Mapping ice thickness for various glacier types on Svalbard

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German Research Foundation
DFG

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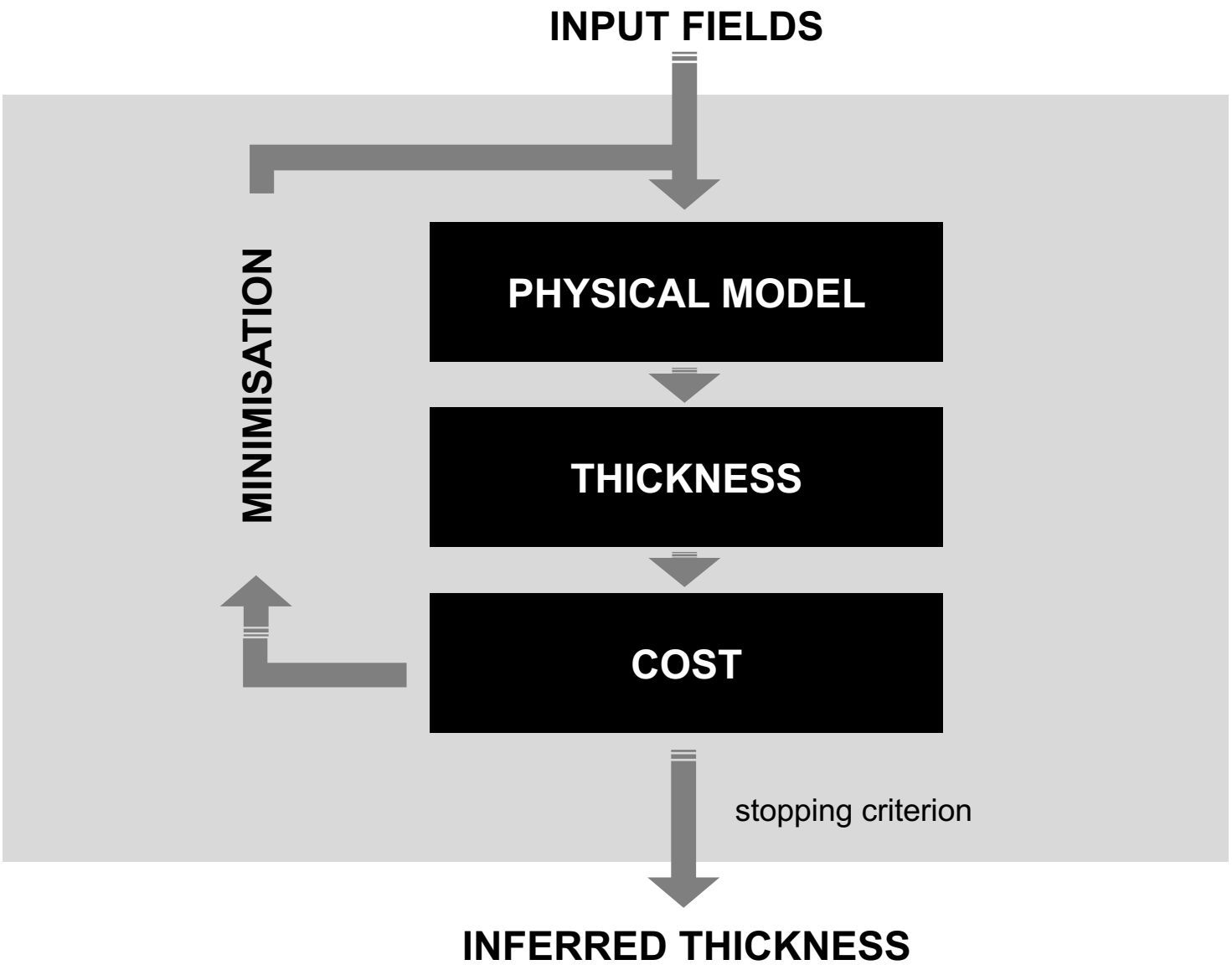
Methodology

Reconstruction approach



Two-fold physical basis

MASS CONSERVATION

1 - ice flux solver

$$\nabla \cdot \mathbf{F} = \dot{b} - \frac{Dh}{Dt} = \dot{a} \qquad \text{in } \Omega$$

- solves for the ice flux **F** assuming that flow directions follow surface slopes

- calculates ice thickness H from **F** assuming no basal sliding and uniform viscosity B

$$\mathbf{F} = -\frac{2}{n+2}(\rho g)^n \cdot B^{-1/n} \cdot |\nabla s|^{n-1} H^{n+2} \cdot \nabla s$$

MINIMUM INPUT

glacier outline
DEM
SMB
DhDt

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$$\nabla \cdot (\bar{\mathbf{u}} H) = \dot{a} \qquad \text{in } \Omega$$

2 – thickness solver

∞ solves for ice thickness H
∞ requires velocity information
∞ assumes vertically uniform flow

EXTRA INPUT surface velocity magnitude and direction

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2 – thickness solver

$$\nabla \cdot (\bar{\mathbf{u}} H) = \dot{a} \qquad \text{in } \Omega$$

MORLIGHEM et al. (2011)

∞ solves for ice thickness H
∞ requires velocity information
∞ assumes vertically uniform flow

EXTRA INPUT surface velocity magnitude and direction

Cost function J

$$J = \Sigma_i \lambda_i J_i$$

1 - ice flux solver

- control parameters: $\dot{a}$
- cost function composition
  - J_1 - penalises negative flux values
  - J_2 - penalises high spatial variability in flux field
  - J_3 - penalises the mismatch to the observed $\dot{a}$

2 - thickness solver

- control parameters: $\dot{a}$ and $\bar{\mathbf{u}}$
- cost function composition
  - J_1 - penalises negative thickness values
  - J_2 - penalises high spatial variability in thickness field
  - J_3 - penalises unrealistic marine ice cliff heights
  - J_4 - penalises mismatch to observed velocities
  - J_5 - penalises mismatch to observed $\dot{a}$

Error estimate

1 - ice flux solver

- assume that erroneous flux field is also a solution of mass conservation

$$\nabla(n\cdot \delta F_1) = \delta\dot{a} - \nabla(F\cdot \delta n) \quad \text{in } \Omega$$
$$\nabla(-n\cdot \delta F_2) = \delta\dot{a} - \nabla(F\cdot \delta n)$$

- upstream and downstream error transmission
- linear error propagation through SIA equation

$$\delta H_i = \frac{1}{n+2}\left(-\frac{n+2}{2}(\rho g)^n \cdot B^{-1/n} \cdot |\nabla s|^n\right)^{-1/(n+2)} \cdot F^{-(n+1)/(n+2)} \cdot \delta F_i$$

- take minimum of δH_1 and δH_2

2 – thickness solver

- assume that erroneous thickness field satisfies mass conservation

$$\nabla(\bar{u}\cdot \delta H) = \delta\dot{a} - \nabla(F\cdot \delta\bar{u}) \quad \text{in } \Omega$$

INPUT UNCERTAINTIES

δn = 20%
$\delta\dot{a}$ = 0.2 m i.e. / yr
$\delta\bar{u}$ = 20 m / yr

Data assimilation

ICE-FLOW MODEL

(Gagliardini & Zwinger 2008; Gillet-Chaulet et al., 2012;
Gagliardini et al., 2013, Mosbeux et al., 2016)

Implemented routines

- mass conservation equation
- adjoint-system defines cost gradients
- M1QN3 minimisation

Automated gridding

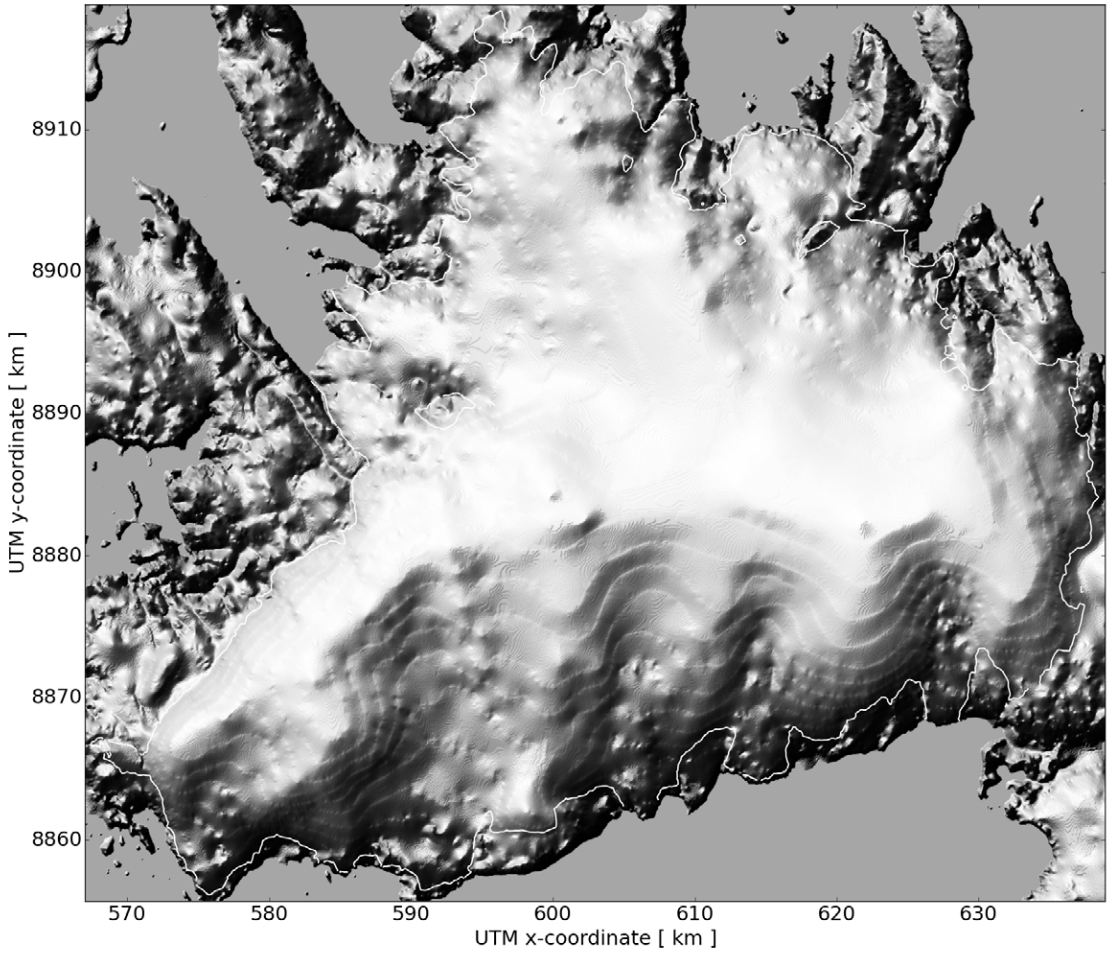
accounts for

- internal boundary conditions (thickness and error at measurements locations)
- external boundary (marine and lake terminated ice fronts)
- nunataks larger than the grid spacing

Input data

Vestfonna

- glacier outlines
  (Nuth et al., 2013)
- surface elevation
  (NPI S0 DTM50)



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- SMB
  regional climate model
  MAR (mean 1979-2015)
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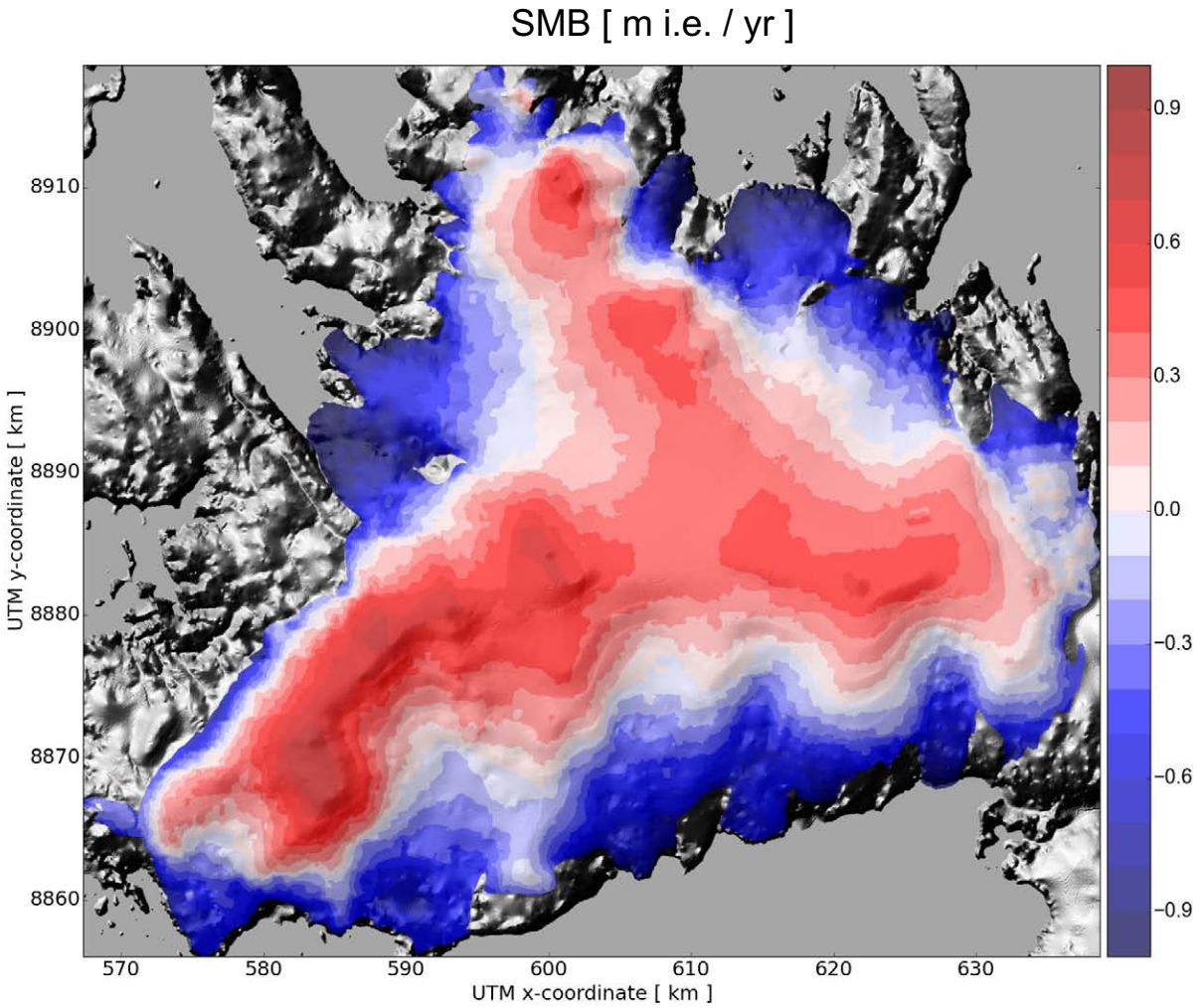

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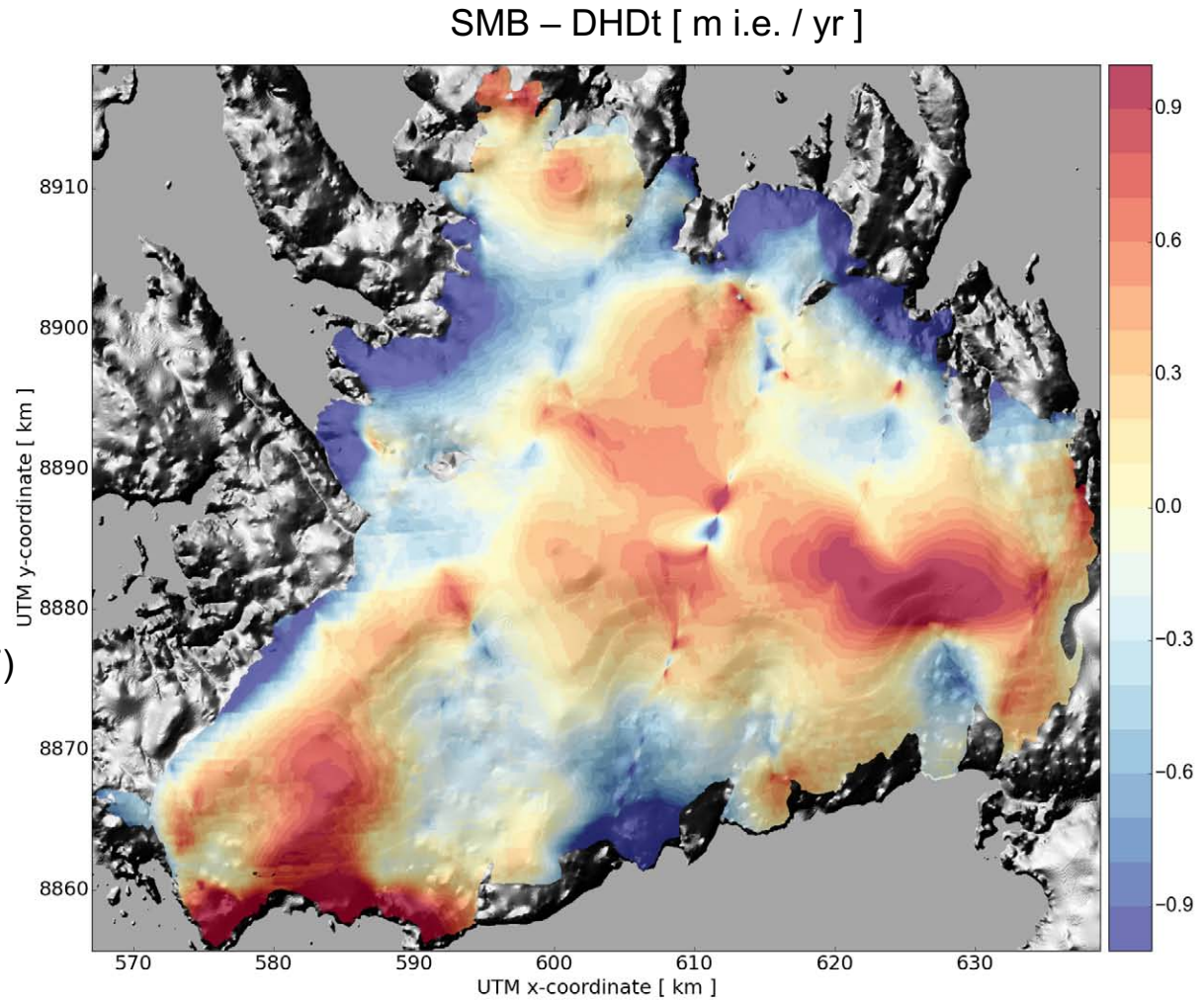

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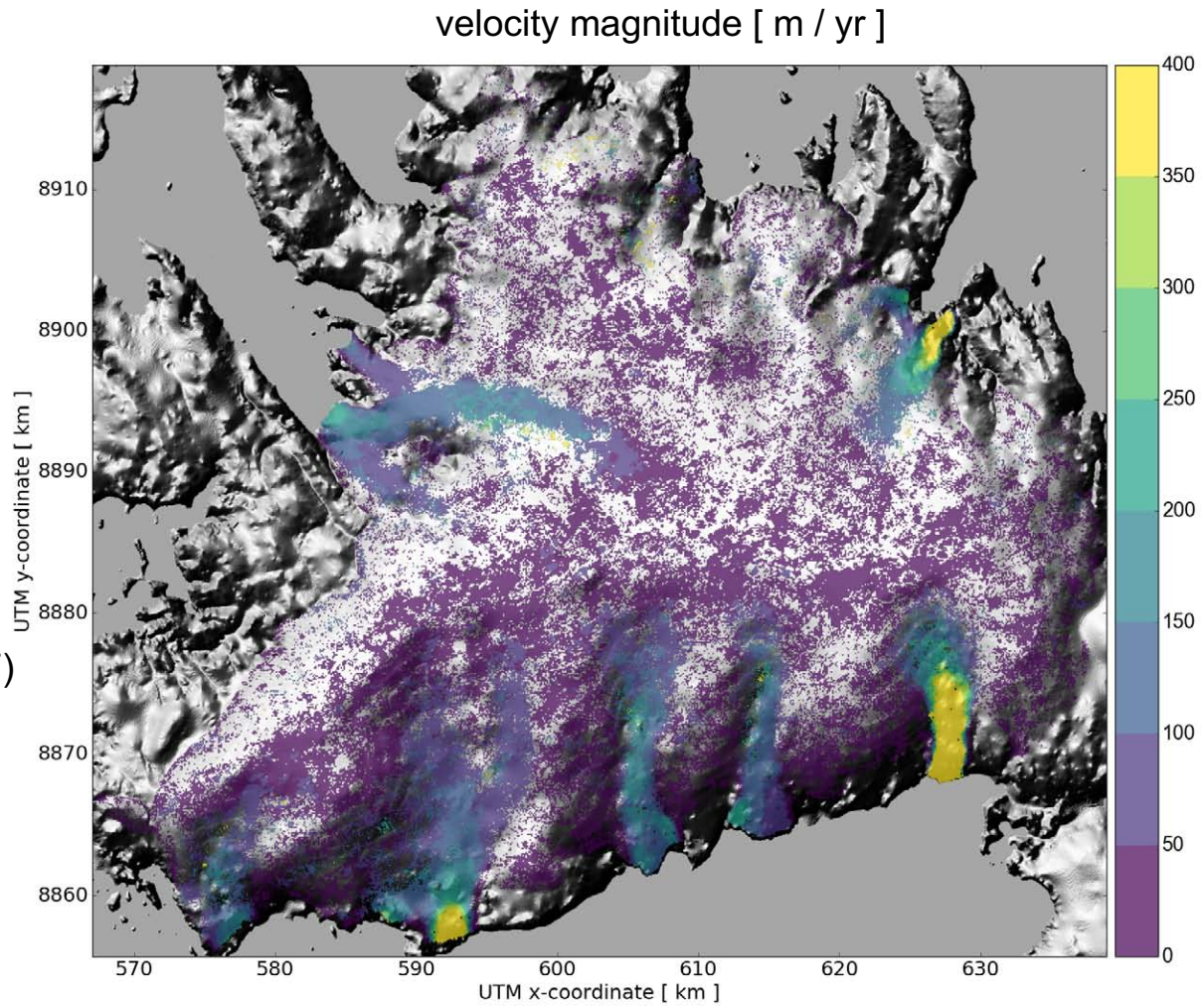
  Sentinel 1 (2015 - 2016)



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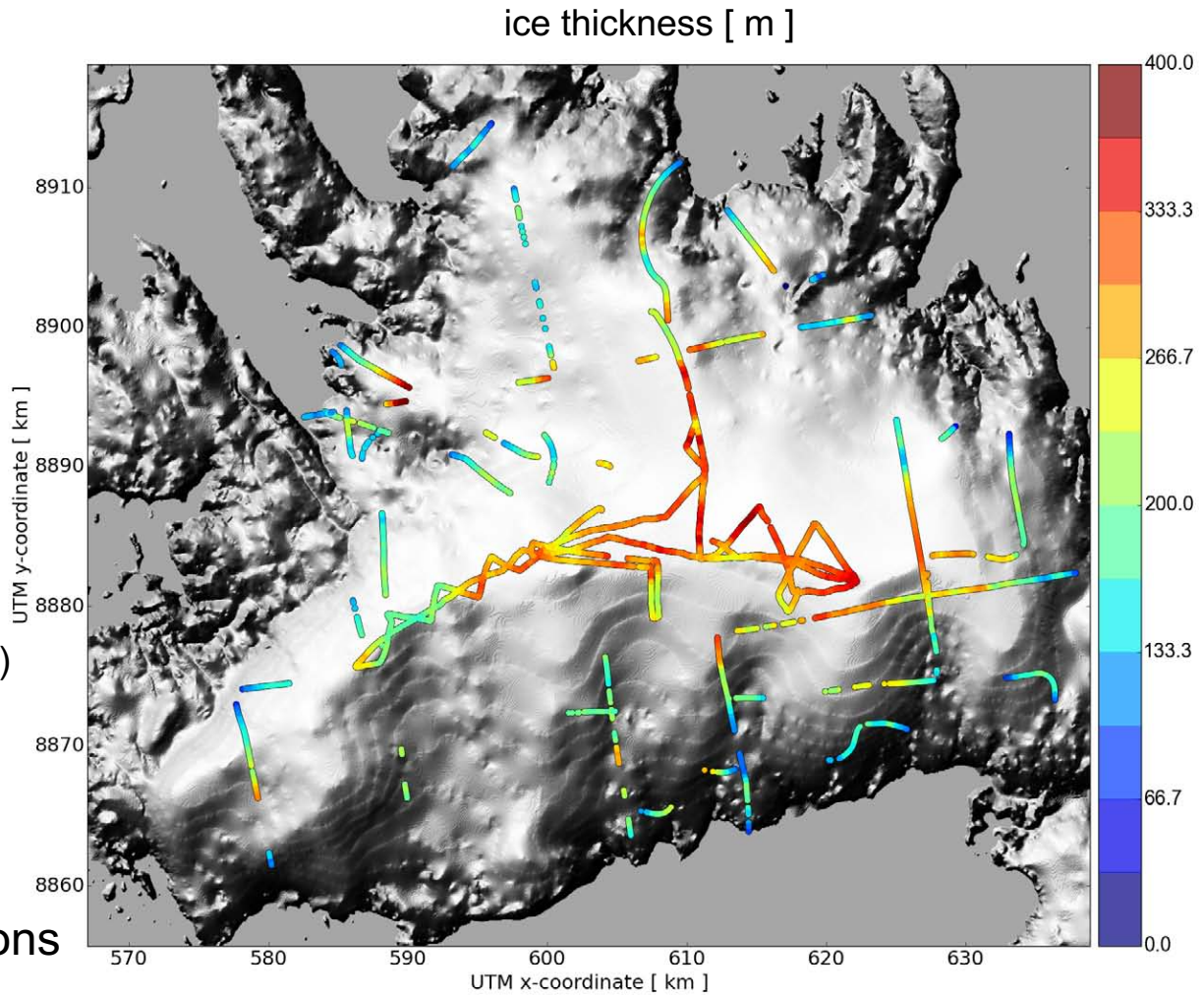
- elevation changes
  ICESat (mean 2003-2007)
  (Nuth et al., 2010)
- surface velocities
  Sentinel 1 (2015 - 2016)
- thickness observations
  airborne and ground RES
  (Dowdeswell et al., 1986; Petterson et al., 2011)



Results

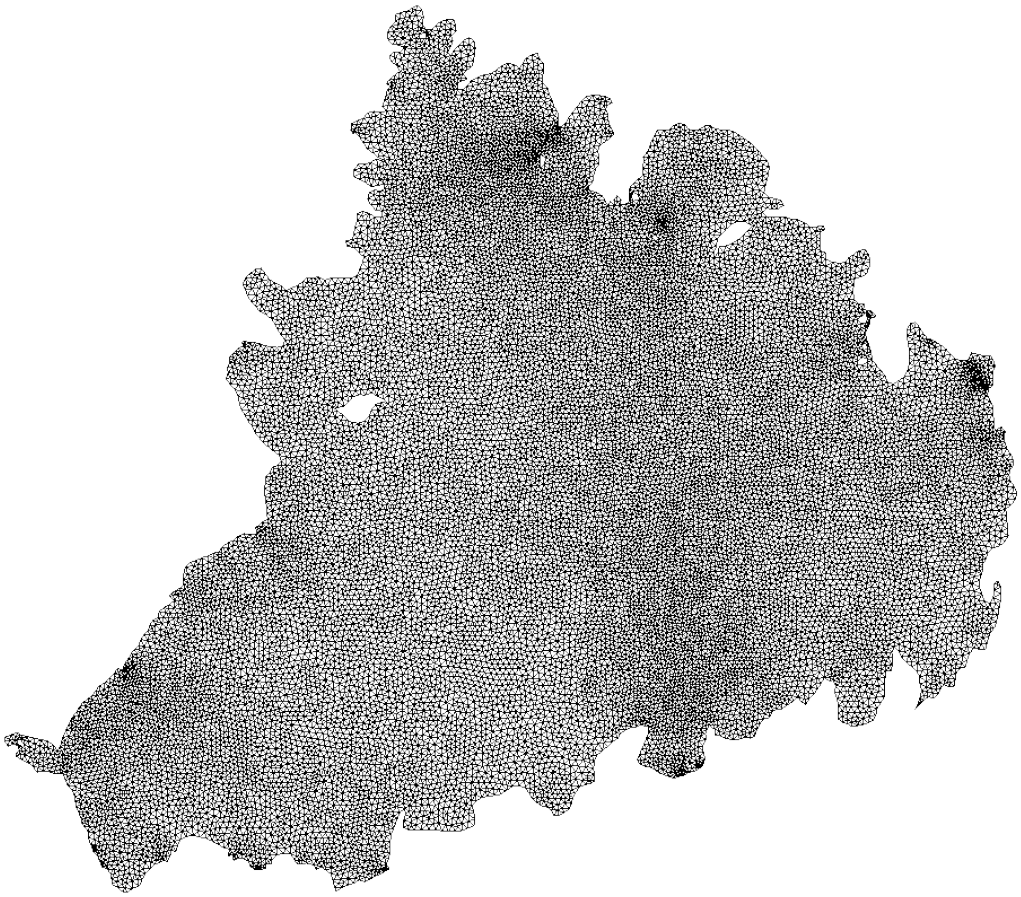
PART 1

Ice flux solver

Reconstruction

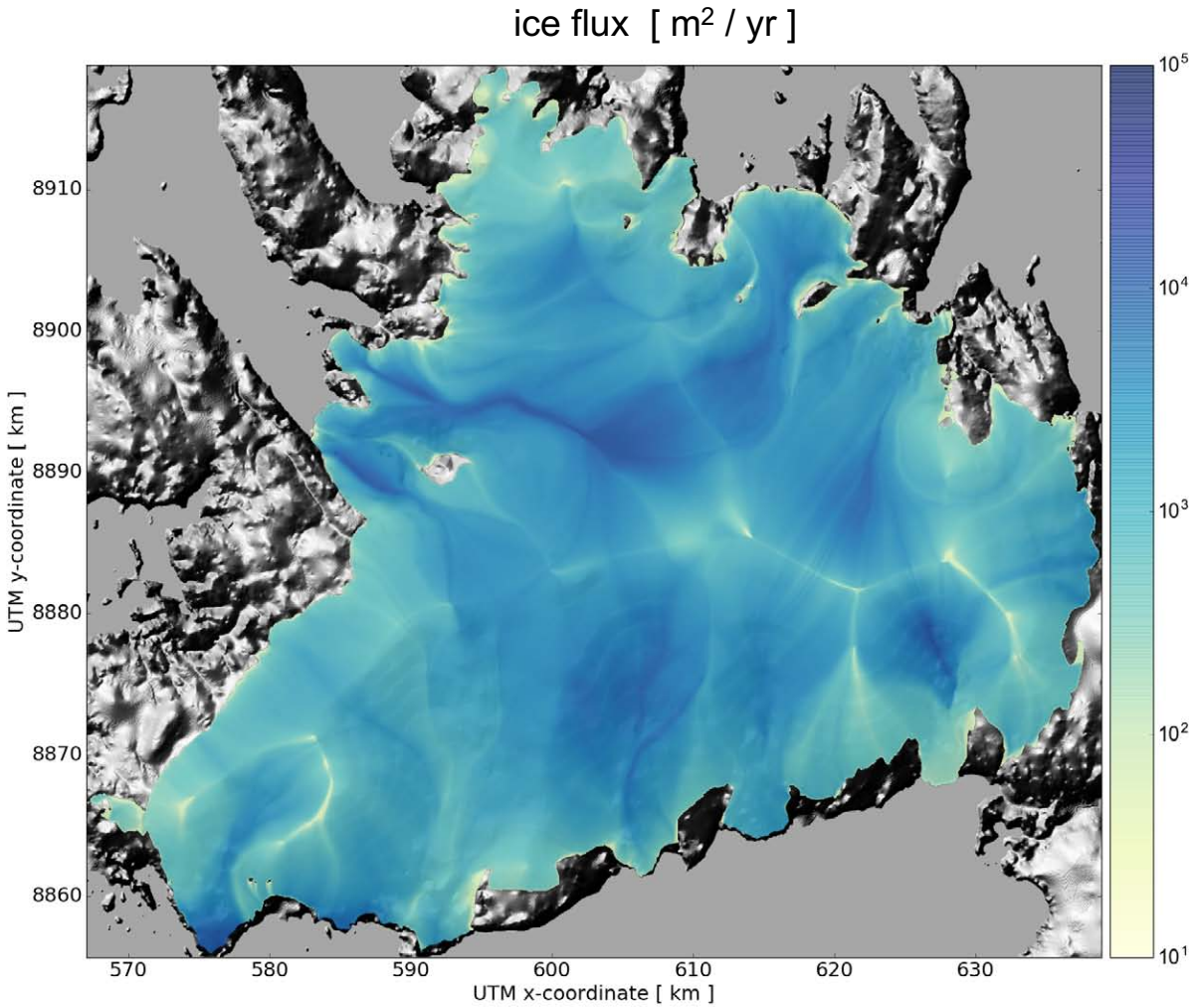
GRIDDING

- 500m nominal resolution



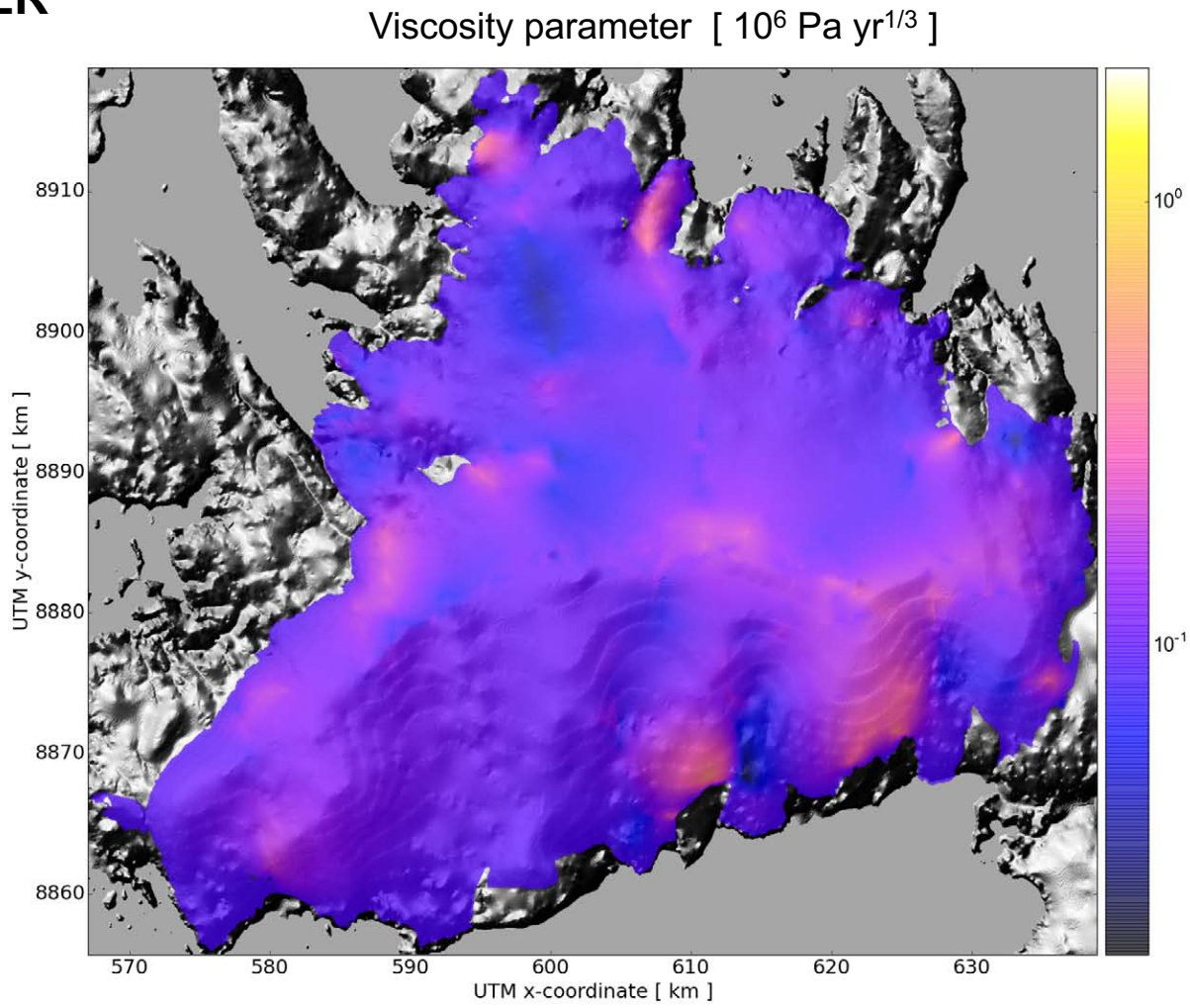
Reconstruction

ICE FLUX



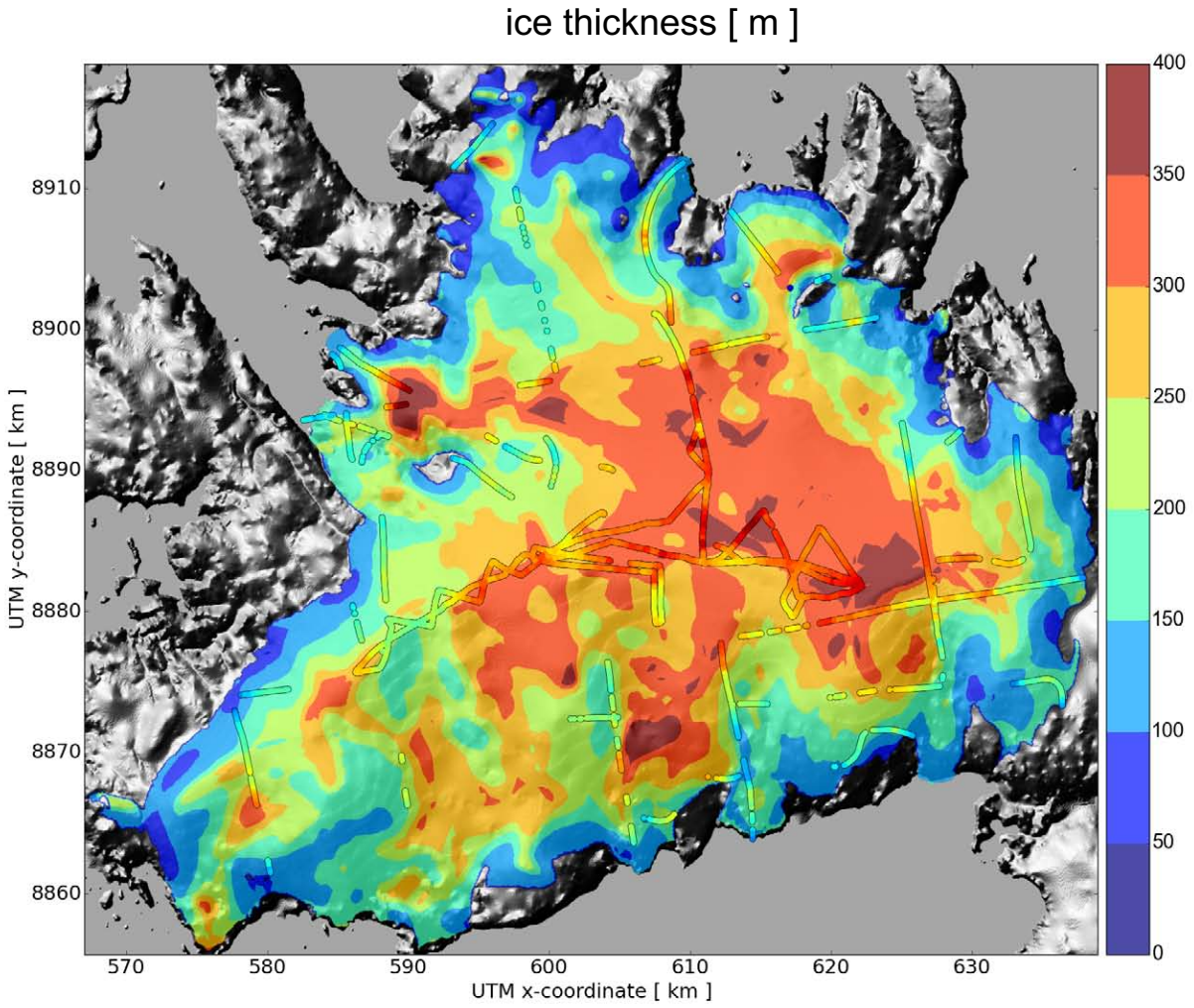
Reconstruction

VISCOSITY PARAMETER



Reconstruction

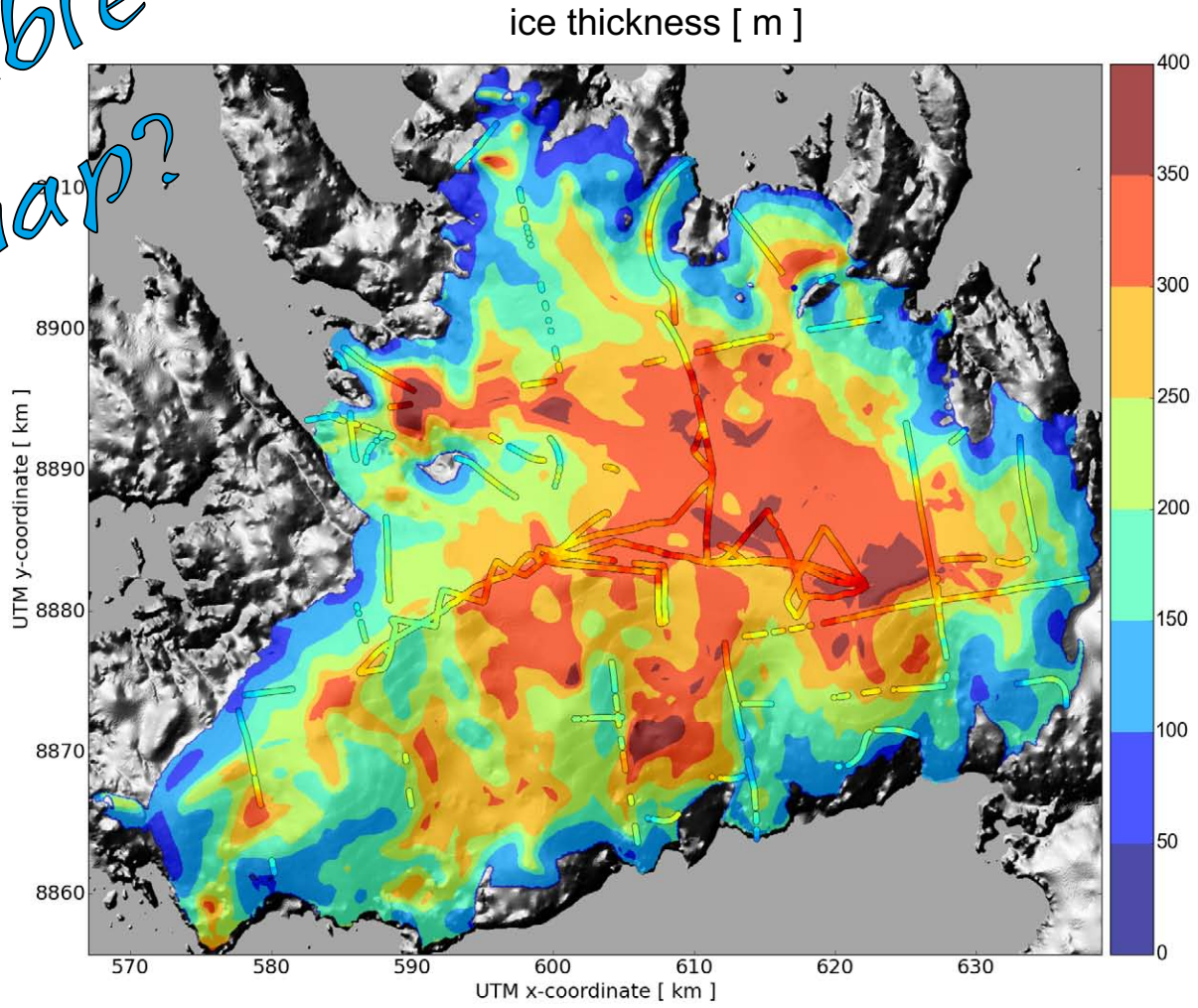
ICE THICKNESS



Reconstruction

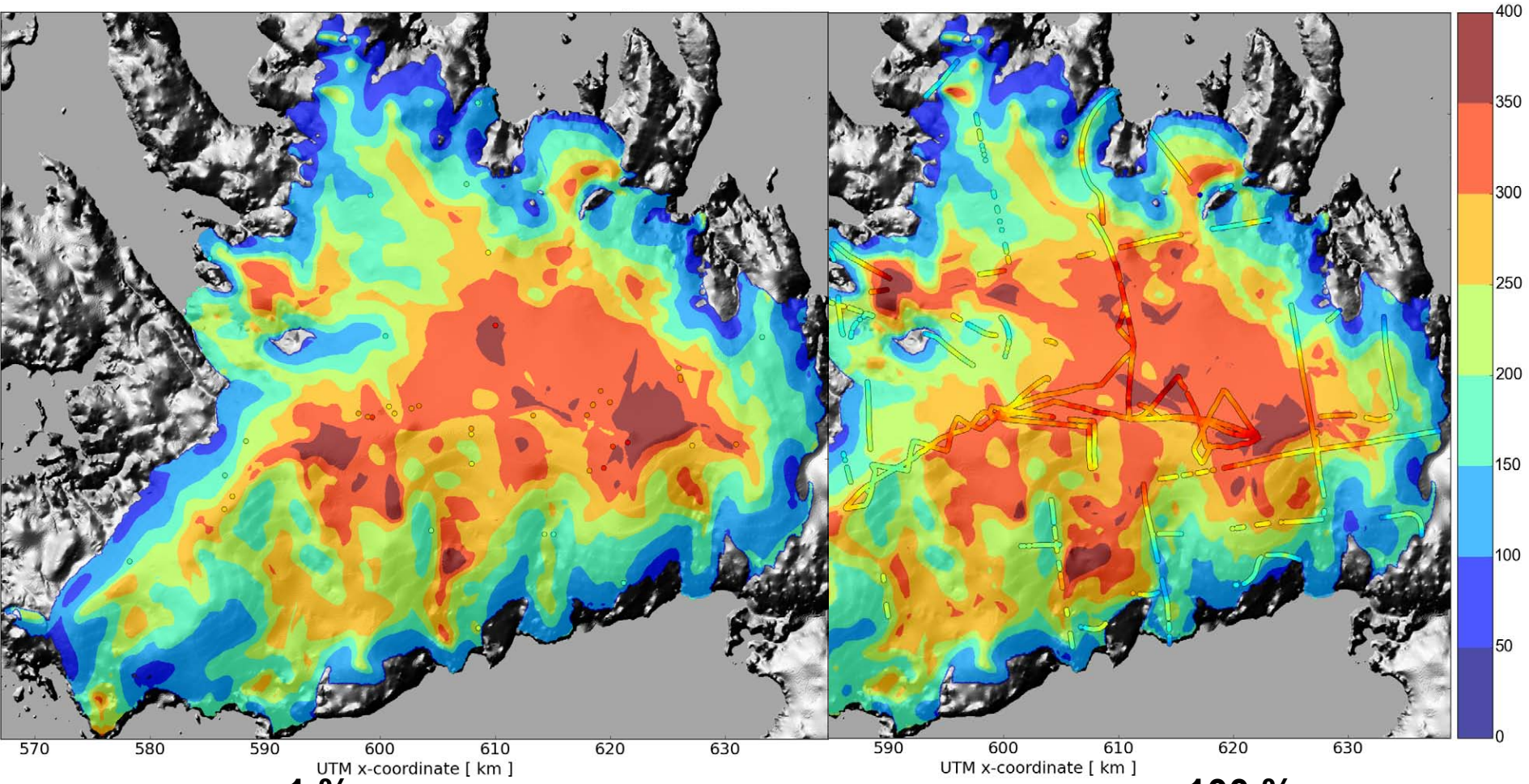
ICE THICKNESS

How reliable is this map?



Reconstruction

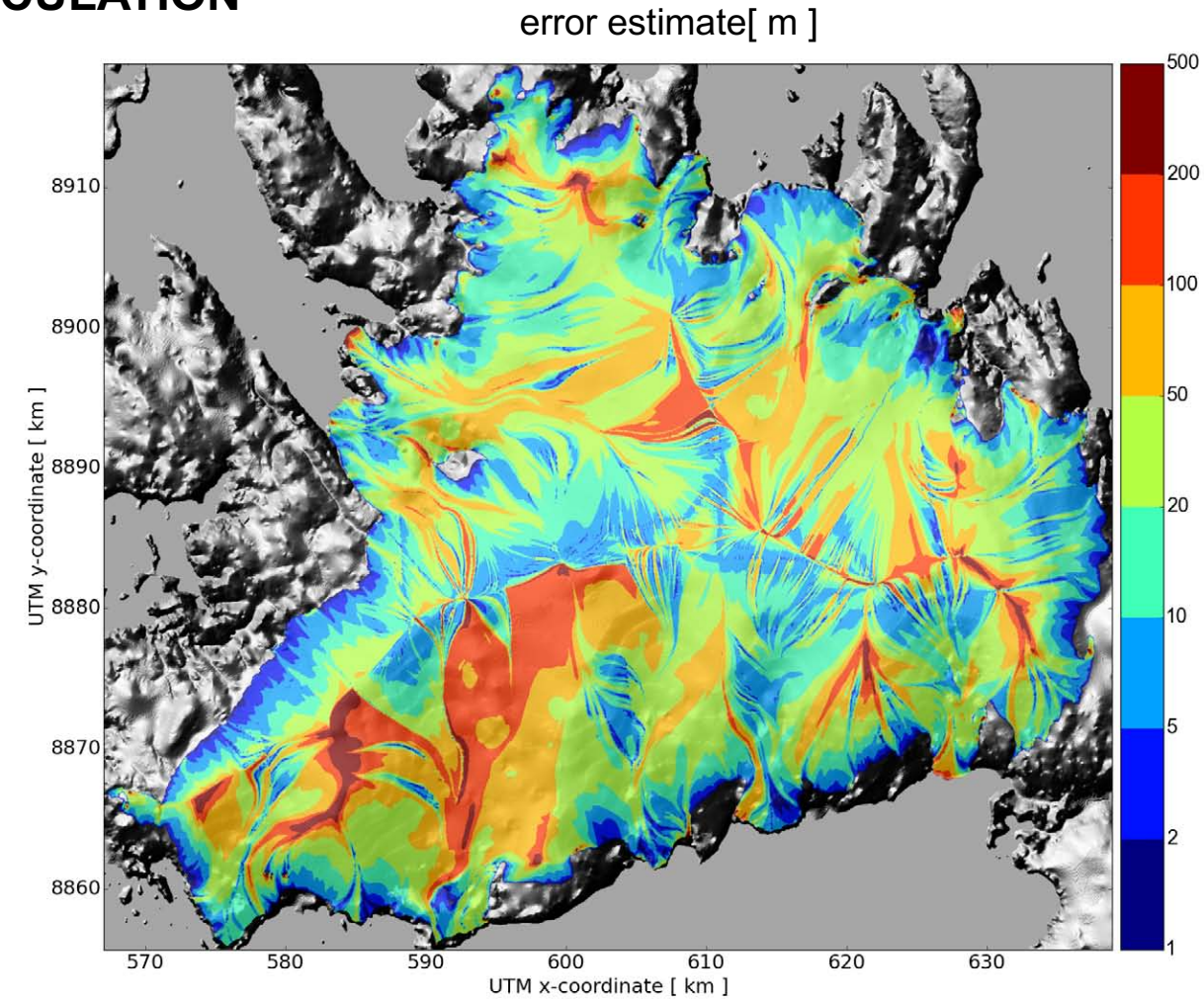
Sensitivity



1 % share of used thickness measurements **100 %**

Reconstruction

FORMAL ERROR CALCULATION



Reconstruction

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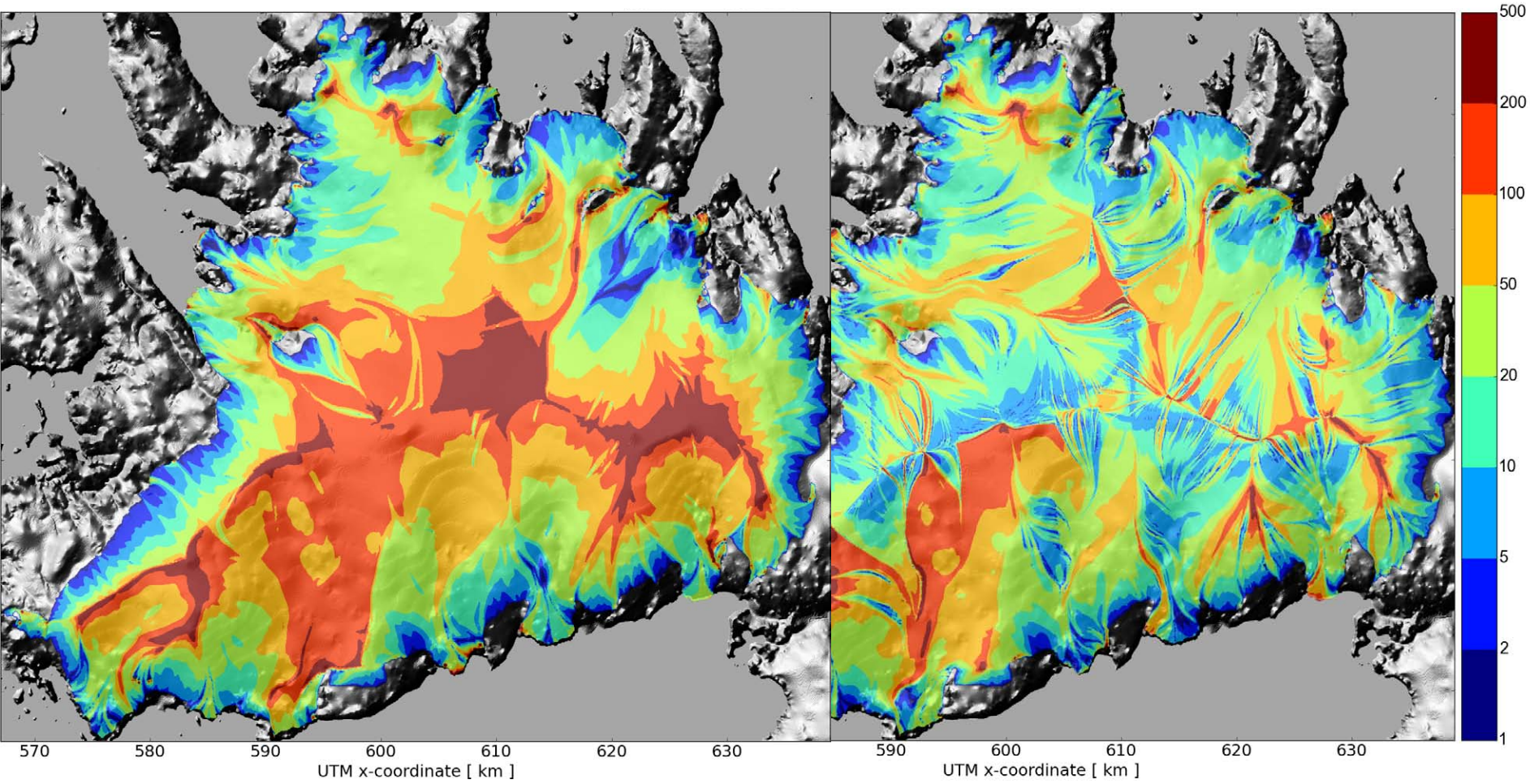

Reconstruction

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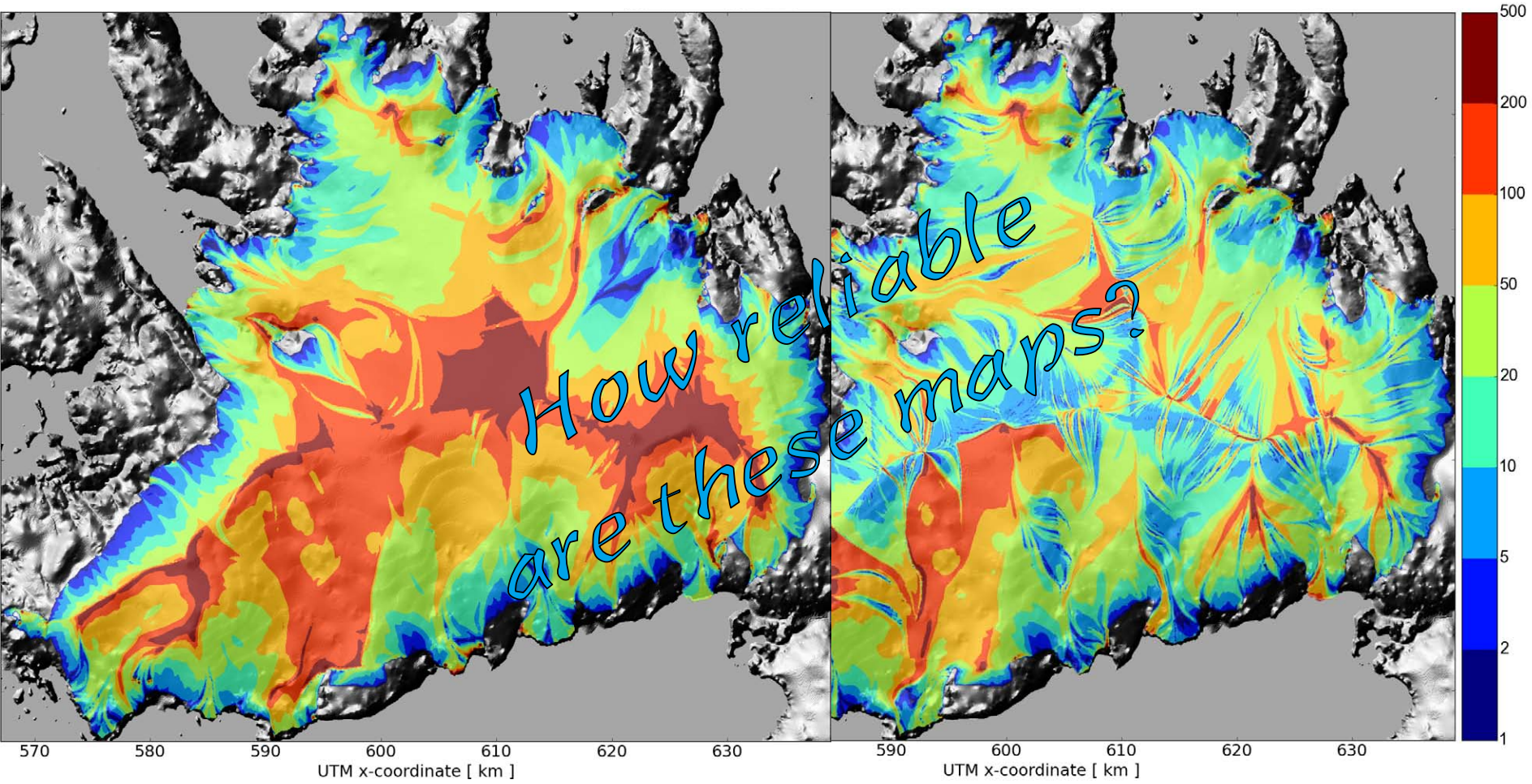
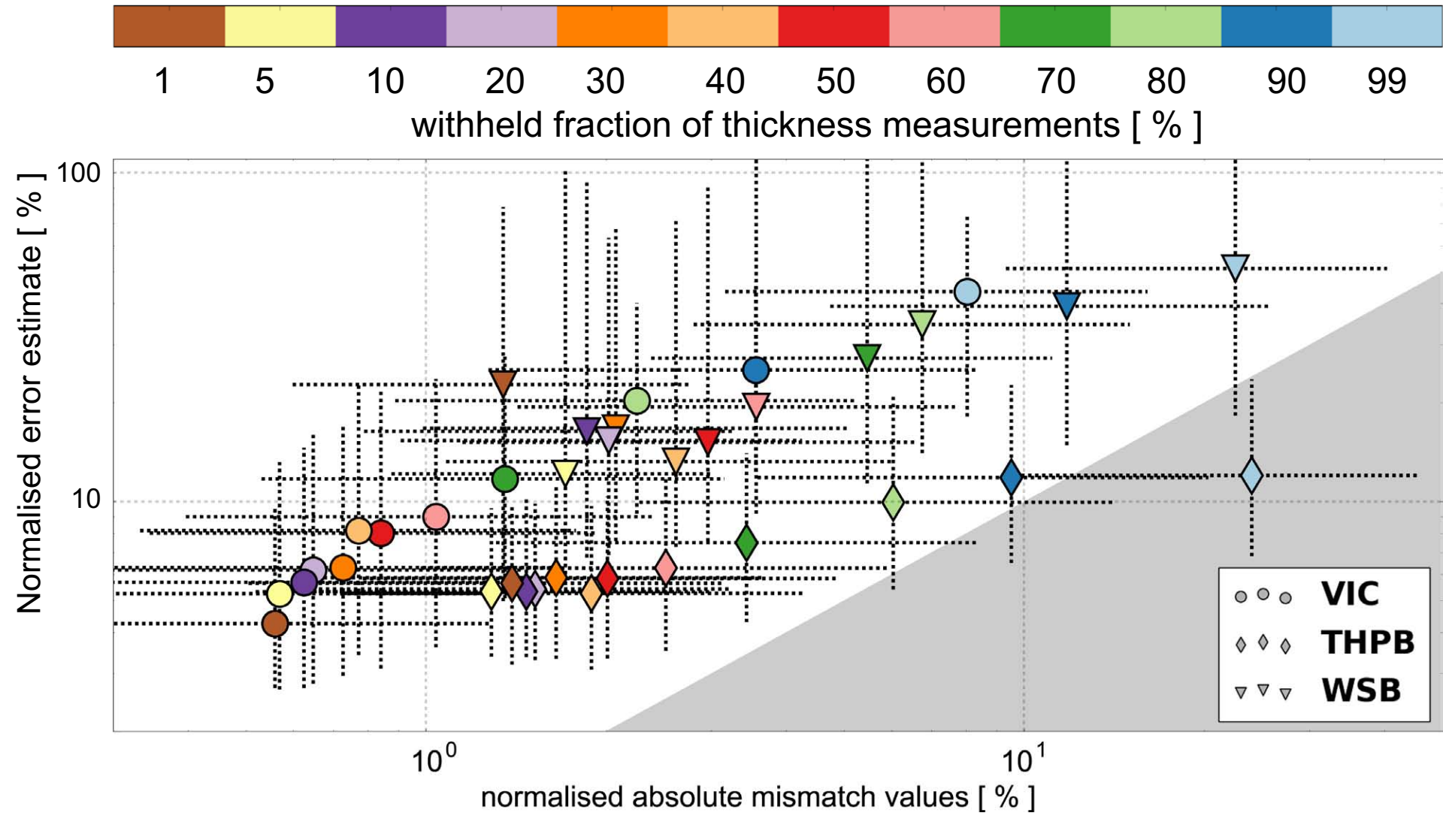


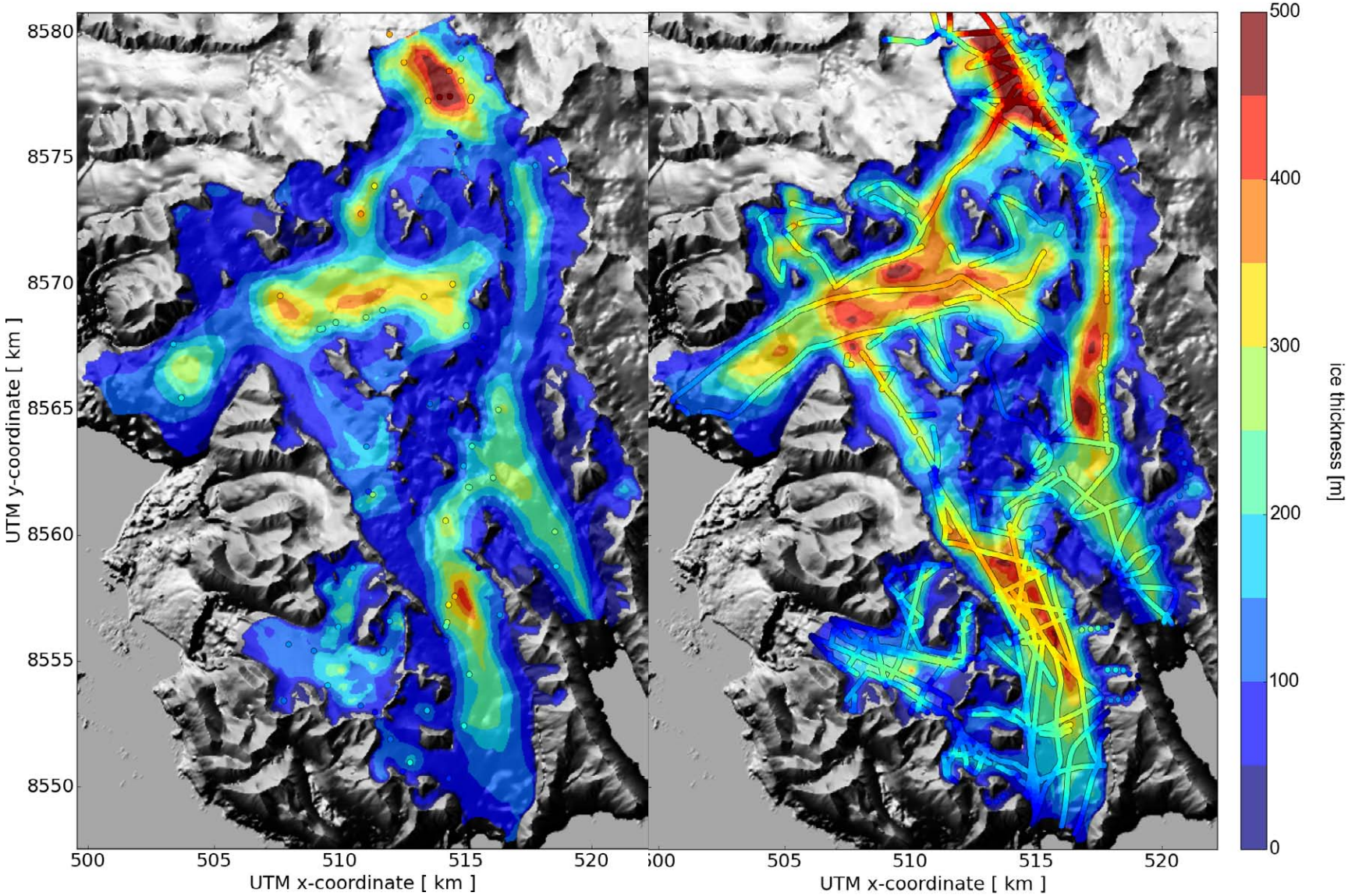
Reconstruction



Reconstruction

ICE THICKNESS

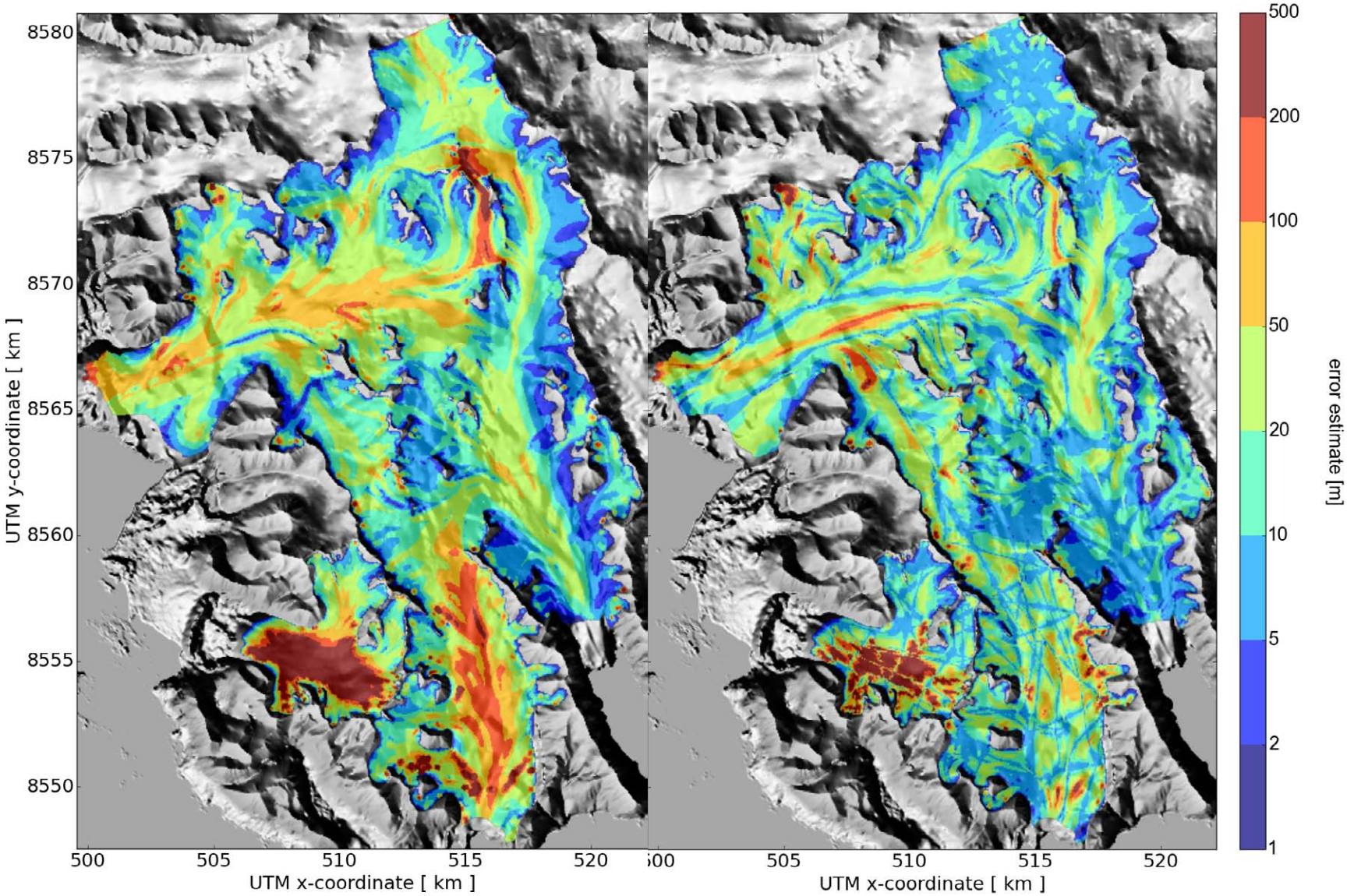
Hans-, Paierl-, Torell-, Werenskioldbreen



Reconstruction

ERROR ESTIMATES

Hans-, Paierl-, Torell-, Werenskioldbreen



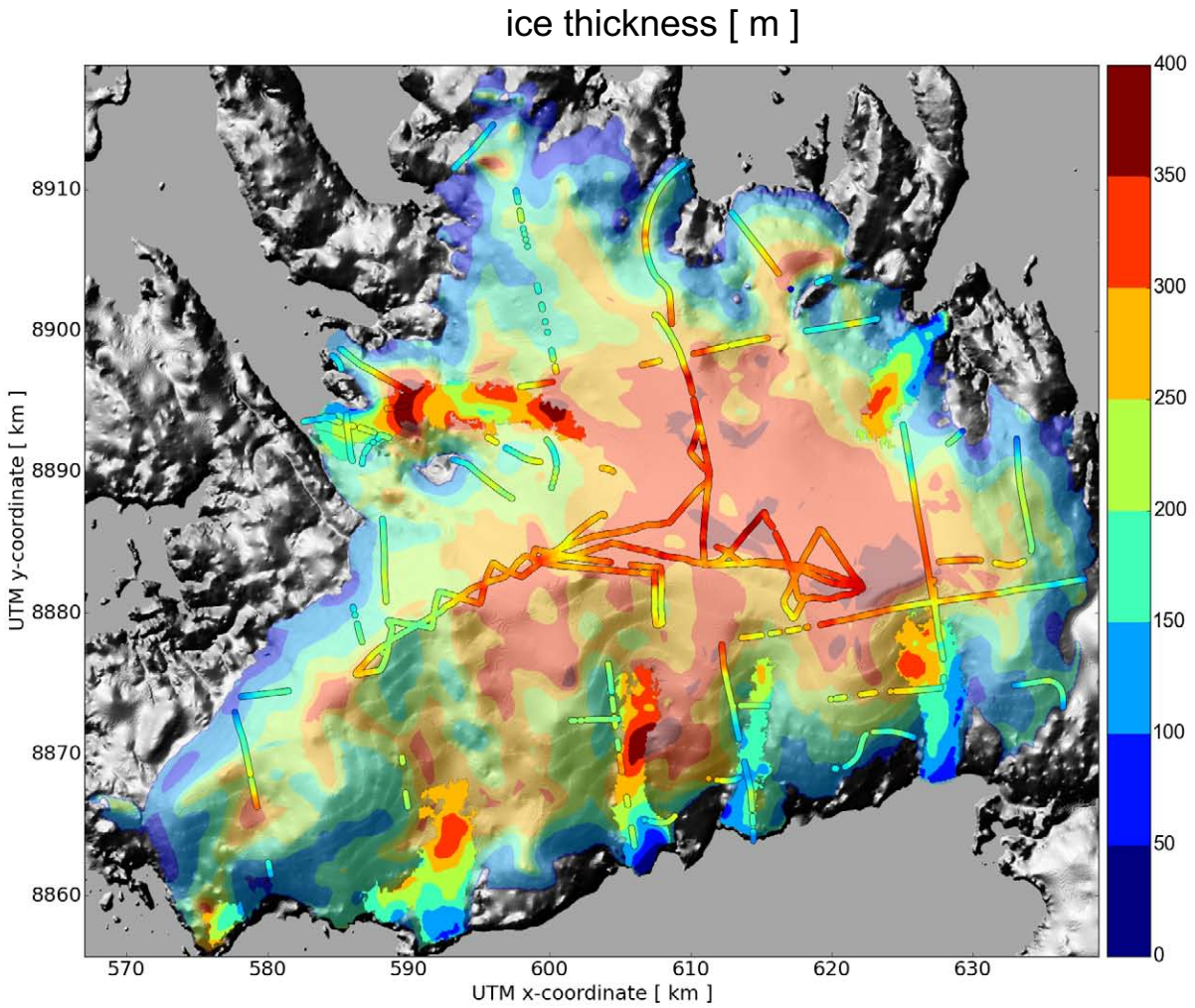
Results

PART II

Thickness solver

Reconstruction

ICE THICKNESS



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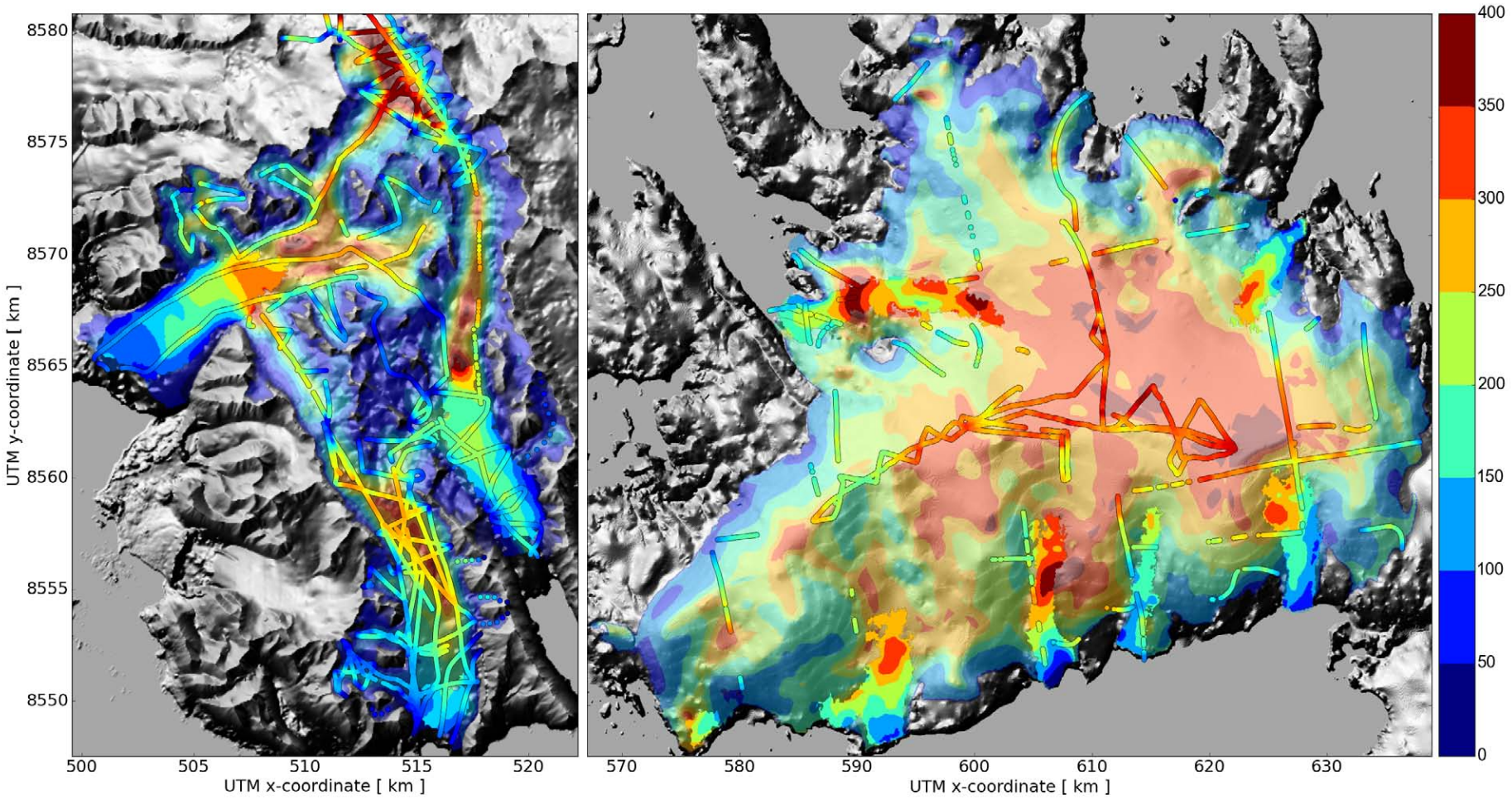
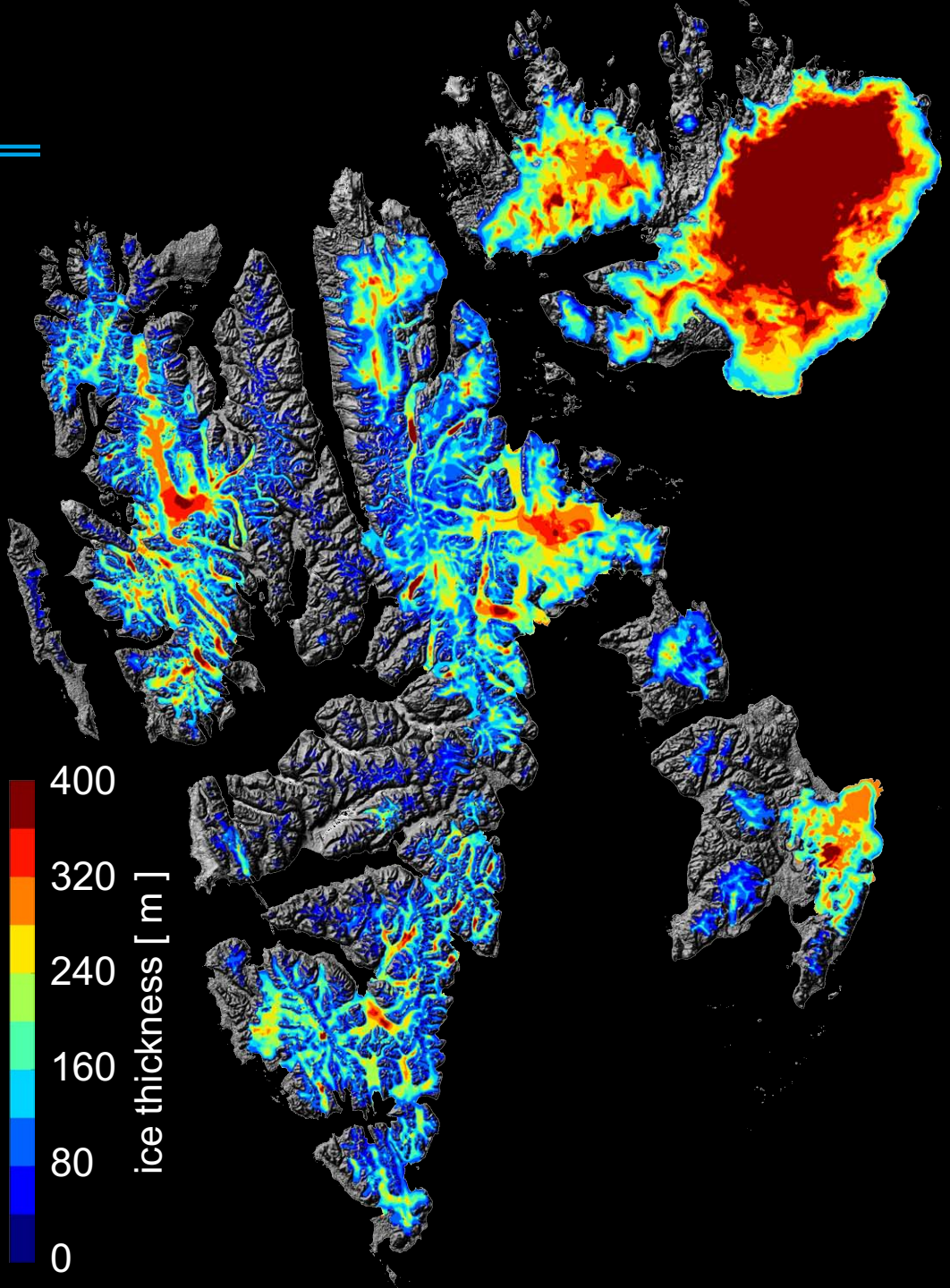


Outlook

Outlook



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