We follow the evolution of the levelset front along the two yellow lines. The bedrock elevation along these lines is plotted at the bottom.

MISMIP - Levelset evolution speed

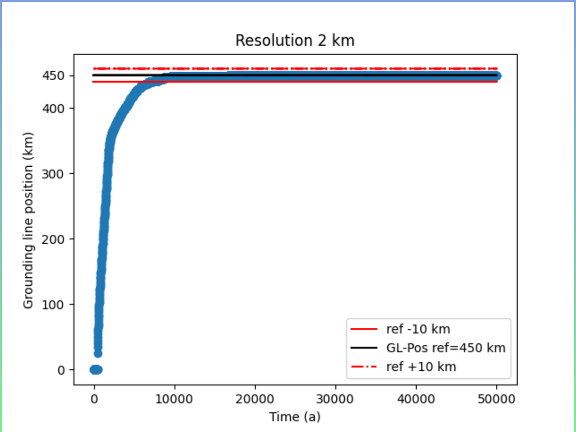


Testing the levelset in the idealized configuration MISMIP+.
$v_{front} = 1000$ m/a, reinitialization frequency 4 a, $\Delta x = 4$ km, so $F_{adv} = 1.0$ grid points.

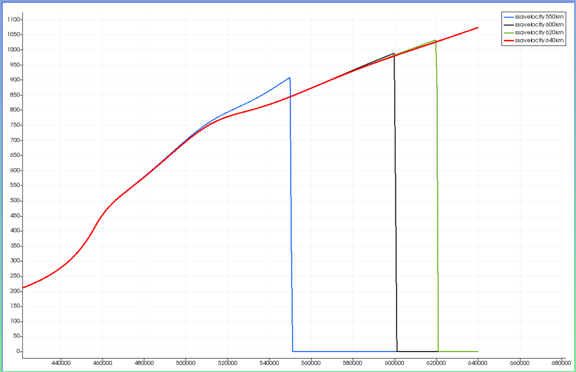
MISMIP - Grounding line position

Figure: Simulation to determine the grounding line position MISMIP+, $x_{res} = 1$ km.

Evolution of the “central” grounding line position vs time



Impact of the calving position on the SSA central velocity value.



Front displacement: realistic case, test with different reinitialization frequencies

Figure: Comparison of different reinitialization frequencies, every 5, 50, and 200 days. For 250, 500, and 1000 m/a; in yellow, gray, and red, resp.

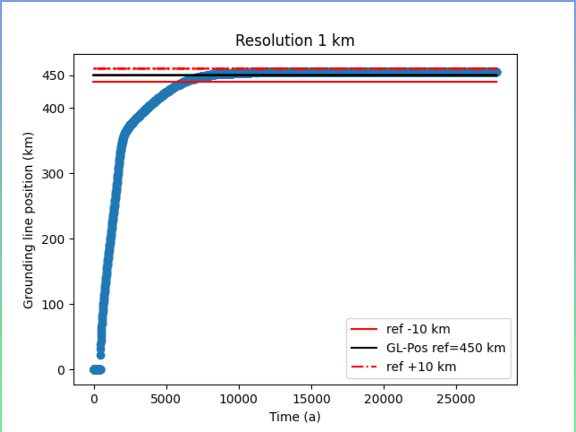
Front displacement: realistic case, test with different reinitialization frequencies

Figure: Comparison of different reinitialization frequencies, every 5, 50, and 200 days. For 250, 500, and 1000 m/a; in yellow, gray, and red, resp.

There is a good agreement when we do a reinitialization of the level set distance every 200 days.

Thank you

Evolution of the “central” grounding line position vs time



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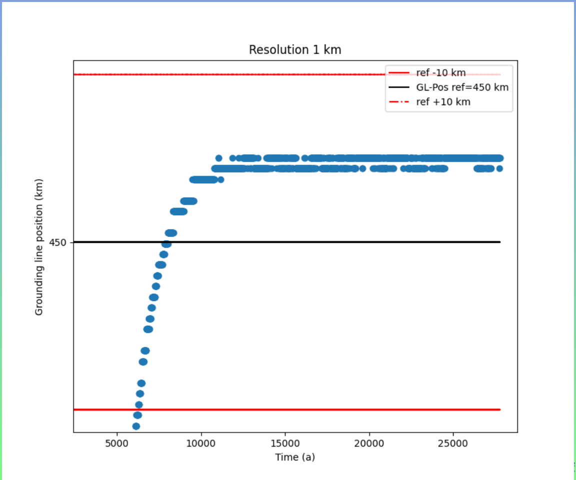

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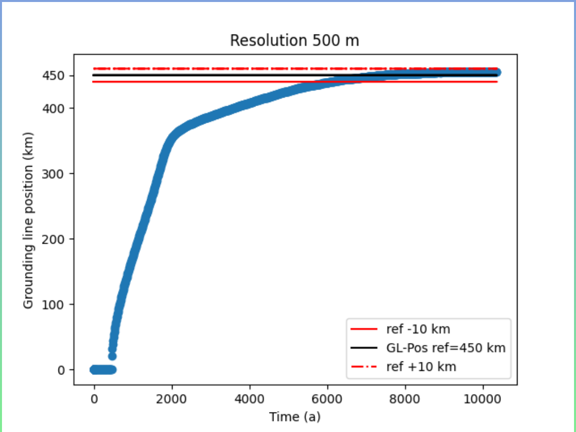

Evolution of the “central” grounding line position vs time