

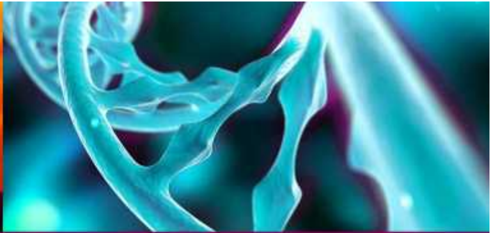

A new visco-elastic Finite Element model for GIA computations

Thomas Zwinger[1], Grace A. Nield[2,3], Juha P. Ruokolainen[1], and Matt A. King[2]

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3 Department of Geography, Durham University, Durham, UK


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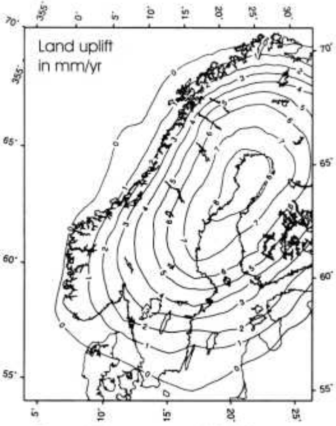


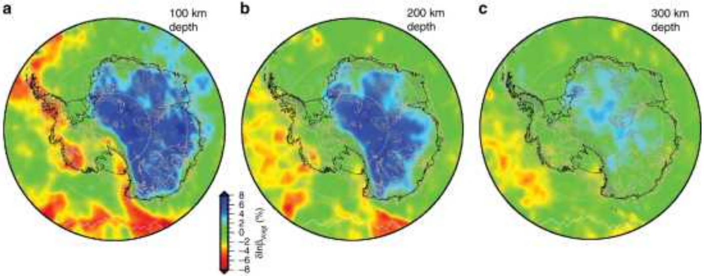
From Thoma and Wolf, 1999

Elmer

CSC

Structure of Earth

- Simple models use constant parameters
- They assume an isotropic spatial distribution of those parameters
- Reality: strong spatial (and also partly temporal) variations

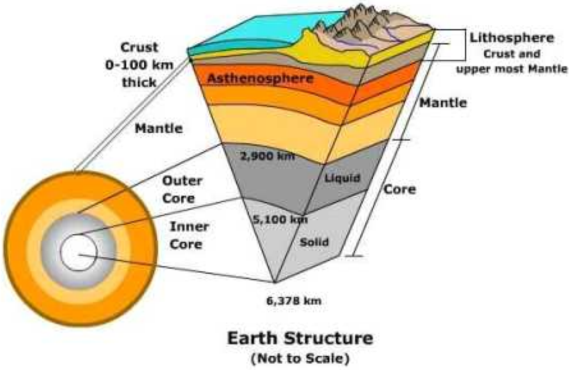


Seismic shear wave velocity perturbations Antarctica

Whitehouse, P. , N. Gomez, M.A. King, D. A. Wiens, Solid Earth change and the evolution of the Antarctic Ice Sheet, Nature Communications, 503, 10, 2019 https://doi.org/10.1038/s41467-018-08068-y

Structure of Earth

- Lithosphere:
  - Earth's crust only in average 20 km thick (~ brittle, elastic plate)
    - Minerals; $\rho \approx 2900 \text{ kg m}^{-1}$
  - Asthenosphere (mineral upper part of mantle):
    - $c; \eta \approx 10^{21} \text{Pa s}$
- Mantle:
  - All in all 2900 km thick; mineral
  - At bottom $\rho \approx 5700 \text{ kg m}^{-1}$
- Core (metallic):
  - $\rho \approx 9400 \longmapsto 13500 \text{ kg m}^{-1}$
  - Outer, 3200 km thick liquid core
  - Inner, 1200 km thick solid core



Picture taken from
http://www.sfu.ca/geog/geog351fall06/group06/Image/Earthquake/EQ%20Pictures/earth_structure.jpg

Structure of Earth

- Earth has a layer type of structure
- Determined by seismic measurements
- PREM (Preliminary Reference Earth Model)

Physics of the Earth and Planetary Interiors, 25 (1981) 297–356
Elsevier Scientific Publishing Company, Amsterdam – Printed in The Netherlands

Preliminary reference Earth model *

Adam M. Dziewonski [1] and Don L. Anderson [2]

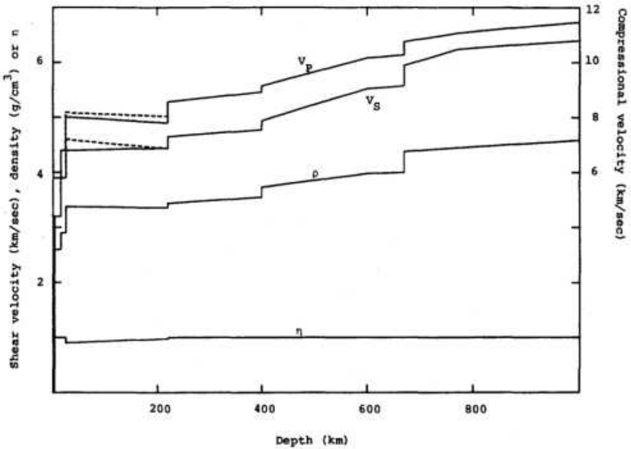
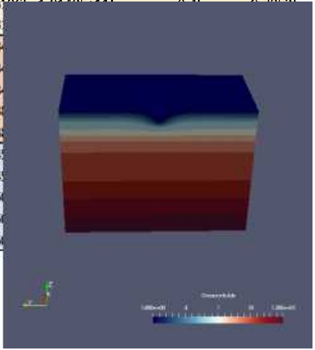


Fig. 7. Upper mantle velocities, density and anisotropic parameter η in PREM. The dashed lines are the horizontal components of velocity. The solid curves are η, ρ and the vertical, or radial, components of velocity.

Structure of Earth

Layer	Layer top (radius, km)	Layer base (radius, km)	Thickness (km)	Viscosity	Density	Young's Modulus	Poisson's Ratio	Gravitational Acceleration
Lithosphere	6371	6336	35	1x10^44	3196	1.8148E+11	0.4	9.7852
		6301	35	1x10^44				
		6251	50	1x10^44				
		6201	50	1x10^18				
		6141	60	1x10^18				
		5971	170	1x10^18				
		5835	136	1x10^18				
		5701	134	1x10^18				
		5450	251	1x10^22				
		4770	680	1x10^22				
		4340	430	1x10^22				
		3910	430	1x10^22				
		3480	430	1x10^22				

Non-self gravitating
Flat-earth model



Implementation in Elmer

- Standard FE linear elasticity: $\nabla \cdot \boldsymbol{\tau} = \mathbf{0}$
- Elastic rheology: stress as a function of reversible deformation
- Visco-elastic – Maxwell rheology :

  (partly non-reversible) deformation as a function of

  viscous and elastic contribution

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Implementation in Elmer

- Introduction of visco-elastic stress (Wu 2004)

$$\frac{\partial \boldsymbol{\tau}}{\partial t} = \frac{\partial \boldsymbol{\tau}_0}{\partial t} + \frac{\mu}{\nu}(\boldsymbol{\tau} - \Pi \mathbf{1})$$

$$\boldsymbol{\tau}_0 = \Pi \mathbf{1} + 2\mu\epsilon$$

  - At the same time we introduce pressure Π to enable incompressibility
- Additional term accounting for restoring force by specific weight gradient (aka. pre-stress advection)

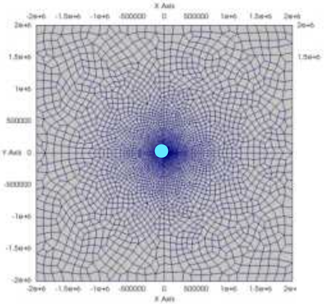
$$\nabla \cdot \boldsymbol{\tau} - \rho g \nabla(\boldsymbol{e}_z \cdot \boldsymbol{d}) = \mathbf{0}$$

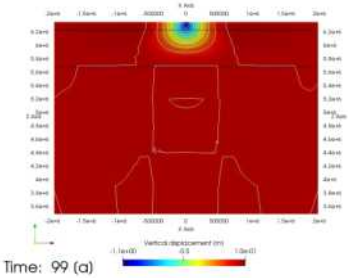
- This is not standard in commercial FE packages, hence needs to be "cheated" around by putting jump-conditions on inter-layer boundaries (Winkler foundations)
- In Elmer we can include this, which introduces the right boundary condition naturally from the third term of the weak formulation

$$\int_{\Omega} \tau(\boldsymbol{u}) \cdot \boldsymbol{\epsilon}(\boldsymbol{v})\, dV - \oint_{\partial\Omega} (\tau(\boldsymbol{u}) \cdot \boldsymbol{n}) \cdot \boldsymbol{v}\, dA - \int_{\Omega} \rho g \nabla(\boldsymbol{e}_z \cdot \boldsymbol{u}) \cdot \boldsymbol{v}\, dV = 0.$$

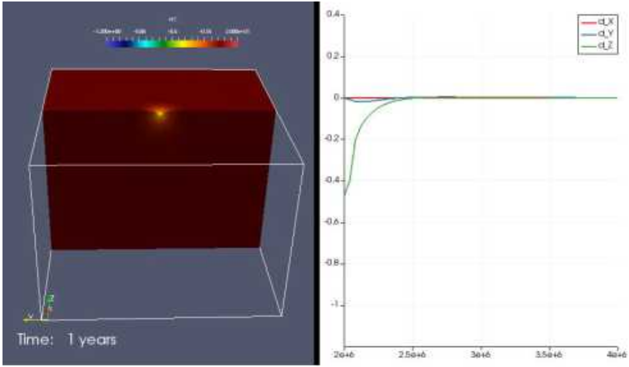
GIA benchmark model

- Total width 4000km (2000km each side of the ice load centre)
- Depth – surface to core (6371 – 3480km)
- Load:
  - Disc radius: 50km (diametre 100km)
  - Disc thickness: 100m
  - Ice density: 917 kg/m3
  - Loading 100 years, unloading 100 years

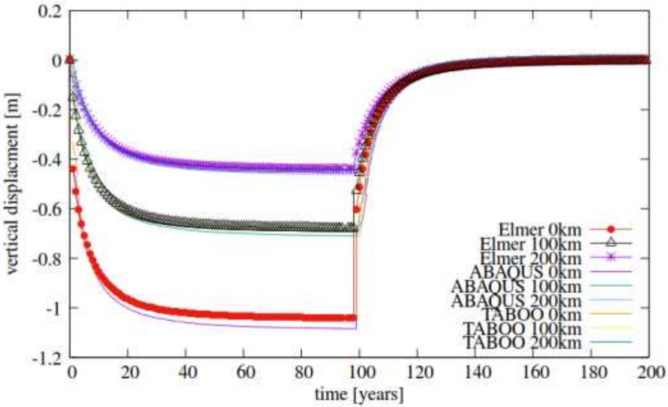




GIA benchmark model



GIA benchmark model – comparison to ABAQUS and TABOO

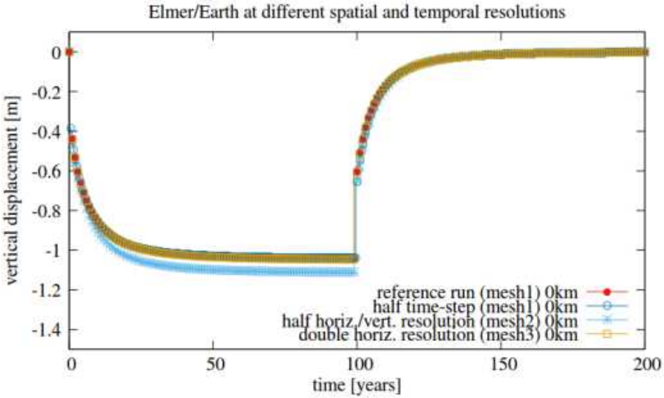


- ABAQUS: FEM model run at constant 25 km horizontal resolution
- TABOO: post-glacial rebound calculator (Spada et al., 2003) using classical visco-elastic normal mode method.
  - o Incompressible, non-rotating, self-gravitational
  - o Spherical harmonics degree 2048 (equivalent to approximately 10km)

GIA benchmark model – accuracy and performance

name	elements	min hor. res.
mesh 1	80k	~10 km
mesh 2 (half)	40k	~25 km
mesh 3 (double)	150k	~5 km

name	partitions	time [s]
mesh 1	16	21 200
mesh 1 (strong)	32	9600 (2.2 speedup)
mesh 3 (weak)	64	15800 (0.61 scaled speedup)



Conclusions

- Developed possibility to account for visco-elastic deformation in linear elasticity solver of Elmer
- Developed layered flat-earth model based on PREM
- Benchmarked against TABOO and ABAQUS
- No restrictions to resolutions; Mesh refinements
- No restrictions to variability of material parameters (can be declared element-wise)

Outlook

- Elmer/Earth, like Elmer/Ice shares the code-basis of Elmer, hence can easily be coupled using residuals of Stoke-solutions (= nodal forces) as load input
- Real world applications will follow (also perhaps visco-elasticity of ice itself)