Laboratoire de Glaciologie et Géophysique de l'Environnement

First Elmer/Ice course

14-15 February 2008 – Updated 2013

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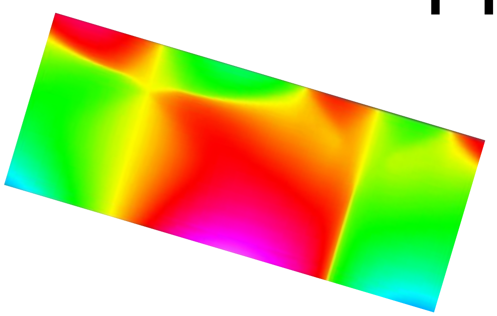
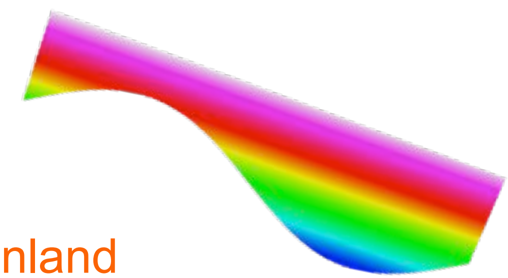
Application to ISMIP HOM(3) tests B and D



Step by step !



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Outline

Step 0 Start from a very simple test case. What are we solving? (Glen's law, …)

Step 1 Move to test ISMIP-HOM B020 (mesh, periodic BC)

Step 2 Add SaveData solver to get output on the BC (SaveLine)

Step 3 Add SaveData solver to get cpu and volume of the domain (SaveScalars)

Step 4 Add ComputeDevStress solver to get the stress field

Step 5 Move to test ISMIP-HOM D020 (sliding law from user function or MATC)

Step 6 Restart from Step 4: Move to Prognostic ISMIP B020.
- Move from a steady to a transient simulation
- Free surface solver
- Mesh Update solver

Step 7 Move to Prognostic ISMIP D020.

Step 0

Create a `My_ISMIP_Appli` directory

Copy the directory *Step0 in* `My_ISMIP_Appli`

- Make the mesh : `> ElmerGrid 1 2 square.grd`

- Run the test : `> ElmerSolver ismip_step0.sif`

- Watch the results : `> ElmerPost` and open `square\ismip_step0.ep`

- What are we solving?

Stokes:

$$\mathrm{div}\,\boldsymbol{\sigma} + \rho \boldsymbol{g} = 0$$

$$u_{i,i} = 0$$

Navier-Stokes with convection and acceleration terms neglected :

```
Flow Model = String Stokes
```

in the Stokes solver section

Step 0 – Glen's law and Elmer

In glaciology, you can find (at least) two definitions for Glen's law:

$$D_{ij} = \frac{B}{2}\tau_e^{n-1} S_{ij} \quad ; \quad S_{ij} = 2B^{-1/n}\dot{\gamma}^{(1-n)/n} D_{ij}$$

$$D_{ij} = A\tau_e^{n-1} S_{ij} \quad ; \quad S_{ij} = A^{-1/n} I_{D_2}^{(1-n)/n} D_{ij}$$ ISMIP notation

where $I_{D_2}^2 = D_{ij}D_{ij}/2$ and $\dot{\gamma}^2 = 2D_{ij}D_{ij}$

The power-law implemented in Elmer writes: $S_{ij} = 2\eta_0 \dot{\gamma}^{m-1} D_{ij}$

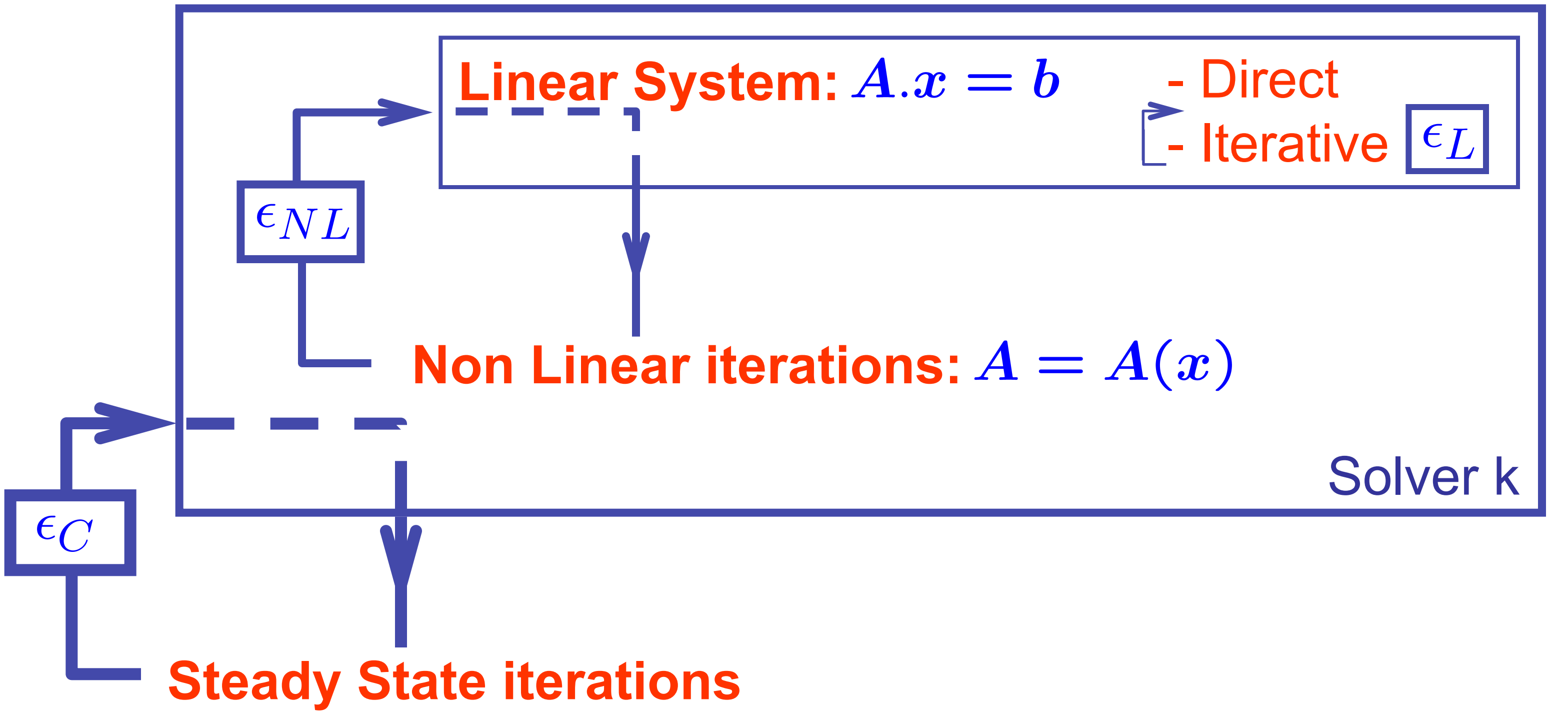
$$\boxed{\begin{aligned} &\eta_0 = B^{-1/n} = (2A)^{-1/n} \\ &m = 1/n \\ &\dot{\gamma}^2 \geq \dot{\gamma}_c^2 \end{aligned}}$$

In Material Section:

```
Viscosity Model = String "power law"
Viscosity = Real η0
Viscosity Exponent = Real m
Critical Shear Rate = Real γ̇c
```

Step 0 – Sketch of a Steady simulation

$\boxed{\text{Geometry + Mesh}} \longrightarrow$ Degrees of freedom



$$\epsilon_L < \epsilon_{NL} < \epsilon_C$$

Step 0 – Numerical methods

In the NS Solver Section: (see Chapters 3 and 4 of Elmer Solver Manual)

- Solution for the Linear System:

```
Linear System Solver = Direct
Linear System Direct Method = umfpack
```

- Non-Linear System :

```
Nonlinear System Max Iterations = 100
Nonlinear System Convergence Tolerance  = 1.0e-5
Nonlinear System Newton After Iterations = 5
Nonlinear System Newton After Tolerance = 1.0e-02
Nonlinear System Relaxation Factor = 1.00
```

$= \epsilon_{NL}$

Picard

Newton

- Coupled problem (not needed here in fact…):

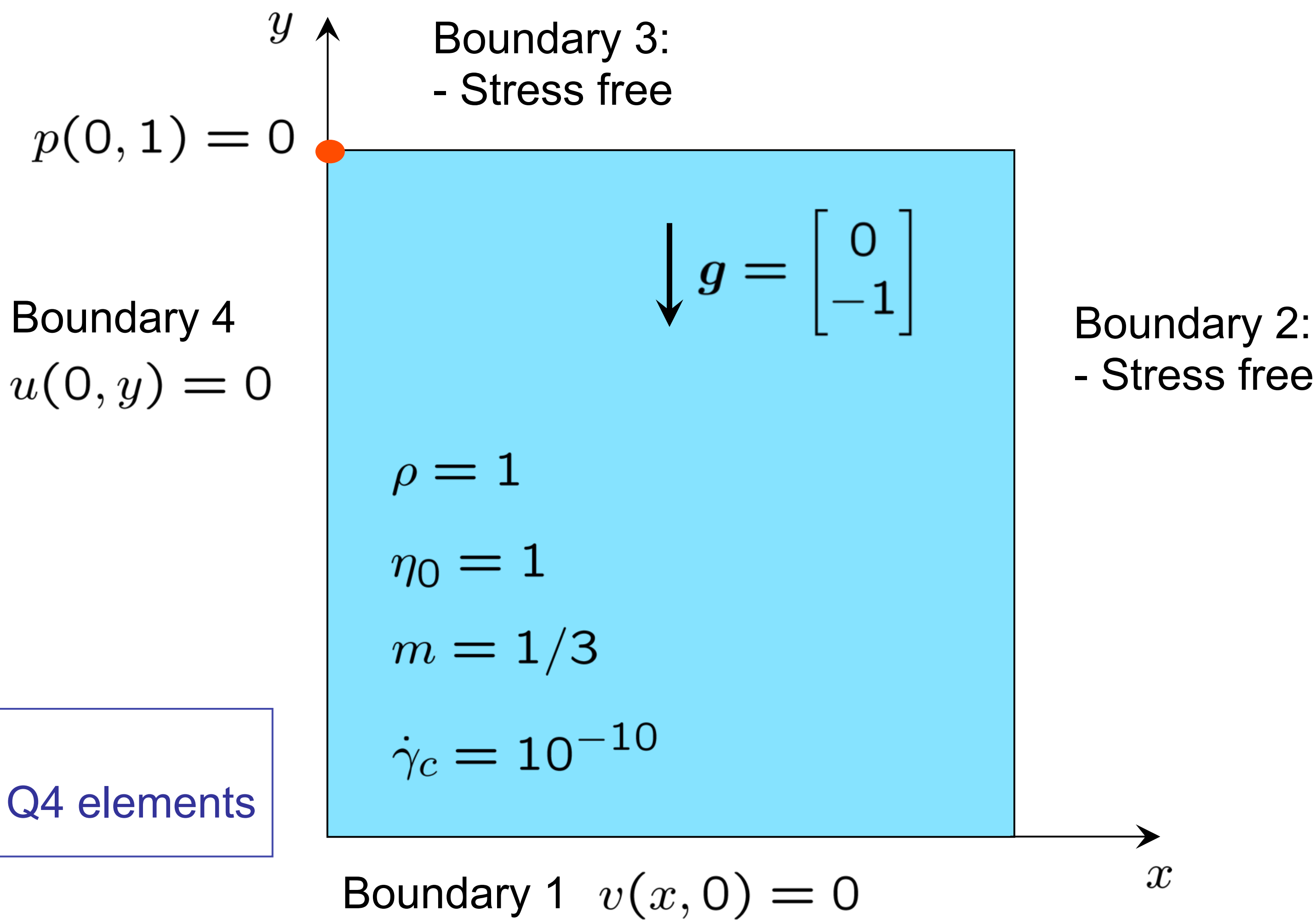
```
Steady State Convergence Tolerance = Real 1.0e-3
```

$= \epsilon_C$

- Stabilization of the Stokes equations:

`Stabilization Method = String Bubbles` (other options: `Stabilized`, `P2P1`)

Step 0 – What are we solving?



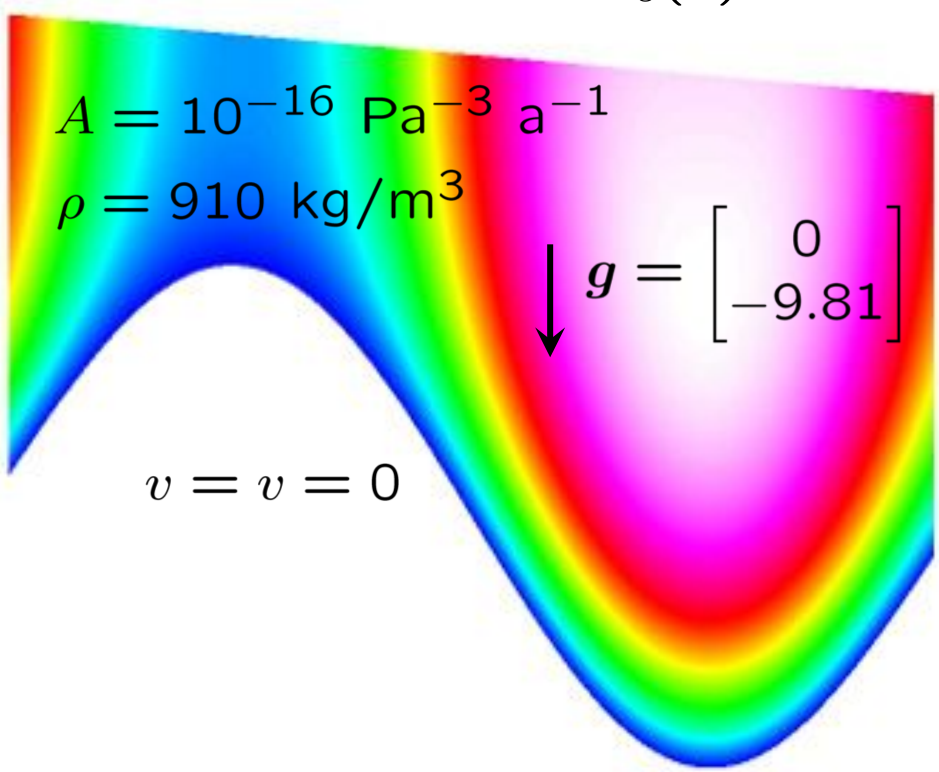
Mesh:
20 x 20 Q4 elements

Step 1 – Move to ISMIP-HOM B020

What we have to solve :

Stress free $z_s(x) = -x\,\mathrm{tan}(0.5°)$

Periodic BC



$A = 10^{-16}\ \mathrm{Pa}^{-3}\ \mathrm{a}^{-1}$

$\rho = 910\ \mathrm{kg/m}^3$

$\boldsymbol{g} = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix}$

Periodic BC

$v = v = 0$

$L = 20 \times 10^3\ \mathrm{m}$

$$z_b(x) = z_s(x) - 1000 + 500\,\mathrm{sin}(\frac{2\pi}{L}x)$$

Step 1 – Changes from Step0

- New directory, new names !

  e.g.: `square.grd -> mesh_B.grd`

- Make the mesh : three (at least) possibilities

- Right values for the different constants (which system of Units ?)

- Add the periodic boundary conditions

Step 1 – Mesh with ElmerGrid

Use the boundary mappings of ElmerGrid (in `mesh_B.grd`)

```
       …
Subcell Limits 1 = 0.0  2.00000000e+04
Subcell Limits 2 = -1000.0 0.0
       …
Geometry Mappings
! mode line limits(2) Np params(Np)
1 1 1000.0 1000.0  4 0.0 0.0  2.00000000e+04   -1.74537356e+02
1 0 1000.0 1000.0  4 0.0 0.0  2.00000000e+04   -1.74537356e+02
5 0 1000.0 1000.0  4 0.0  2.00000000e+04  1.0 500.0
End
```

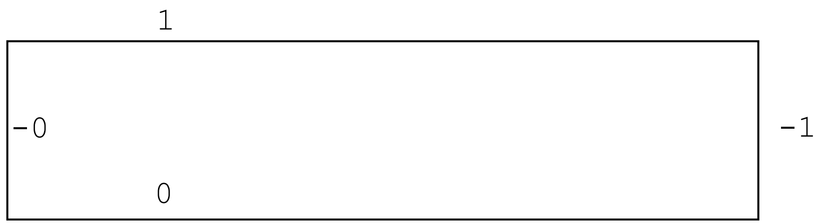
distance of the mapping effect (up and down)

line number : >0 horizontal , < 0 vertical

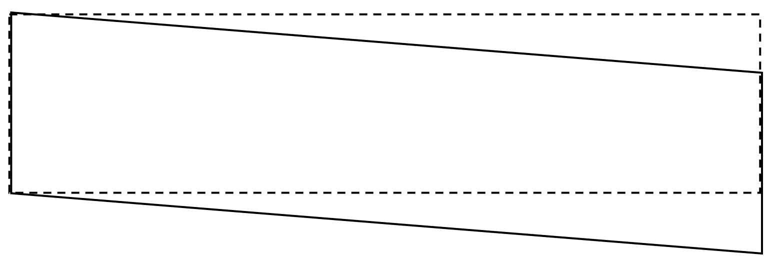
mode: 1 = piecewise linear $(x_0, dy_O),\ (x_1, dy_1)$

5 = sinus x_0, x_1, c, dy $\quad x = dy\,\sin(2\pi c(x - x_0)/(x_1 - x_0))$

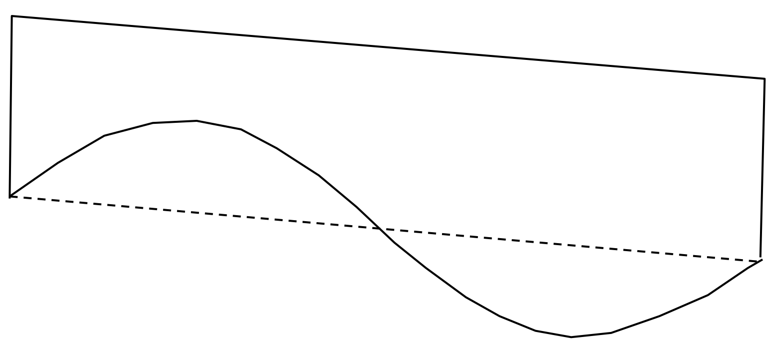
Step 1 – Mapping with ElmerGrid



1 1 1000.0 1000.0 4 0.0 0.0 2.00000000e+04   -1.74537356e+02



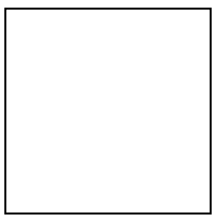
1 0 1000.0 1000.0 4 0.0 0.0 2.00000000e+04   -1.74537356e+02



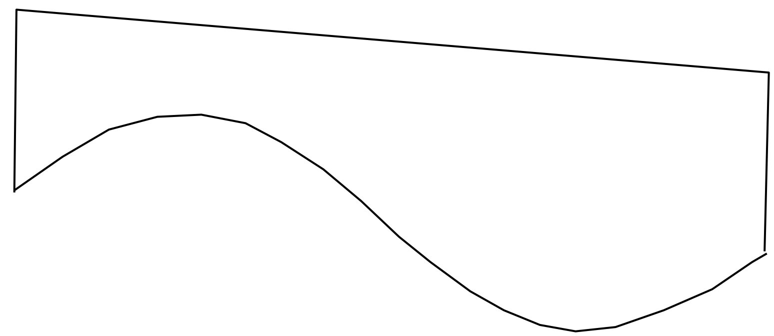
5 0 1000.0 1000.0 4 0.0 2.00000000e+04 1.0 500.0

Step 1 – Mesh with your own script

- Start from the square 1 x 1 of Step 0 and deform it to obtain the mesh of ISMIP B



The script of your choice !
(awk, f90, matlab, …)



- Open the file `mesh.nodes`

- Read (x, y)

- Make the scaling :

$$\left| \begin{array}{l} x \rightarrow L \times x \\ y \rightarrow z_b + (z_s - z_b) \times y \end{array} \right.$$

$$\begin{cases} z_s(x) = -x \tan(0.5^\circ) \\ z_b(x) = z_s(x) - 1000 + 500 \sin(\frac{2\pi}{L} x) \\ L = 20 \times 10^3 \text{ m} \end{cases}$$

- Re-write (x, y) in `mesh.nodes`

Step 1 – Mesh with your own script

Format of the file `mesh.nodes`:

`Node number , -1, x, y, z` (`z=0` in 2D)

Examples:

- f90: see file `MSH_ismipB.f90`

```
PROGRAM  MSH_ismipB
IMPLICIT NONE
REAL(KIND=8)  ::   x, y, z, xnew, ynew, zb, zs, L, Pi
REAL(KIND=8), ALLOCATABLE  :: xnode(:), ynode(:)
CHARACTER :: NameMsh*20
INTEGER :: NtN, i, j, N
      Pi = ACOS(-1.0)
      WRITE(*,*)'Name of the Elmer mesh directory ?'
      READ(*,*)NameMsh
      WRITE(*,*)'Lenght of the mesh L [m] :'
      READ(*,*)L
      OPEN(10,file=TRIM(NameMsh)//"/mesh.header")
        READ(10,1000)NtN
      CLOSE(10)
      ALLOCATE(xnode(NtN), ynode(NtN))
      OPEN(12,file=TRIM(NameMsh)//"/mesh.nodes")
      READ(12,*)(N, j, xnode(i), ynode(i), z, i=1,NtN)
      REWIND(12)
      DO N=1, NtN
        x = xnode(N)
        y = ynode(N)
        xnew = x * L
        zs = -xnew*TAN(0.5*Pi/180.0)
        zb = zs - 1000.0 + 500.0*SIN(2.0*Pi*xnew/L)
        ynew = zb + y * (zs - zb)
        WRITE(12,1200)N,j,xnew,ynew,z
      END DO
      DEALLOCATE(xnode, ynode)
1000 FORMAT(I6)
1200 FORMAT(i5,2x,i5,3(2x,e22.15))
END PROGRAM MSH_ismipB
```

- See other examples :

  awk: see file `script_awk`

  matlab: see file `dilat_mesh.m`

Step 1 – Mesh with your own mesher

Possible input format for ElmerGrid:

```
4)  .ansys     : Ansys input format
5)  .inp       : Abaqus input format by Ideas
6)  .fil       : Abaqus output format
7)  .FDNEUT    : Gambit (Fidap) neutral file
8)  .unv       : Universal mesh file format
9)  .mphtxt    : Comsol Multiphysics mesh format
10) .dat       : Fieldview format
11) .node,.ele: Triangle 2D mesh format
12) .mesh      : Medit mesh format
13) .msh       : GID mesh format
14) .msh       : Gmsh mesh format
```

Examples with gmsh: see file `mesh_Bgmsh.geo`

```
> gmsh mesh_Bgmsh.geo -2 -o mesh_Bgmsh.msh
> ElmerGrid 14 2 mesh_Bgmsh.msh -autoclean
```

→ mesh_Bgmsh/mesh.*

```
> ElmerGrid 14 3 mesh_Bgmsh.msh -autoclean
```

→ mesh_Bgmsh.ep (ElmerPost)

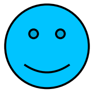
Step 1 – Elmer and Units

The choice of Units have to be coherent.
But you are free because the Stiff matrix is normalized.

For the Stokes problem, one should give values for:

- the density: $\rho \quad (= 910 \text{ kg/m}^3)$
- the gravity: $g \quad (= 9.81 \text{ m s}^{-2})$
- the viscosity: $\eta_0 \quad (\text{Pa s}^{1/n}) \qquad (1 \text{ Pa} = 1 \text{ kg s}^{-2} \text{ m}^{-1})$

kg – m – s [USI] : velocity in m/s and timestep in secondes 

kg – m – a : velocity in m/a and timesteps in years  $1 \text{ a} = 31\,557\,600 \text{ s}$

MPa – m – a : velocity in m/a and Stress in MPa 

(What I will use in the following)

Step 1 – Value of the ISMIP constants

For ISMIP tests A-D, the value for the constants are

- the density: $\rho = 910\ \mathrm{kg/m^3}$
- the gravity: $g = 9.81\ \mathrm{m\ s^{-2}}$
- the fluidity: $A = 10^{-16}\ \mathrm{Pa^{-3}\ a^{-1}}$

	USI kg - m - s		kg - m - a		MPa - m - a	
g =	9.81	m / s^2	9.7692E+15	m / a^2	9.7692E+15	m / a^2
ρ =	910	kg / m^3	910	kg / m^3	9.1380E-19	MPa m^{-2} a^2
A =	3.1689E-24	kg^{-3} m^3 s^5	1.0126E-61	kg^{-3} m^3 a^5	100	MPa^{-3} a^{-1}
η =	5.4037E+07	kg m^{-1} s$^{-5/3}$	1.7029E+20	kg m^{-1} a$^{-5/3}$	0.1710	MPa a$^{1/3}$

$$\eta_0 = B^{-1/n} = (2A)^{-1/n}$$

$$m = 1/n$$

$$\dot\gamma^2 \geq \dot\gamma_c^2$$

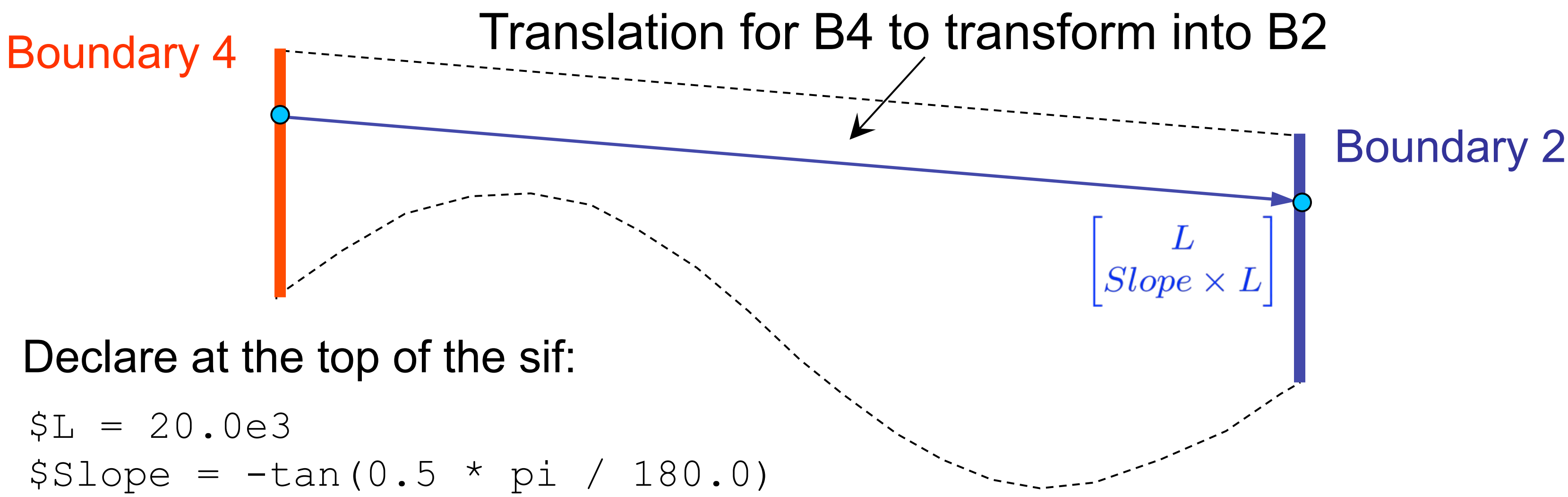
Step 1 – Value of the ISMIP constants

One can use MATC coding to get the correct value of the parameters

```
$yearinsec = 365.25*24*60*60
$rhoi = 900.0/(1.0e6*yearinsec^2)
$gravity = -9.81*yearinsec^2
$n = 3.0
$eta = (2.0*100.0)^(-1.0/n)
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Body Force 1
  Flow BodyForce 1 = Real 0.0
  Flow BodyForce 2 = Real $gravity
End

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Material 1
  Density = Real $rhoi

  Viscosity Model = String "power law"
  Viscosity = Real $eta
  Viscosity Exponent = Real $1.0/n
  Critical Shear Rate = Real 1.0e-10
End
```

Step 1 – Periodic boundary conditions



$$\begin{bmatrix} L \\ Slope \times L \end{bmatrix}$$

Declare at the top of the sif:

```
$L = 20.0e3
$Slope = -tan(0.5 * pi / 180.0)
        …
Boundary Condition 2
  Target Boundaries = 2
  Periodic BC = 4
  Periodic BC Translate(2) = Real $ (L) (Slope*L)
  Periodic BC Velocity 1  = Logical True
  Periodic BC Velocity 2  = Logical True
  Periodic BC Pressure  = Logical True
End
Boundary Condition 4
  Target Boundaries = 4
End
```

Nothing to declare for BC4 !

Step 2 – Add SaveData Solver (SaveLine)

Objective: save the variables on the top surface (ASCII matrix file)

- Add a new solver

```
Solver 2
  Exec Solver =  After All
  Procedure = File "SaveData" "SaveLine"
  Filename =  "ismip_surface.dat"
  File Append = Logical False
End
```

- Add this solver in the Equation Section

```
 Active Solvers(2) = 1 2
```

- Tell in which BC you want to save the data

```
Boundary Condition 3
  Target Boundaries = 3
  Save Line = Logical True
End
```

- Ordering of the variables: see file `ismip_surface.dat.names`

Step 2 – Add SaveData Solver (SaveLine)

SaveLine can also be used to save data at a ‘drilling site’ (a line which is not a boundary). Here, the data are saved at x = 10km.

- Add a new solver

```
Solver 2
  Exec Solver =  After All
  Procedure = File “SaveData” “SaveLine”
  Filename =  “ismip_drilling.dat”
  Polyline Coordinates(2,2) = Real $ (0.5*L) -1000. (0.5*L) 0.0
  File Append = Logical False
End
```

- Add this solver in the Equation Section

```
        Active Solvers(2) = 1 2
```

- And don’t forget to comment the `Save line = Logical True` in BC3

Step 3 – Add SaveScalars

SaveScalars allows to save scalars and derived quantities. Here, we will save:

1/ the volume of the domain (surface),
2/ the maximum value of the absolute horizontal velocity,
3/ the flux on the 3 boundaries 2, 3 and 4.
4/ the CPU time,
5/ the CPU memory

Step 3 – Add SaveScalars

-Add a new solver

```
Solver 3
  Exec Solver =  After TimeStep   !! For transient simulation
  Procedure = "SaveData" "SaveScalars"
  Filename = "ismip_scalars.dat"
  File Append = Logical True     !! For transient simulation
  Variable 1 = String "flow solution"
  Operator 1 = String "Volume"
  Variable 2 = String "Velocity 1"
  Operator 2 = String "max abs"
  Variable 3 = String "flow solution"
  Operator 3 = String "Convective flux"
  Operator 4 = String "cpu time"
  Operator 5 = String "cpu memory"
End
```

- Add this solver in the Equation Section

```
Active Solvers(3) = 1 2 3
```

- Tell at which boundaries you want to save the flux

```
Flux Integrate = Logical True
```

Step 4 – Add ComputeDevStress

Objective: compute the stress field as

$$\int_V S_{ij}\Phi \, \mathrm{d}V = 2\int_V \eta D_{ij}\Phi \, \mathrm{d}V$$

where D_{ij} and η are calculated from the nodal velocities using the derivative of the basis functions

- Add a Solver

```
Solver 2
  Equation = Sij
  Variable = -nooutput "Sij"
  Variable DOFs = 1
  Exported Variable 1 = Stress[Sxx:1 Syy:1 Szz:1 Sxy:1]
  Exported Variable 1 DOFs = 4
  Procedure = ”ElmerIceSolvers" "ComputeDevStress"

  Flow Solver Name = String “Flow Solution”

  Linear System Solver = Direct
  Linear System Direct Method = umfpack
End
```

Step 4 – Add ComputeDevStress

- Add this solver in the Equation Section

```
Active Solvers(4) = 1 2 3 4
```

-Add the 4 stress components in the periodic BC

```
Boundary Condition 2
…
  Periodic BC Sxx  = Logical True
  Periodic BC Syy  = Logical True
  Periodic BC Szz  = Logical True
  Periodic BC Sxy  = Logical True
End
```

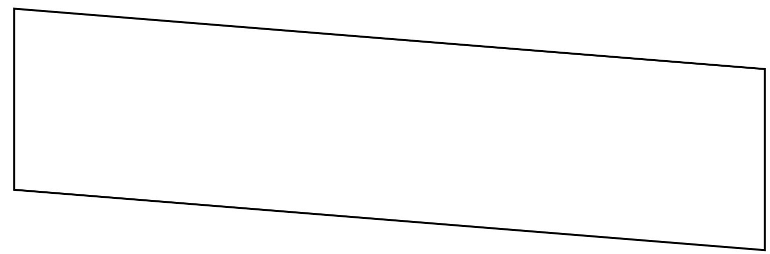
- Add in the material section:

```
Cauchy = Logical False
```

Step 5 – Move to ISMIP-HOM D020

Changes from B020:

- geometry of domain



$$\begin{cases} z_s(x,y) = -x\,\tan(0.1^\circ) \\ z_b(x,y) = z_s(x,y) - 1000 \end{cases}$$

→ modify the mesh and the Boundary Condition 2 (Easy !)

- boundary condition at the bedrock interface

$$\begin{cases} \tau_{nt} = \beta^2 u_t \\ u_n = \boldsymbol{u} \cdot \boldsymbol{n} = 0 \end{cases}$$

with $\beta^2(x) = 1000 + 1000\,\sin(\frac{2\pi}{L}x)$

in [Pa a m^{-1}] !

→ modify the Boundary Condition 1

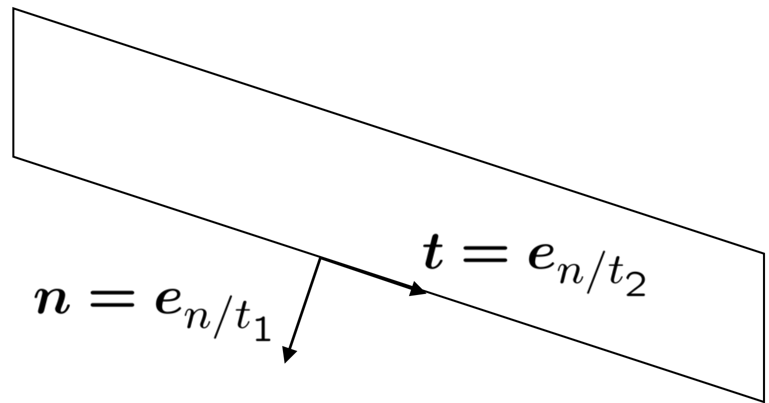
Step 5 – Move to ISMIP-HOM D020

Friction law in Elmer:

$$C_i u_i = \sigma_{ij} n_j \ (i = 1, 2)$$

$$\Rightarrow \quad C_t u_t = \sigma_{nt} \ ; \ C_n u_n = \sigma_{nn}$$

where $\boldsymbol{n}$ is the surface normal vector



Modification of the Boundary Condition 1:

- First Solution: MATC definition of Ct

```
Boundary Condition 1
  Target Boundaries = 1
  Flow Force BC = Logical True
  Normal-Tangential Velocity = Logical True

  Velocity 1 = Real 0.0e0

  Slip Coefficient 2 = Variable coordinate 1
    Real  MATC "1.0e-3*(1.0 + sin(2.0*pi* tx / L ))
End
```

Stress condition defined in a normal-tangential coordinate system

$u_n = 0$

$C_t = \ldots$

in [MPa a m^{-1}] !

Step 5 – Move to ISMIP-HOM D020

- Second Solution: User Function to define Ct

```
Boundary Condition 1
  …
  Slip Coefficient 2 = Variable coordinate 1
     Real Procedure "./ISMIP_D" "Sliding"
End
```

where Sliding is a User Function defined in the file `ISMIP_D.f90` (see next slide)

Compilation:

```
> elmerf90 ISMIP_D.f90 -o ISMIP_D
```

Step 5 – Move to ISMIP-HOM D020

```
FUNCTION Sliding ( Model, nodenumber, x) RESULT(C)
   USE Types

   IMPLICIT NONE
   TYPE(Model_t) :: Model
   INTEGER :: nodenumber, i
   REAL(KIND=dp) :: x, C, L
   LOGICAL :: FirstTime=.True.

   SAVE FirstTime, L

   IF (FirstTime) THEN
       FirstTime=.False.
       L = MAXVAL(Model % Nodes % x)
   END IF

   x = Model % Nodes % x(nodenumber)
   C  =  1000.0e-6_dp*(1.0_dp + SIN(2.0_dp * Pi * x/ L))
                         ! in MPa a /m
END FUNCTION Sliding
```

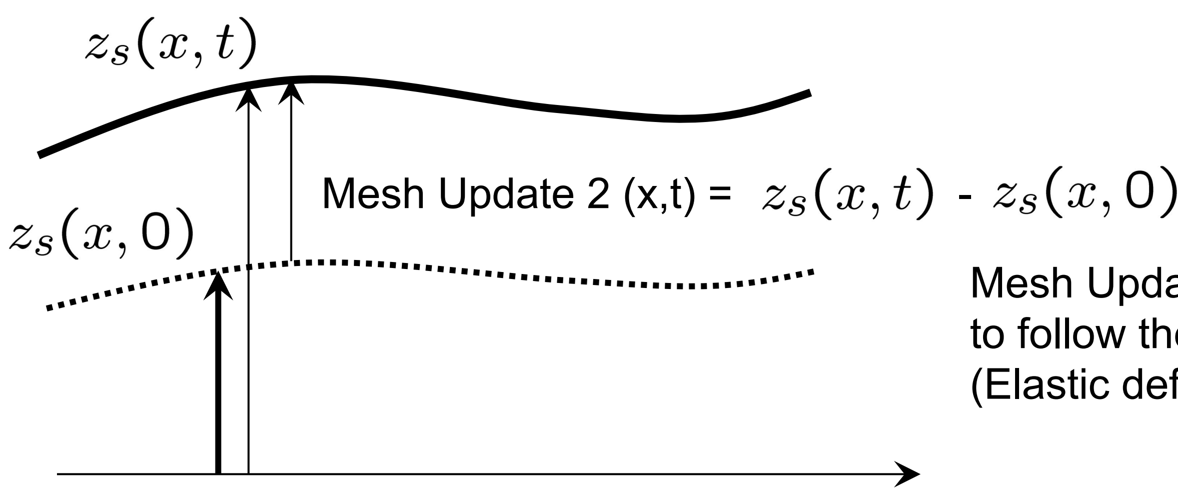
Step 6 – Move to prognostic B020

Move from a Diagnostic to a prognostic simulations:

- Steady to transient

- Add two solvers: the **free surface** and the **Mesh Update** solvers

$$\frac{\partial z_s}{\partial t} + u_x \frac{\partial z_s}{\partial x} - u_z = a$$



Mesh Update solver deforms the mesh to follow the moving boundary (Elastic deformation)

Step 6 – Steady to transient

The simulation Section has to be modified:

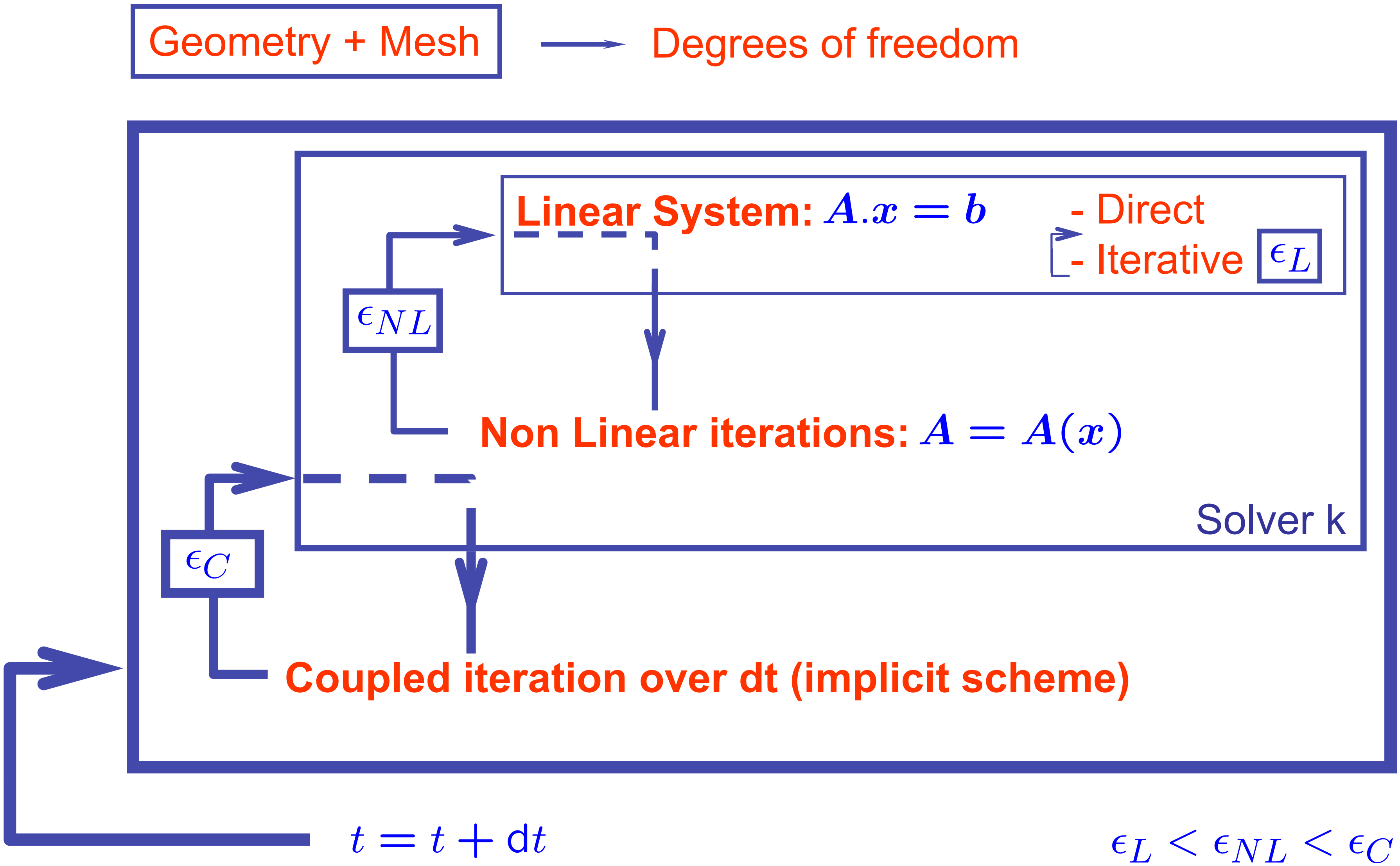
```
Simulation Type = Transient

Timestepping Method = “bdf”
BDF Order = 1
Output Intervals = 1
Timestep Intervals = 200
Timestep Sizes = 1.0

Steady State Min Iterations = 1
Steady State Max Iterations = 10
```

Timestepping Method = “bdf” → Backward Differences Formulae

Output Intervals = 1 → Save in .ep file

Steady State Max Iterations = 10 → To control the “implicity” of the solution over one time step (see example bellow).

Step 6 – Sketch of a transient simulation



Step 6 – Free surface Solver

The free surface solver only apply to the boundary 3 (top surface)

➡ Define a 2nd body which is the boundary 3.

```
Body 2
  Equation = 2
  Body Force = 2
  Material = 1
  Initial Condition = 2
End
```

where `Equation 2, Body Force 2 and Initial Condition 2` are defined for the free surface equation.

Tell in BC3 that this is the body 2:

```
Boundary Condition 3
  Target Boundaries = 3
        …
  !!! this BC is equal to body no. 2 !!!
  Body Id = 2
        …
End
```

Step 6 – Free surface Solver

Add the Free Surface Solver:

The minimum is presented here, you can add limits not to be penetrated by the free surface

```
Solver 2
   Equation = "Free Surface"
   Variable = String Zs
   Variable DOFs =  1
   Exported Variable 1 = String “Zs Residual"
   Exported Variable 1 DOFs = 1

   Procedure = "FreeSurfaceSolver" "FreeSurfaceSolver"
   Before Linsolve = “EliminateDirichlet” “EliminateDirichlet”

   Linear System Solver = Iterative
   Linear System Max Iterations = 1500
   Linear System Iterative Method = BiCGStab
   Linear System Preconditioning = ILU0
   Linear System Convergence Tolerance = Real 1.0e-5
   Linear System Abort Not Converged = False
   Linear System Residual Output = 1

   Steady State Convergence Tolerance = 1.0e-03

   Relaxation factor = Real 1.0
   Stabilization Method = Bubbles
   End
          …
```

Step 6 – Free surface Solver

Body Force 2:

```
Body Force 2
  Zs Accumulation Flux 1 = Real 0.0e0
  Zs Accumulation Flux 2 = Real 0.0e0
End
```

Equation 2:

```
Equation 2
  Active Solvers(1) = 2
  Flow Solution Name = String "Flow Solution"
  Convection = String Computed
End
```

Initial Condition 2: (tell that $z_s(x,0)$ = ordinate of the initial top surface)

```
Initial Condition 2
  Zs = Variable Coordinate 1
     Real MATC "tx*Slope"

End
```

OR

```
Initial Condition 2
  Zs = Variable Coordinate 1
     real Procedure "ElmerIceUSF" "ZsIni"
End
```

Step 6 – Mesh Update Solver

Add the Mesh Update Solver:

```
Solver 3
  Equation = “Mesh Update”
  Linear System Solver = “Direct”
  Linear System Direct Method = umfpack
  Steady State Convergence Tolerance = 1.0e-04
End
```

Material parameter for this solver:

```
   Mesh Youngs Modulus = Real 1.0
    Mesh Poisson Ratio = real 0.3
```

Force that Mesh Update 1 = 0 everywhere:

```
Body Force 1
   Flow BodyForce 1 = Real 0.0
   Flow BodyForce 2 = Real -9.7696e15  !MPa - a - m
   Mesh Update 1 = real 0.0
End
```

Step 6 – Mesh Update Solver

Boundary condition:

BC1:

```
Mesh Update 1 = real 0.0
Mesh Update 2 = real 0.0
```

BC2:

```
Periodic BC Mesh Update 2 = Logical True
Mesh Update 1 = real 0.0
```

BC3:

```
Mesh Update 1 = real 0.0
```

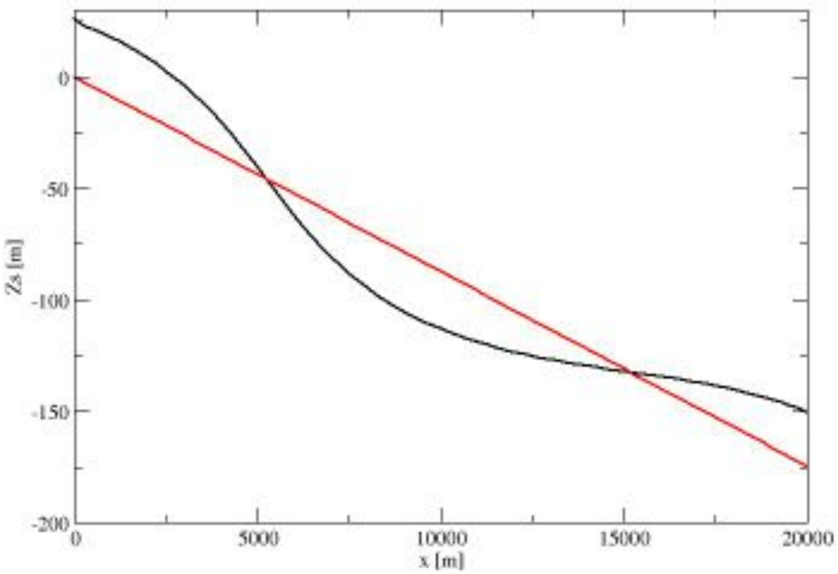
```
Mesh Update 2 = Variable Zs, Coordinate 1
  Real MATC "tx(0)-Slope*tx(1)"
```

OR

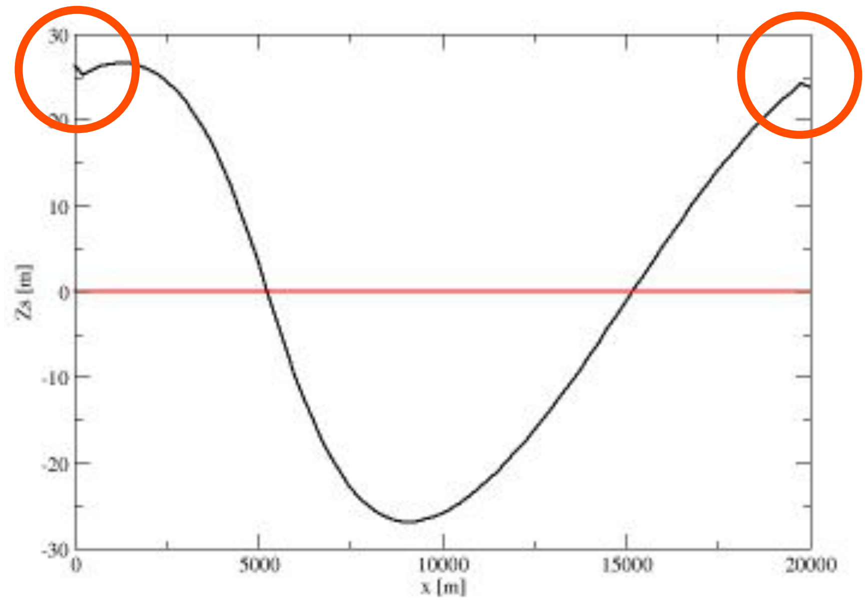
```
Mesh Update 2 = Variable Zs
  Real Procedure "ElmerIceUSF" "ZsMZsIni"
```

Step 6 – Results !

Comparison of the initial and steady surface of the prognostic run



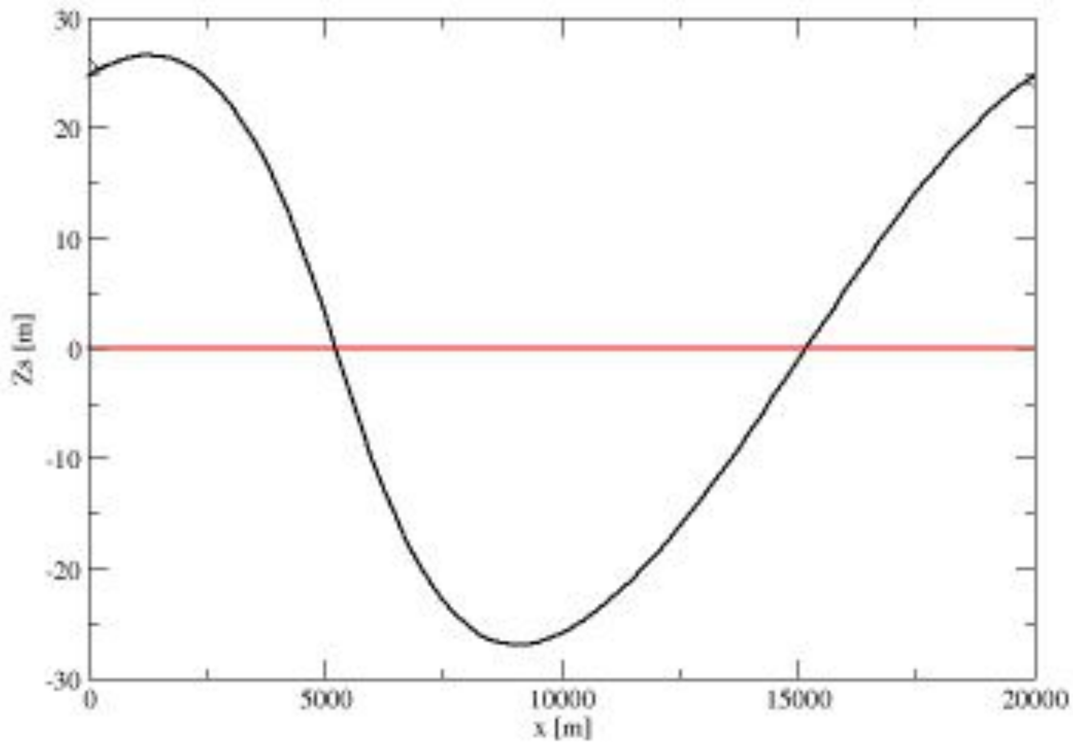
Not periodic



Step 6 – better results !

Turn the mesh so that zs = 0 (turn the gravity vector also !)
Force zs to be periodic

See Step6_hori



Step 7 – Move to prognostic D020

Merge Step 5 and Step 6 and it should work !