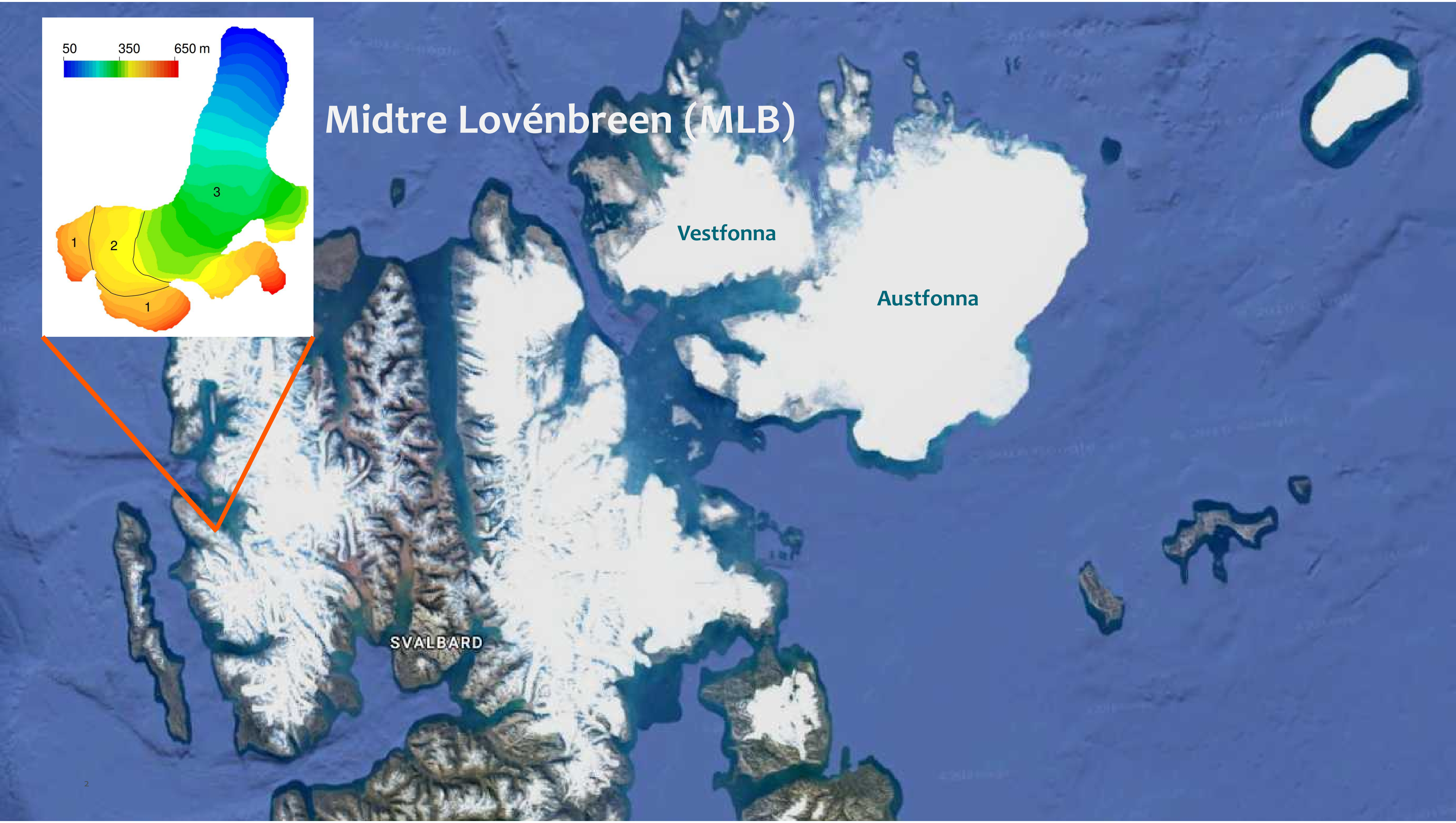
Arctic Glacier Example

Midtre Lovénbreen, Svalbard

Thomas Zwinger

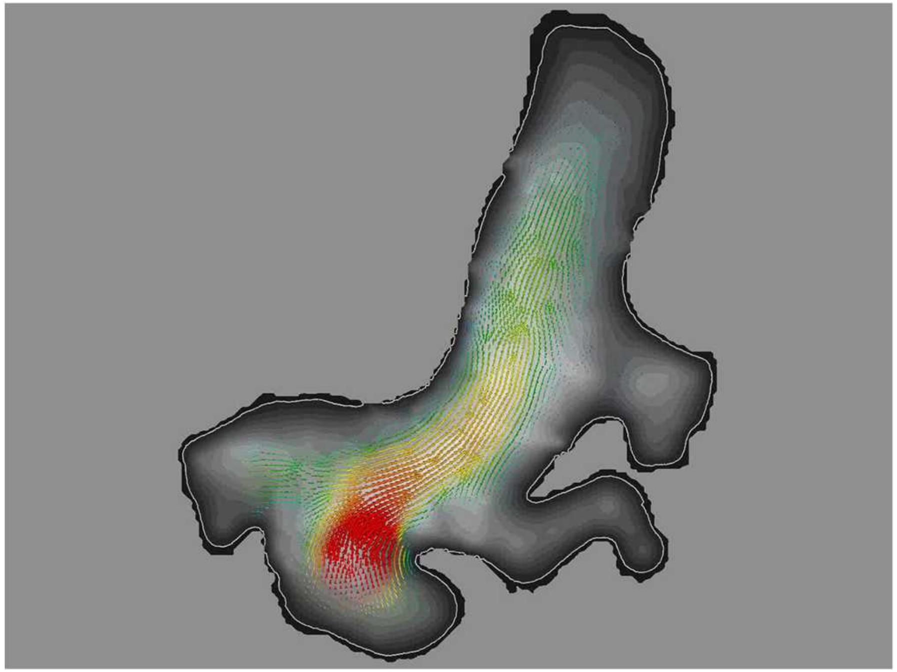
ElmerTeam

data contributions from Jack Kohler (NPI)


Midtre Lovénbreen (MLB)
50
350
650 m
1
2
3
1
Vestfonna
Austfonna
SVALBARD

Midtre Lovénbreen

- Midtre Lovénbreen (MLB) is a small glacier very close to the research station in Ny Ålesund (https://www.swisseduc.ch/glaciers/svalbard/midtre_lovenbreen/index-en.html)
- Very well monitored glacier
  - Part of World Glacier Monitoring Service (WGMS) (https://wgms.ch/products_ref_glaciers/midtre-lovenbreen-svalbard/)
- It is a cold-ice glacier in the lower parts, but in the upper parts temperate
- From 50 – 650 m a.s.l., ~5 km²
- Like most glaciers under constant retreat



Zwinger T. and J.C. Moore, 2009. *Diagnostic and prognostic simulations with a full Stokes model accounting for superimposed ice of Midtre Lovénbreen, Svalbard*, The Cryosphere, 3, 217-229, doi:10.5194/tc-3-217-2009

Reconstructing SMB: Midtre Lovénbreen, Svalbard

Pictures and data provided by Jack Kohler, NPI, NOR (2005 DEM from NERC)

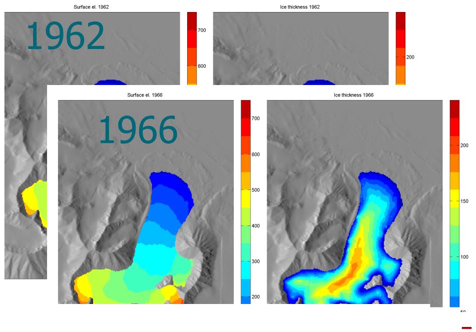
- DEM's obtained at different times
- Using 2 consecutive time-levels
  - Obtaining averaged DEM
    - $h_{2000} = (h_{2005} - h_{1995})/2$
  - and local elevation change

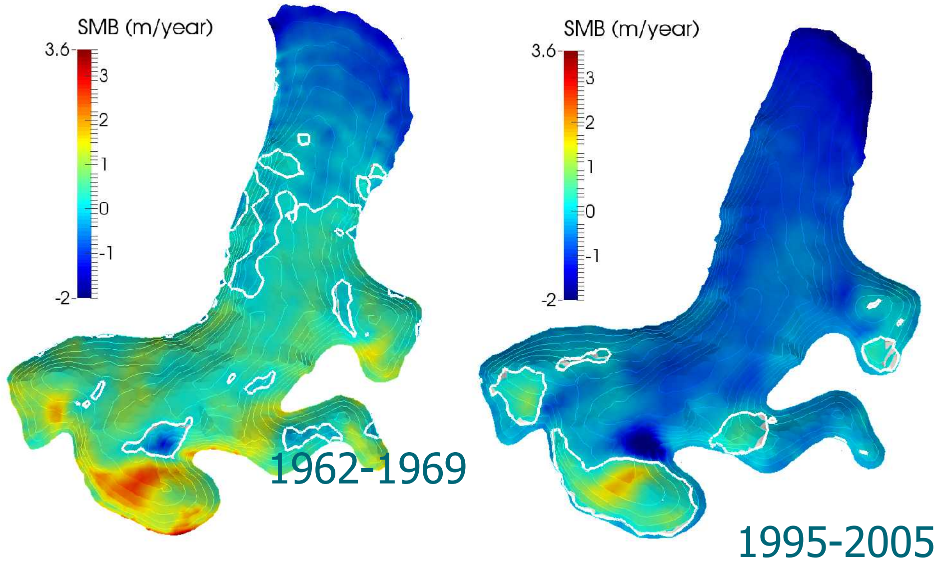
$$\frac{\partial h}{\partial t}|_{2000} = (h_{2005} - h_{1995})/11$$

- Elmer/Ice FS diagnostic simulations → $\boldsymbol{u} = (u,\, v,\, w)^{\mathrm{T}}$
- Spatial distribution of SMB:

$$\mathrm{SMB} = \left(\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} - w\right)$$

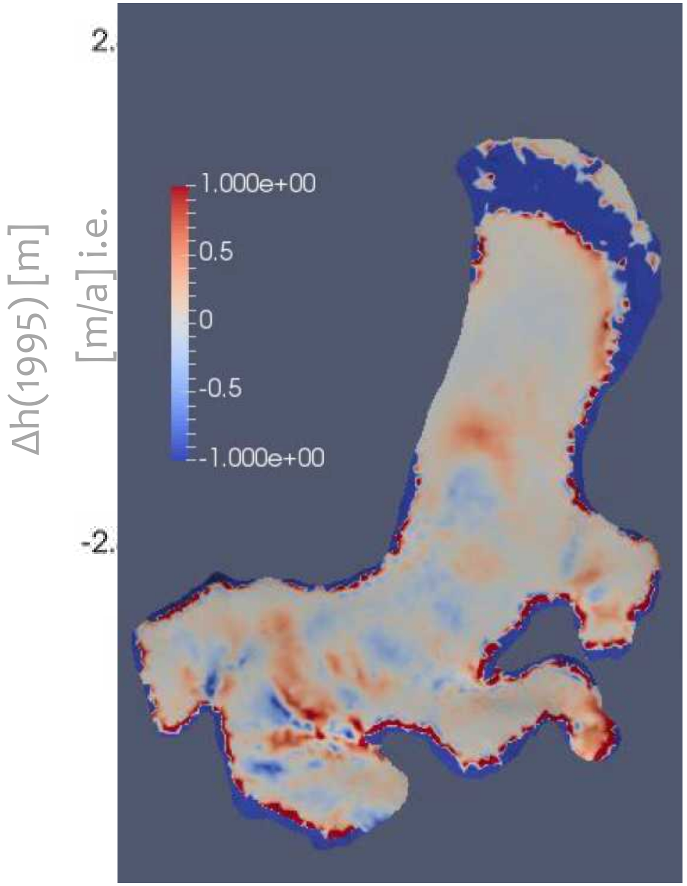
Välisuo, I., T. Zwinger and J. Kohler (2017): ***Inverse solution of surface mass balance of Midtre Lovénbreen, Svalbard***, Journal of Glaciology, 1-10, doi:10.1017/jog.2017.26.





Reconstructing Climate: Midtre Lovénbreen, Svalbard

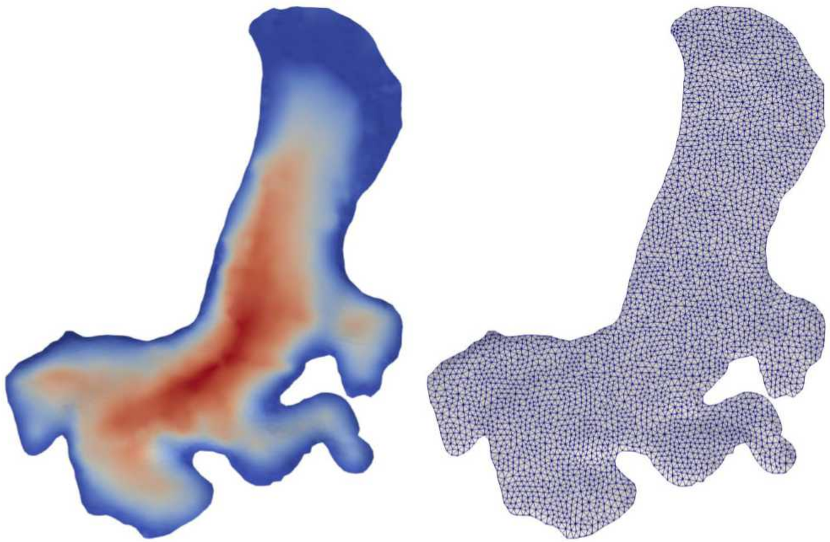
- This works nicely on MLB, because it is a relatively slow flowing glacier, dominated by SMB
    - Quite simple method, as it only needs a single diagnostic run for one time-interval
- <u>BUT</u>: It will fail on any ice-mass that shows significant amount of basal sliding
- Test run: using the 1977-1995 time interval
    - Using $SMB_{77\text{-}95}$ obtained with DEM method (to the right)
    - Starting with 77 DEM, integrating for 18 years, obtaining surface change by solving kinematic free surface equation
    - Comparison of computed 1977-95 result and 1995 DEM



Result produced by 2017 student class at Univ. Helsinki

This exercise

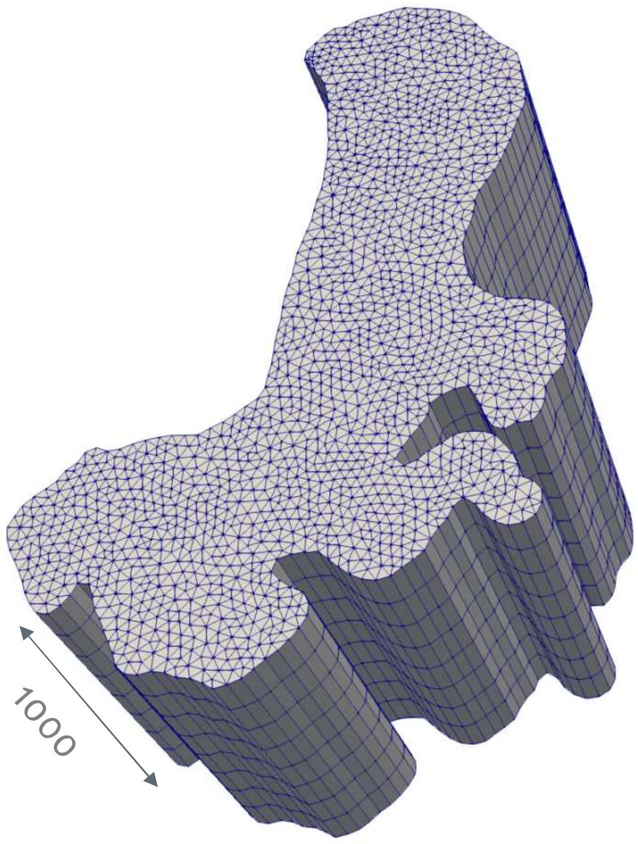
- We take the DEM of 1995
  - If running standard virtual machine settings, use the 75m DEM
- We will first run a diagnostic simulation on the given geometry
- Emphasis on some special features
  - 3D mesh generation using extrusion
  - Restart from 2D data
  - Utilizing extruded structure in mesh deformation
  - Vectorized & threaded version of Navier-Stokes
  - Block preconditioning
  - Semi-Lagrangian solver for purely advective transport i.e. of age
- Users are free to try out different things
  - Solution strategies
  - Parallel runs
  - …



Finalizing mesh using internal extrusion

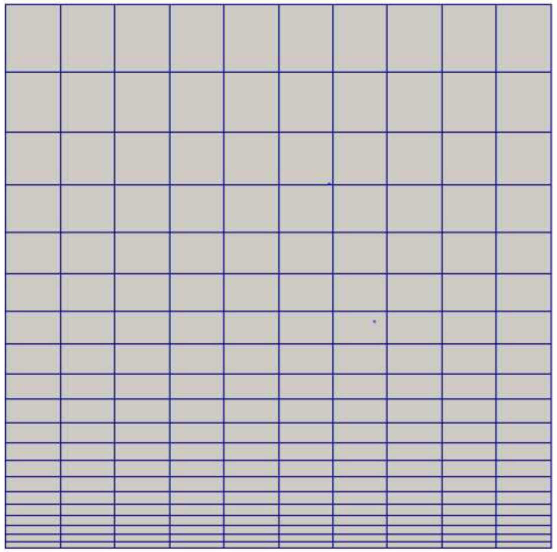
- The mesh is finalized in memory starting from 2D footprint
- Mind! Here the extruded height does not play any role
    - Mesh is further adapted to follow true bottom and top DEM

```
Simulation
   Extruded Mesh Levels = Integer 9
   Extruded Max Coordinate = Real 1000
```



Internal mesh extrusion

- Start from an initial 2D (1D) mesh and then extrude into 3D (2D)
  - Mesh density may be given a geometric ratio and even an arbitrary function
- Implemented also for partitioned meshes
  - Extruded lines belong to the same partition by construction!
- Effectively eliminates meshing bottle-necks
- Side boundaries get a BC constraint so that
  - 2D constraint BC = 1D constraint BC + offset
  - offset is set if the baseline BCs are preserved
- Top and bottom boundaries get the next free BC constraint indexes
  - Note that the BCs refer directly to the "Boundary Condition"
  - "Target Boundaries" is used only when reading in the mesh in the 1st place and they are not available any more at this stage



```
Extruded Mesh Levels = 21
Extruded Mesh Density = Variable Coordinate 1
Real MATC "1+10*tx"
```

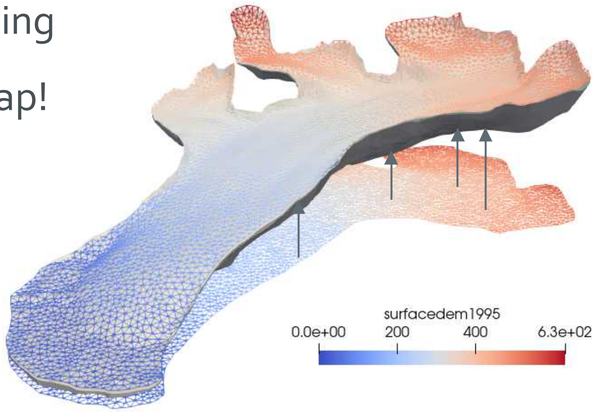
Restart from 2D data: Mesh2MeshSolver

- We can take 2D data and interpolate it to top/bottom layers of 3D mesh
  - 2D interpolation task with z-coordinate neglected
- Makes workflow easier since the data needs to be interpolated only once to an Elmer mesh
- 2D file is read in full to all processes
  - Same restart file can be used for any number of cores!
- **We have precomputed restart files for you!**

```
Solver 1
  Exec Solver = "before all"
  Equation = "InterpSolver"
  Procedure = "Mesh2MeshSolver" "Mesh2MeshSolver"
  ! Restart is here always from a serial mesh
  Mesh = -single $restartdir
  Restart File = $restartfile
  ! We use the primary 2D mesh with local copy
  Mesh Enforce Local Copy = Logical True
  ! These are the variables for restart
  Restart Position = Integer 0
  Restart Variable 1 = String "bedrockDEM"
  Restart Variable 2 = String "surfaceDEM1995"
  ! Ensures that we perform interpolation
  ! on plane
  Interpolation Passive Coordinate = Integer 3
End
```

Utilizing extruded structure: `StructuredMeshMapper`

- The shape of the mesh needs to be accommodated
  - Bottom of ice follows bedrock
  - Top of ice follows ice surface
- This could be done using generic 3D techniques
  - `MeshSolve` (version of linear elasticity equation)
  - Expensive and unnecessary!
- We can apply to each vertically extruded node 1D mapping
- Very cheap!

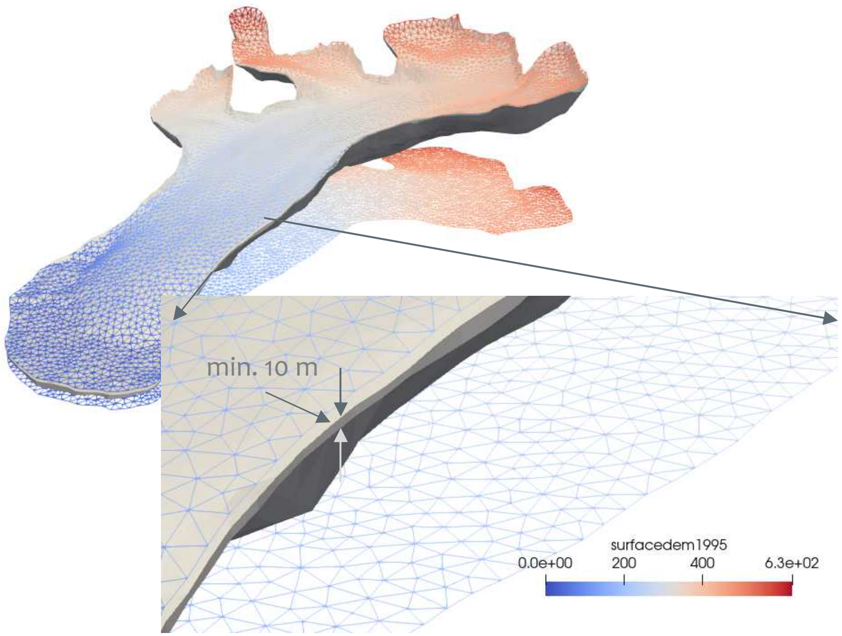



```
! Maps the constant-thickness mesh
! between given bedrock and surface topology
Solver 2
  Exec Solver = "before simulation"
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"
  Active Coordinate = Integer 3
  Displacement Mode = Logical False
  Correct Surface = Logical True
  Minimum Height = Real 10.0
  Correct Surface Mask = String "Glaciated"
  Dot Product Tolerance = 1.0e-3
  ! Allocate some fields here
  Variable = MeshUpdate
  Exported Variable 1 = "bedrockDEM"
  Exported Variable 1 Mask = String "BedRock"
  Exported Variable 2 = "surfaceDEM1995"
  Exported Variable 2 Mask = String "Surface"
End
```


Midtre Lovénbreen

Utilizing extruded structure: `StructuredMeshMapper`

- Imposing a minimum extrusion depth

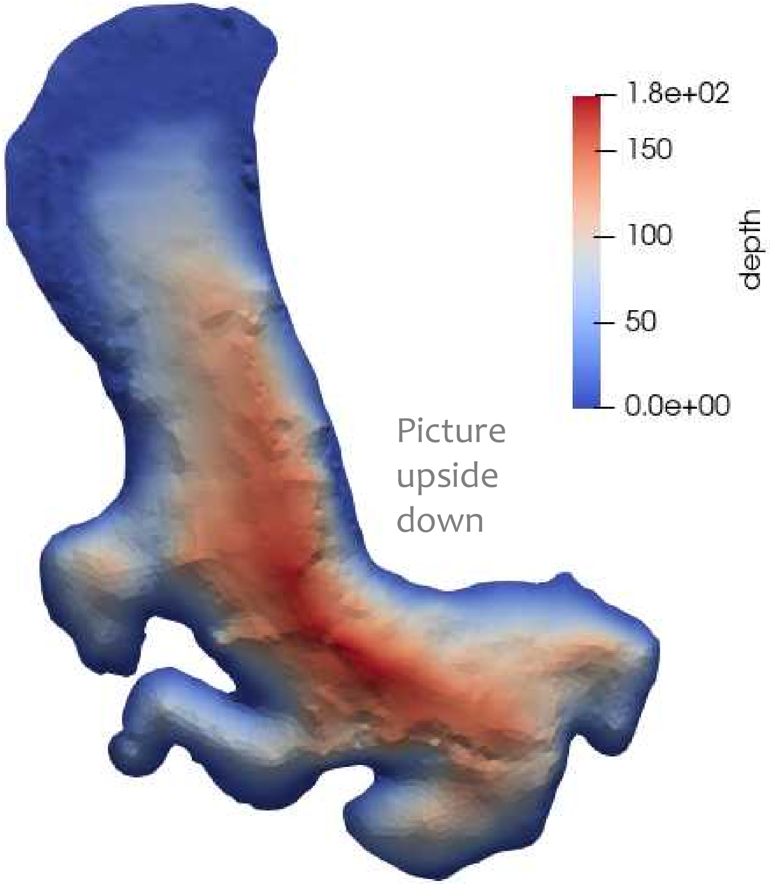



```
! Maps the constant-thickness mesh
! between given bedrock and surface topology
Solver 2
  Exec Solver = "before simulation"
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"
  Active Coordinate = Integer 3
  Displacement Mode = Logical False
  Correct Surface = Logical True
  Minimum Height = Real 10.0
  Correct Surface Mask = String "Glaciated"
  Dot Product Tolerance = 1.0e-3
  ! Allocate some fields here
  Variable = MeshUpdate
  Exported Variable 1 = "bedrockDEM"
  Exported Variable 1 Mask = String "BedRock"
  Exported Variable 2 = "surfaceDEM1995"
  Exported Variable 2 Mask = String "Surface"
End
```

Using extruded structure for mapping: `StructuredProjectToPlane`

- We may perform various operations
- Along the extruded 1D (vertical) lines
  - Computation of height & depth
  - Computation of integrals over the depth etc.

```
! Computes height and depth assuming an
! extruded mesh.
Solver 3
  Exec Solver = "before simulation"
  Equation = "HeightDepth"
  Procedure = "StructuredProjectToPlane"   \ ↓
          "StructuredProjectToPlane"       ←
  Active Coordinate = Integer 3
  Operator 1 = depth
  Operator 2 = height
End
```



Picture upside down

Optimized Stokes solver: `IncompressibleNSVec`

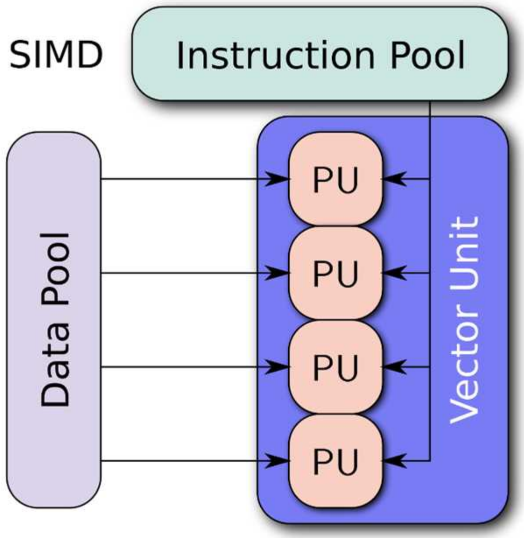
- Legacy module `FlowSolve` is one of the oldest in Elmer
  - Has a lot of extra baggage
  - Cannot ideally utilize modern CPU architectures
- `IncompressibleNSVec`:
  - Includes vectorization and threading
  - Takes use of code modernization in many places
  - Unfortunately vectorization and threading make the code less readable
- Performance boost depends heavily on the length of the vectors = Number of Gaussian integration points

IncompressibleNSVec.F90 - emacs

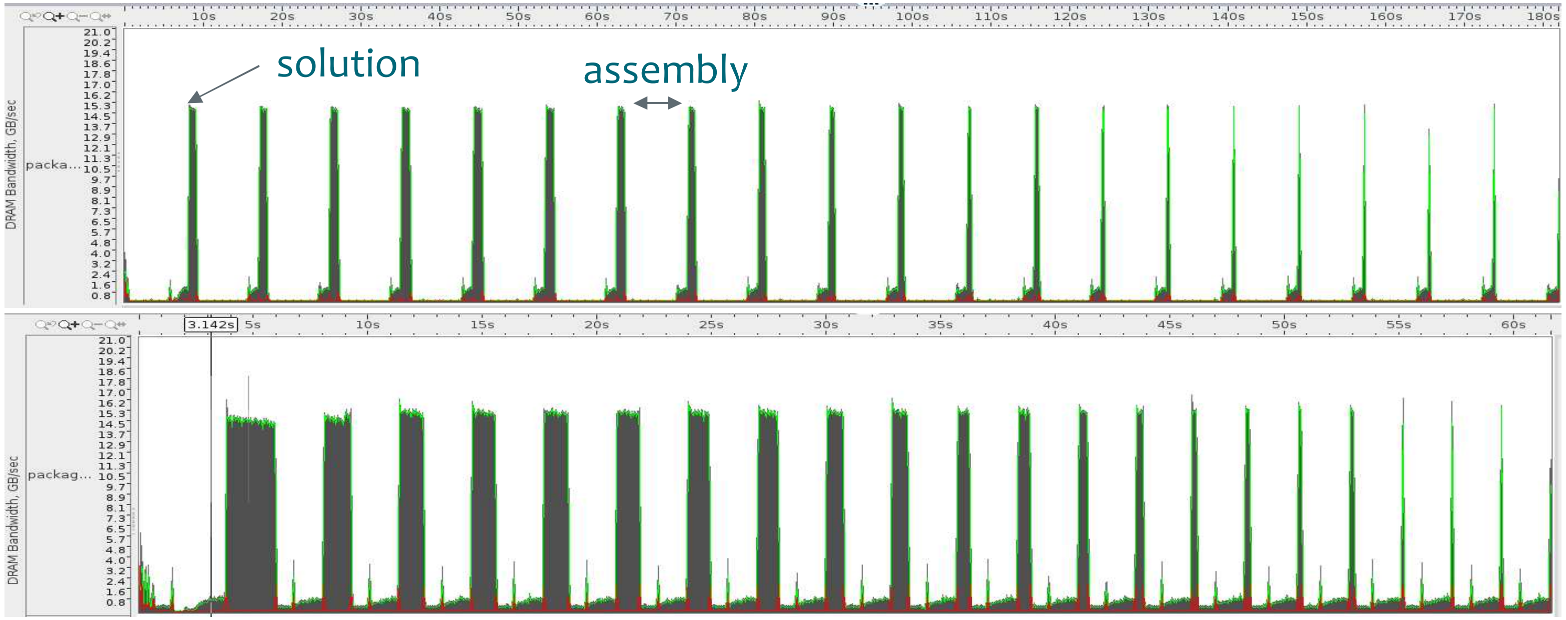
```
!            dBasisdxVec(1:ngp,1:ntot,i), dBasisdxVec(1:ngp,1:ntot,j), weight_c, stif
ford(1:ntot,1:ntot,i,j))
          END DO
        END DO
      END IF

      IF (GradPVersion) THEN
        ! b(u,q) = (u, grad q) part
          DO i = 1, dim
          CALL LinearForms_UdotV(ngp, ntot, elemdim, &
              BasisVec, dbasisdxvec(:,:,i), detJVec, stifford(:,:,i,dofs))
          StiffOrd(:,:,dofs,i) = transpose(stifford(:,:,i,dofs))
        END DO
      ELSE
         DO i = 1, dim
           CALL LinearForms_UdotV(ngp, ntot, elemdim, &
              dBasisdxVec(:, :, i), BasisVec, -detJVec, StiffOrd(:,:,i,dofs))
          StiffOrd(:,:,dofs,i) = transpose(stifford(:,:,i,dofs))
        END DO
      END IF

      ! Masses (use symmetry)
      ! Compute bilinear form G=G+(alpha u, u) = u .dot. (grad u)
      IF ( .NOT. StokesFlow ) THEN
        CALL LinearForms_UdotU(ngp, ntot, elemdim, BasisVec, DetJVec, VelocityMass, rhov
ec)

        ! Scatter to the usual local mass matrix
        DO i = 1, dim
          mass(i::dofs, i::dofs) = mass(i::dofs, i::dofs) + VelocityMass(1:ntot, 1:ntot)
        END DO
        !CALL LinearForms_UdotU(ngp, ntot, elemdim, BasisVec, DetJVec, PressureMass, -ka
ppavec)

        !mass(dofs::dofs, dofs::dofs) = mass(dofs::dofs, dofs::dofs) + PressureMass(1:nt
ot,1:ntot)
 U:---  IncompressibleNSVec.F90    28% L370   Git-devel  (F90 AC Abbrev)
```

Optimized Stokes solver: `IncompressibleNSVec`

- New CPU architectures use vector units (SIMD) to do fast computations (AVX2/AVX512)
- Cache misses (=memory performance) governed by data-layout
  - If you (on an Intel chip) ignore this, you easily loose ¾ of your performance by not utilizing whole cache line
- Until quite recently, assembly procedures in Elmer did not utilize SIMD and did not have a cache-friendly data layout, neither where they threaded (OMP)
- `IncompressibleNSVec` does!
- Interface to block-preconditioner functionality to increase solution efficiency
  - Or to allow for Krylov methods at all
- SIMD first step to enable code for accelerators



Optimized Stokes solver: `IncompressibleNSVec`



Mind the different timescales!

Comparison vectorised/legacy Solver using Intel VTune

Optimized Stokes solver: `IncompressibleNSVec`

- We have to specify that this is a Stokes model
  - Inertia terms neglected
- Number of integration points affects the accuracy of discretization
  - Has significant effect on performance!
- We may use different solution techniques for linear solver
  - Iterative method
  - Direct method
  - Block preconditioning (next topic)

```
Solver 4
 Equation = "Stokes-Vec"
 Procedure = "IncompressibleNSVec" "IncompressibleNSSolver"
 Flow Model = Stokes
 ! 1st iteration viscosity is constant
 Constant-Viscosity Start = Logical True
 ! Accuracy of numerical integration (on wedges)
 Number of Integration Points = Integer 44 ! 21, 28, 44, 64
 ! Iterative approach:
 Linear System Solver = Iterative
 Linear System Iterative Method = "GCR"
 Linear System Max Iterations = 500
 Linear System Convergence Tolerance = 1.0E-08
 Linear System Preconditioning = "ILU1"
 Linear System Residual Output = 10
 ! Direct approach (as alternative to above):
 !Linear System Solver = Direct
 !Linear System Direct Method = MUMPS
 !Non-linear iteration settings:
 Nonlinear System Max Iterations = 50
 Nonlinear System Convergence Tolerance  = 1.0e-5
 Nonlinear System Newton After Iterations = 10
 Nonlinear System Newton After Tolerance = 1.0e-1
 Nonlinear System Consistent Norm = True
 ! Nonlinear System Relaxation Factor = 1.00
End
```

Optimized Stokes solver: `IncompressibleNSVec`

- We can start with constant viscosity
  - Eliminates need for initial guess – takes value of `Viscosity` in `Material`
- Nonlinear solver takes use of Newton linearization
- Starts with Picard iteration that has larger radius of convergence

```
Solver 4
 Equation = "Stokes-Vec"
 Procedure = "IncompressibleNSVec" "IncompressibleNSSolver"
 Flow Model = Stokes
 ! 1st iteration viscosity is constant
 Constant-Viscosity Start = Logical True
 ! Accuracy of numerical integration (on wedges)
 Number of Integration Points = Integer 44 ! 21, 28, 44, 64
 ! Iterative approach:
 Linear System Solver = Iterative
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 !Non-linear iteration settings:
 Nonlinear System Max Iterations = 50
 Nonlinear System Convergence Tolerance  = 1.0e-5
 Nonlinear System Newton After Iterations = 10
 Nonlinear System Newton After Tolerance = 1.0e-1
 Nonlinear System Consistent Norm = True
 ! Nonlinear System Relaxation Factor = 1.00
End
```

Optimized Stokes solver: `IncompressibleNSVec`

- **`Material`** section stays quite the same as in legacy solver
- Remember from previous slide: start from constant, Newtonian Viscosity (here $1\ \mathrm{MPa\ a}^{-1}$) using keyword

```
Constant-Viscosity Start = Logical True
```

```
Material 1
 Name = "Ice"
 Density = Real $rhoi
 ! First round viscosity with Newtonian fluid
 ! happens to give velocities of proper size
 Viscosity = Real 1.0
 ! Non-Newtonian viscosity
 Viscosity Model = String Glen
 Glen Exponent = Real 3.0 !(yes, you may also try 4.0)
 Critical Shear Rate =  Real 1.0E-10
 ! Paterson value in MPa^-3a^-1
 Limit Temperature = Real -10.0
 Rate Factor 1 = Real $A1
 Rate Factor 2 = Real $A2
 Activation Energy 1 = Real $Q1
 Activation Energy 2 = Real $Q2
 Glen Enhancement Factor = Real 1.0
 Relative Temperature = Real $Tc
End
```

Advecting scalars with ice: ParticleAdvector

- Uses ability to follow particles in the mesh
  - Initially implemented for true physical particles
- Particles are made to travel backward in time along the flowlines
- Values may be integrated along the path or registered at the initial location


```
Solver 5
  Equation = ParticleAdvector
  Procedure = "ParticleAdvector" "ParticleAdvector"
  ! Initialize particles at center of elements
  Advect Elemental = Logical True
  ! Timestepping strategy
  Particle Dt Constant = Logical False
  Max Timestep Intervals = Integer 1000
  Timestep Unisotropic Courant Number = 0.25
  Max Timestep Size = 1.0e3
  Max Integration Time = Real 1.0e4
  ! Integration forward in time
  Runge Kutta = Logical False
  Velocity Gradient Correction = Logical True
  Velocity Variable Name = String "Flow Solution"
  ! The internal variables for this solver
  Variable 1 = String "Particle Distance"
  Variable 2 = String "Particle Time Integral"
  ! The field variables being advected
  Variable 3 = String "Coordinate 1"
  Result Variable 3 = String "Advected Z"
End
```

 Midtre Lovénbreen

Running initial case

- In serial:

```
ElmerSolver mlb.sif
```

- In parallel, here with 2 processes:

```
ElmerGrid 2 2 outline62_lc50 -partdual -metiskway 2
```

```
mpirun -np 2 ElmerSolver_mpi mlb.sif
```

- Try both of the above and check the timings at the end of the run

Postprocessing

- Load the case into `ParaView`
- Display the advected properties
    - which also provide a nice way to determine which tributary the ice comes from
    - This structure is also reflected in the surface of the glacier (compare to picture)

Picture: Midtre Lovénbreen in 1999 (taken by Michael Hambrey)
Source: https://wgms.ch/products_ref_glaciers/midtre-lovenbreen-svalbard/

Block-preconditioner in `IncompressibleNSVec`

- In parallel runs a central challenge is to have good **preconditioners** that work in parallel
- This problem is increasingly difficult for PDEs with vector fields
  - Navier-Stokes, elasticity, acoustics,…
  - Strongly coupled multi-physics problems
- **Preconditioner** need not to be just a matrix, it **can be a procedure!**
- Use as **block-preconditioner** a procedure where the components are solved one-by-one and the solution is used as a search direction in an outer Krylov method
- Number of outer iterations may be shown to be bounded
- Individual blocks may be solved with optimally scaling methods
  - E.g. multilevel methods

What is a preconditioner ?:

Instead of solving

$$\mathbf{K} \cdot \boldsymbol{x} = \boldsymbol{b}$$

Identify a preconditioner, $\mathbf{P}$,

which makes solution of

$$\mathbf{K}\mathbf{P}^{-1} \cdot \boldsymbol{x}' = \boldsymbol{b}$$

with $\boldsymbol{x}' = \mathbf{P} \cdot \boldsymbol{x}$

(much) easier than the original problem.

Block-preconditioner in `IncompressibleNSVec`

- Utilizing block-preconditioner

```
Solver 4
 Equation = "Stokes-Vec"
 Procedure = "IncompressibleNSVec" "IncompressibleNSSolver"
 Flow Model = Stokes
 ! 1st iteration viscosity is constant
 Constant-Viscosity Start = Logical True
 ! Accuracy of numerical integration (on wedges)
 Number of Integration Points = Integer 44 ! 21, 28, 44, 64
 ! Iterative approach using BPC:
 include "linsys/block4_gcr.sif"
!Non-linear iteration settings:
 Nonlinear System Max Iterations = 50
 Nonlinear System Convergence Tolerance  = 1.0e-5
 Nonlinear System Newton After Iterations = 10
 Nonlinear System Newton After Tolerance = 1.0e-1
 Nonlinear System Consistent Norm = True
 ! Nonlinear System Relaxation Factor = 1.00
End
```

- Given a block system:

$$\begin{bmatrix} \mathbf{K}_{11} & \cdots & \mathbf{K}_{1N} \\ & \cdots & \\ \mathbf{K}_{N1} & \cdots & \mathbf{K}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_N \end{bmatrix}$$

- Preconditioner is operator which produces new search directions, $\boldsymbol{s}$, for $\boldsymbol{x}$
- Block-Gauss-Seidel or Block Jacobi

$$\mathrm{P} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{0} & \cdots \\ \cdots & & & \end{bmatrix} \qquad \mathrm{P} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{K}_{22} & \mathbf{0} & \cdots \\ \cdots & & & \end{bmatrix}$$

- Minimization of residual

$$r = \|\boldsymbol{b} - \mathbf{K} \cdot \boldsymbol{x}^{(k)}\|$$

with outer GCR loop over space

$$\mathcal{V}_k = \mathbf{x}^{(0)} + \mathrm{span}\{\mathbf{s}^{(1)}, \mathbf{s}^{(2)}, \ldots, \mathbf{s}^{(k)}\}$$

Block-preconditioner in `IncompressibleNSVec`

- Recommended "natural" outer iterative method is GCR

$k = 0$

$\mathbf{r}^{(k)} = \mathbf{f} - \mathbf{K}\mathbf{u}^{(k)}$

while $(\|\mathbf{r}^{(k)}\| < TOL\|\mathbf{f}\|$ and $k < m)$

$\quad \boxed{\begin{array}{l}\text{Generate the search direction } \mathbf{s}^{(k+1)} \\ \mathbf{v}^{(k+1)} = \mathbf{K}\mathbf{s}^{(k+1)}\end{array}}$

$\quad$ do $j = 1, k$

$\qquad \mathbf{v}^{(k+1)} = \mathbf{v}^{(k+1)} - \langle \mathbf{v}^{(j)}, \mathbf{v}^{(k+1)} \rangle \mathbf{v}^{(j)}$

$\qquad \mathbf{s}^{(k+1)} = \mathbf{s}^{(k+1)} - \langle \mathbf{v}^{(j)}, \mathbf{v}^{(k+1)} \rangle \mathbf{s}^{(j)}$

$\quad$ end do

$\quad \mathbf{v}^{(k+1)} = \mathbf{v}^{(k+1)} / \|\mathbf{v}^{(k+1)}\|$

$\quad \mathbf{s}^{(k+1)} = \mathbf{s}^{(k+1)} / \|\mathbf{v}^{(k+1)}\|$

$\quad \mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \langle \mathbf{v}^{(k+1)}, \mathbf{r}^{(k)} \rangle \mathbf{s}^{(k+1)}$

$\quad \mathbf{r}^{(k+1)} = \mathbf{r}^{(k)} - \langle \mathbf{v}^{(k+1)}, \mathbf{r}^{(k)} \rangle \mathbf{v}^{(k+1)}$

$\quad k = k + 1$

end while

```
Linear System Solver = "Block"
Block Gauss-Seidel = Logical True
Block Matrix Reuse = Logical False
Block Scaling = Logical False
Block Preconditioner = Logical True
! Default is [1 2 3 4]
Block Structure(4) = Integer 1 2 3 4
!  Block Order(2) = Integer 2 1
! Linear System Scaling = False
! Linear system solver for outer loop
!----------------------------------------
  Outer: Linear System Solver = "Iterative"
  Outer: Linear System Iterative Method = GCR
  Outer: Linear System GCR Restart =  250
  Outer: Linear System Residual Output = 1
  Outer: Linear System Max Iterations = 200
  Outer: Linear System Abort Not Converged = False
  Outer: Linear System Convergence Tolerance = 1e-8
```

Block-preconditioner in `IncompressibleNSVec`

- Stokes problem block-structure

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} \begin{bmatrix} V \\ P \end{bmatrix} = \begin{bmatrix} F \\ G \end{bmatrix}$$

from stabilization

- Optimal pre-conditioner with Pressure-Schur complement, $\mathbf{Q}$,

$$\mathbf{P} = \begin{bmatrix} \mathbf{A} & \mathbf{B}^{\mathbf{T}} \\ \mathbf{0} & \mathbf{Q} \end{bmatrix}$$

  - Either split velocity block, **A**, into 3x3 (recommended!)

    `Block Structure(4)=Integer 1 2 3 4`

  - Or as one

    `Block Structure(4)=Integer 1 1 1 4`

```
Linear System Solver = "Block"
Block Gauss-Seidel = Logical True
Block Matrix Reuse = Logical False
Block Scaling = Logical False
Block Preconditioner = Logical True
! Default is [1 2 3 4]
Block Structure(4) = Integer 1 2 3 4
!  Block Order(2) = Integer 2 1
! Linear System Scaling = False
! Linear system solver for outer loop
!-----------------------------------------
  Outer: Linear System Solver = "Iterative"
  Outer: Linear System Iterative Method = GCR
  Outer: Linear System GCR Restart =  250
  Outer: Linear System Residual Output = 1
  Outer: Linear System Max Iterations = 200
  Outer: Linear System Abort Not Converged = False
  Outer: Linear System Convergence Tolerance = 1e-8
```

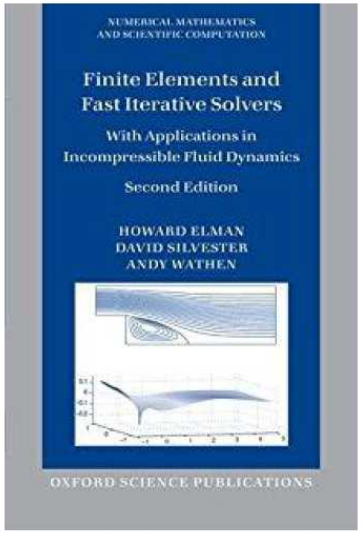
Block-preconditioner in `IncompressibleNSVec`

- Inner solutions (of blocks)
- Blocks 1,2,3 here associated with velocity components 1,2,3

$$\mathbf{P} = \begin{bmatrix} \boldsymbol{A}_1 & \mathbf{0} & \mathbf{0} & \\ \boldsymbol{A}_{12} & \boldsymbol{A}_2 & \mathbf{0} & \mathbf{B}^{\mathrm{T}} \\ \boldsymbol{A}_{31} & \boldsymbol{A}_{23} & \boldsymbol{A}_3 & \\ & \mathbf{0} & & \mathbf{Q} \end{bmatrix}$$

- Block 4 associated with pressure (preconditioned with scaled mass matrix is suggested by Elman)

$$\mathbf{Q} = \mu^{-1}\mathbf{1}$$




```
block 11: Linear System Convergence Tolerance = $blocktol
block 11: Linear System Solver = "iterative"
block 11: Linear System Scaling = false
block 11: Linear System Preconditioning = ilu
block 11: Linear System Residual Output = 100
block 11: Linear System Max Iterations = 500
block 11: Linear System Iterative Method = idrs

block 22: Linear System Convergence Tolerance = $blocktol
block 22: Linear System Solver = "iterative"
block 22: Linear System Scaling = false
block 22: Linear System Preconditioning = ilu
block 22: Linear System Residual Output = 100
block 22: Linear System Max Iterations = 500
block 22: Linear System Iterative Method = idrs

block 33: Linear System Convergence Tolerance = $blocktol
block 33: Linear System Solver = "iterative"
block 33: Linear System Scaling = false
block 33: Linear System Preconditioning = ilu
block 33: Linear System Residual Output = 100
block 33: Linear System Max Iterations = 500
block 33: Linear System Iterative Method = idrs

block 44: Linear System Convergence Tolerance = $blocktol
block 44: Linear System Solver = "iterative"
block 44: Linear System Scaling = true
block 44: Linear System Preconditioning = ilu
block 44: Linear System Residual Output = 100
block 44: Linear System Max Iterations = 500
block 44: Linear System Iterative Method = idrs
```

Run case with BPC – compare to other methods

- Running the initial case (cl75)
  - You may try to run the larger cases (cl50, cl25) – might exceed available memory
- Altering number of integration points
  - Does it have an affect on simulation results: ...,21, 28, 44, 64,..?
- Trying out different linear system strategies
  - GCR vs. block precondtioner vs. direct solver
  - `mlb_linsys.sif` contains linear system recipes with (for GCR, MUMPS and BPC)

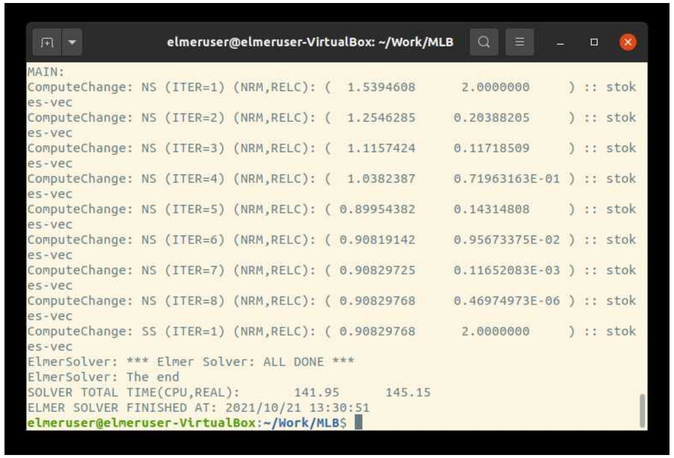
    `include linsys/`*`method`*`.sif`
- Trying effect of Courant number in particle advection
- NB: You may want to turn off `ParticleAdvector` if testing Stokes by adding

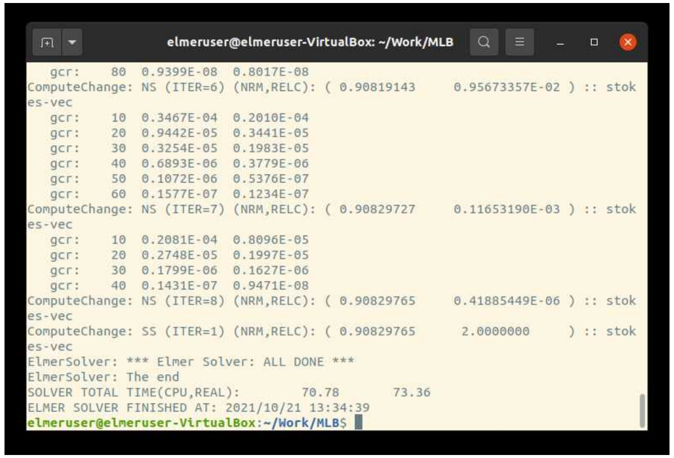
```
Exec Solver = never
```

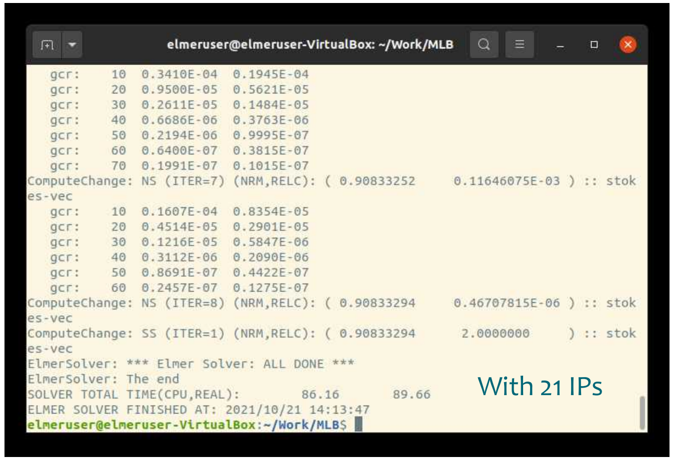
into the corresponding Solver section

Run case with BPC – compare to others

`linsys/mumps.sif` `linsys/gcr.sif` `linsys/block4_idrs.sif`







- Mind, that BPC on small cases (here we ran the 75 m mesh) are not necessarily performing faster than the same GCR with a simple pre-conditioner (e.g., ILU)
- Nevertheless, for many cases, Krylov methods with not-optimal pre-conditioners do not work at all
- Direct methods (MUMPS, cPardiso) stop scaling beyond a few hundred cores

Computing steady state mass balance

- Add solver for computing **emergence velocity**

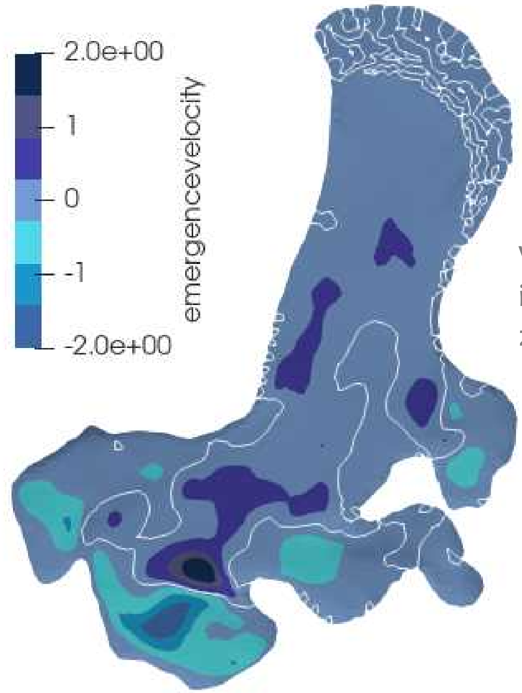
```
! Computing emergence velocity
Solver 6
  Equation = "SMB"
  Exec Solver = "After Timestep"
  Procedure = "ElmerIceSolvers" "GetEmergenceVelocity"
  Variable = -dofs 1 EmergenceVelocity
End
```

  - Needs also surface normal to be computed
  - Needs to be run on free-surface boundary, only

```
Body 2
  Name = "surface"
  Equation = 2
  Material = 1 ! Not used, but needed
End
Equation 2
  Name = "Surface Equations"
  Active Solvers(1) = 6
  Convection = String "Computed"
  Flow Solution Name = String "Flow Solution"
End
Boundary Condition 4 !free surface boundary
  Body ID = 2
…
End
```

Run with `mpirun -np 2 ElmerSolver mlb_emergence.sif`

$$v_{em} = -u\frac{\partial h}{\partial x} - v\frac{\partial h}{\partial y} + w$$



White line indicates zero SMB (ELA)

Prognostic relaxation run

- The kinematic BC at the free surface

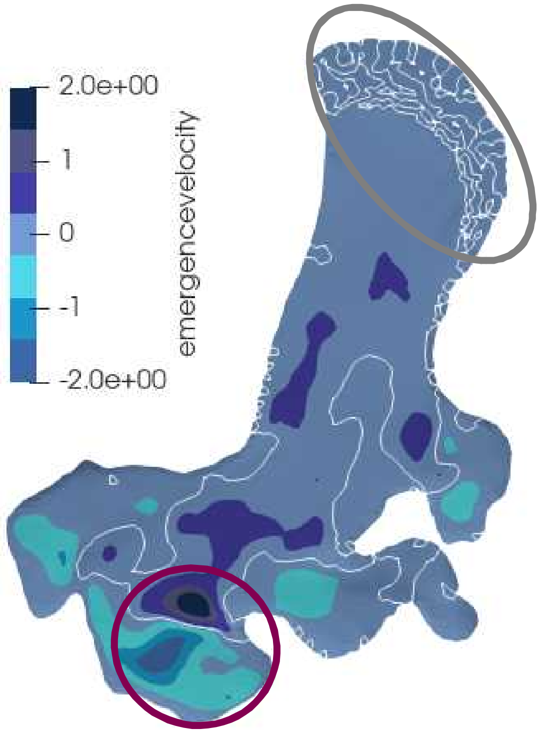
$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} - w = a_{\perp}$$

- In case of equilibrium ($\frac{\partial h}{\partial t} \approx 0$) reduces to

$$u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} - w = -v_{em} = a_{\perp equ.}$$

- That means, if we restart a prognostic run **using the negative emergence velocity as SMB**, we can use it as a relaxation run to even out flaws in the DEMs or the interpolation of those
  - o There is an obvious issue (most likely from bedrock data) with strong accumulation and ablation around the ELA (see red marker in picture)
  - o Also spot the chaotic area in front of the glacier (the part kept at minimum glaciation) with partly slight positive and slight negative SMB – that will cause troubles in the long run (see grey marker in picture)

$$v_{em} = -u\frac{\partial h}{\partial x} - v\frac{\partial h}{\partial y} + w$$



Prognostic relaxation run

- Following adaptations to previous run:
  - Move from `steady state` to `transient` in `Simulation`-section
  - Restart from previous file in `Simulation`-section
  - All Exec Solver commands change from `before simulation` to `before timestep`

```
Simulation
  Coordinate System  = "Cartesian 3D"
  Simulation Type = "Transient"

…
  ! Time-stepping settings
  !-----------------------------------------------
  Steady State Max Iterations = 1
  Timestepping Method = "BDF" ! implicit Euler
  BDF Order = 1
  Timestep Sizes = $1.0/52 ! 1 week
  Timestep Intervals = 52
  Output Intervals = 1

  Restart File = $restartfile
  Restart Before Initial Conditions = Logical True
  Interpolation Passive Coordinate = Integer 3
…
End
```

  - The variable, `Zs`, of the new introduced `FreeSurfaceSolver` (=computation of BC on surface; see next slide) is initialized to the surface DEM
  - And the upper elevation in the `StructuredMeshMapper` is following `Zs`

```
Initial Condition 1
  Zs = Equals "surfaceDEM1995"
End

Boundary Condition 4
  Body ID = 2
  Name = "surface"
  Top Surface = Equals "Zs"
…
END
```

Prognostic relaxation run

- Free surface solver is run on `Body 2` (the upper surface):
  - The keyword `Apply Dirichlet` triggers the contact problem to be evaluated. It demands
    1. `Zs Residual` to be declared as `Exported Variable`
    2. `Nonlinear System Max Iterations` to be set to a value >1 (i.e., non-linear iterations are needed for the contact problem)
    3. The value `Min Zs to be` given in the `Material`-section declares the lowest possible value (here 10 m above the bedrock) for the contact problem

```
Material 1
…
 Min Zs = Variable "BedrockDem"
     Real MATC "tx + 10.0"
…
End
```

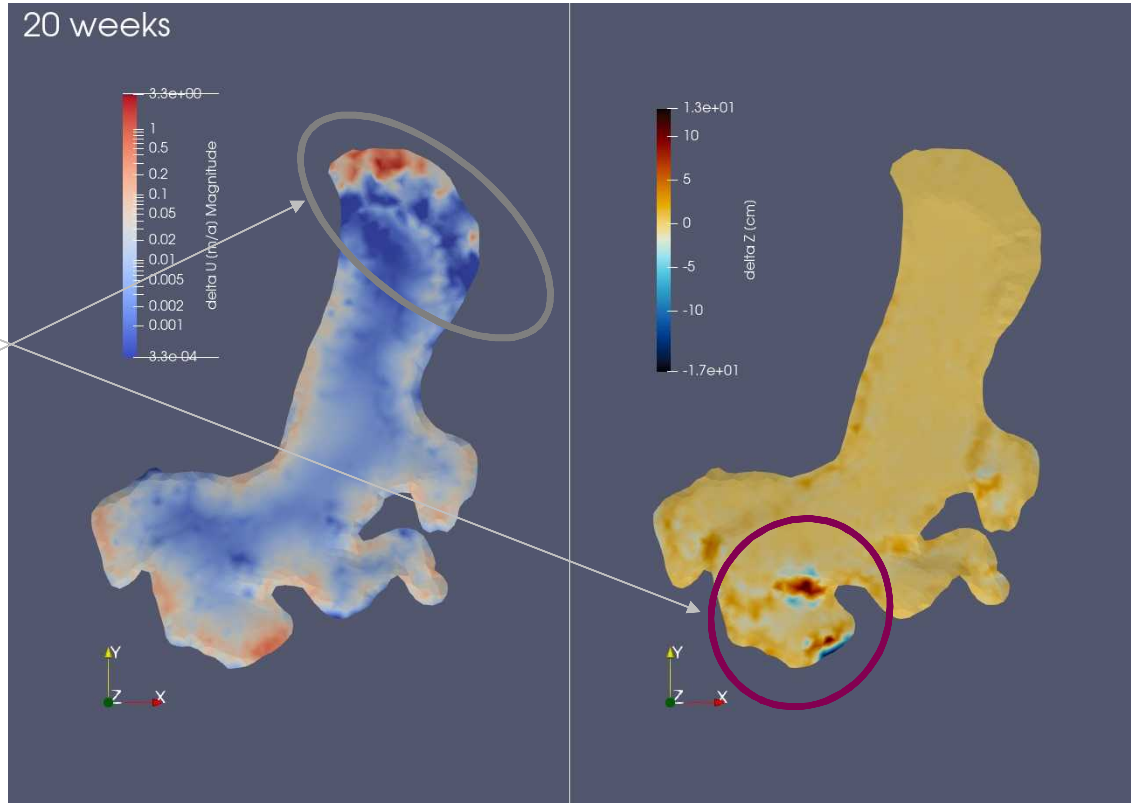
```
Solver 4
   Exec Solver = "after timestep"
   Equation = "Free Surface"
   Variable = String "Zs"
   Variable DOFs =  1
   Procedure = "FreeSurfaceSolver" "FreeSurfaceSolver"
  ! This would take the contrained points out of solution
  ! Use in serial run, only
 !  Before Linsolve = "EliminateDirichlet" "EliminateDirichlet"
   Linear System Solver = Iterative
   Linear System Max Iterations = 1500
   Linear System Iterative Method = BiCGStab
   Linear System Preconditioning = ILU0
   Linear System Convergence Tolerance = Real 1.0e-8
   Linear System Abort Not Converged = True
   Linear System Residual Output = 10
   Nonlinear System Max Iterations = 100
   Nonlinear System Convergence Tolerance  = 1.0e-7
   !Nonlinear System Relaxation Factor = 0.60
   Steady State Convergence Tolerance = 1.0e-04
   ! Apply contact problem
   Apply Dirichlet = Logical True
   ! needed for evaluating the contact pressure
   Exported Variable 1 = -dofs 1 "Zs Residual"
   ! needed to host imported emergence velocity
   Exported Variable 2 = -dofs 1 "EmergenceVelocity"
 ! How much the free surface is relaxed (default is no
relaxation)
  ! Relaxation Factor = Real $1.0/3.0
End
```

Prognostic relaxation run

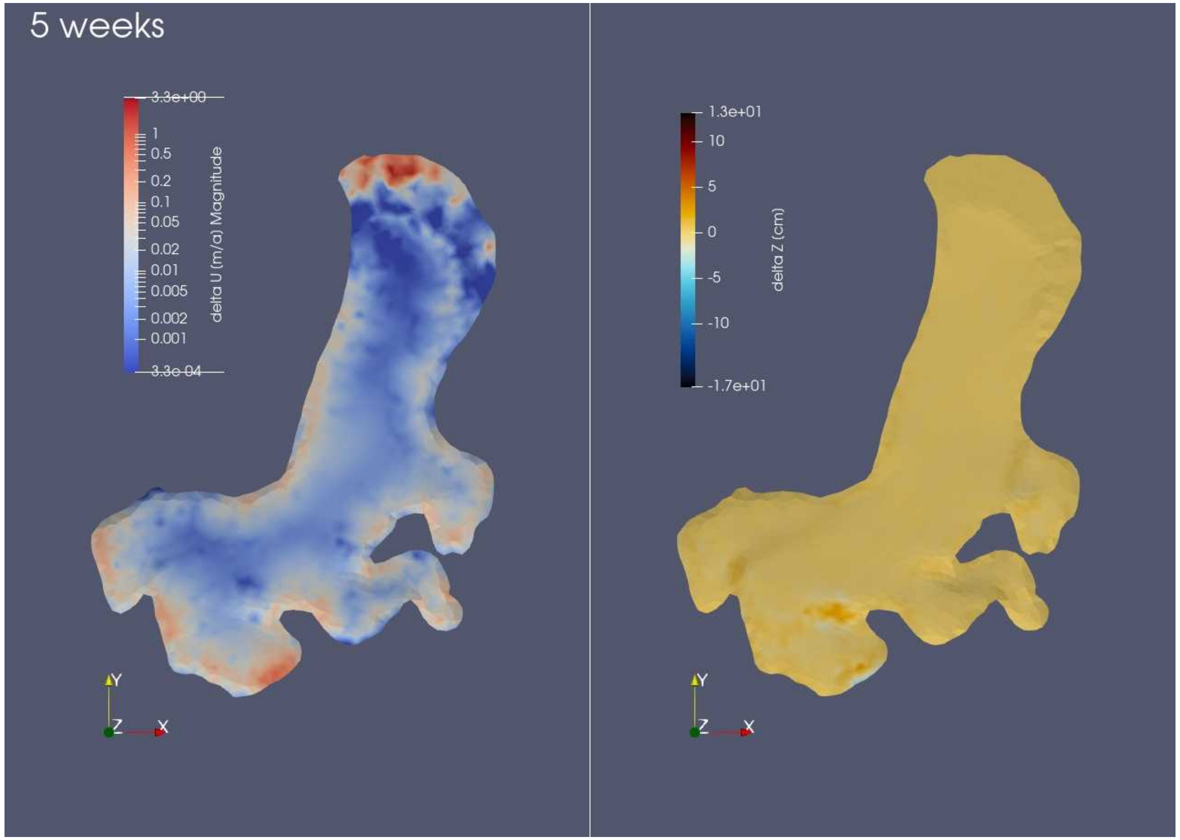
- Run with

```
mpirun -np 2 ElmerSolver mlb_relax.sif
```

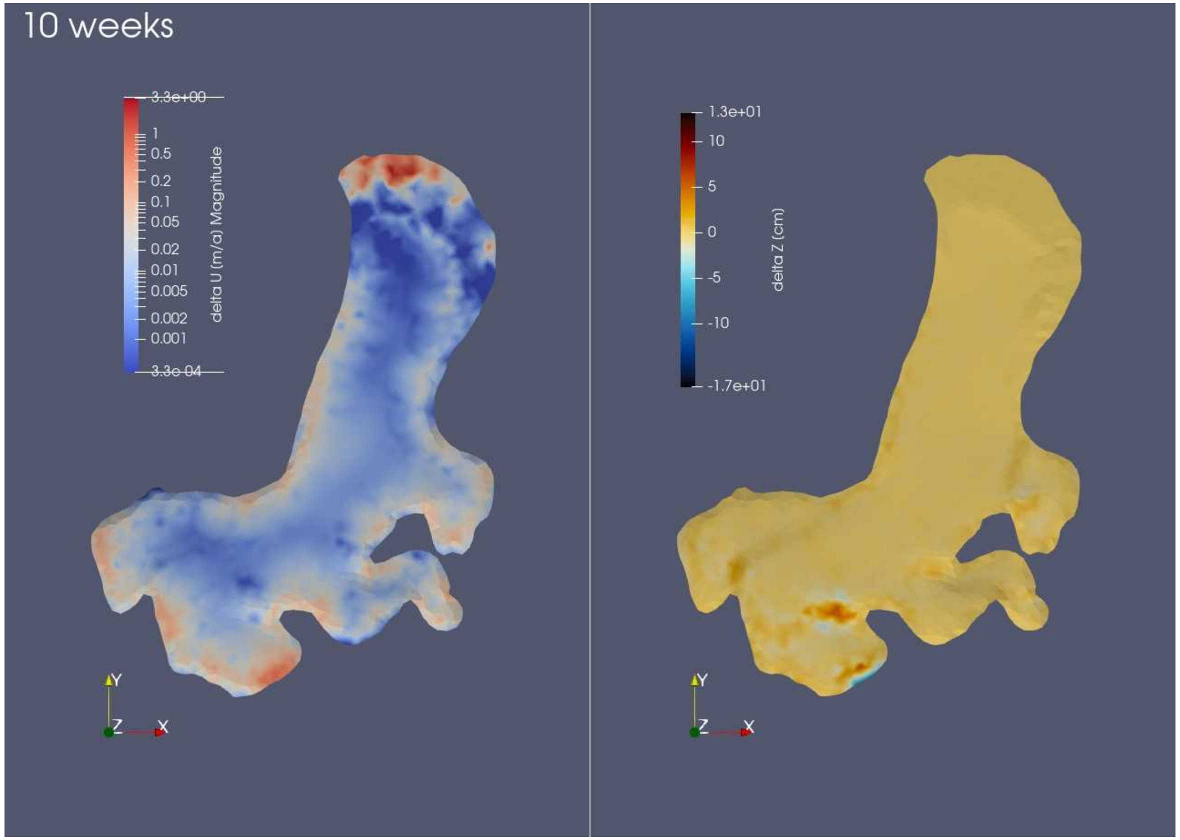
- We see changes of the free surface (here relative to the initial surface DEM), particular in the earlier discussed place
- We also see an artificial velocity field in the deglaciated area in front of the glacier tongue
    - This would cause a problem if run longer



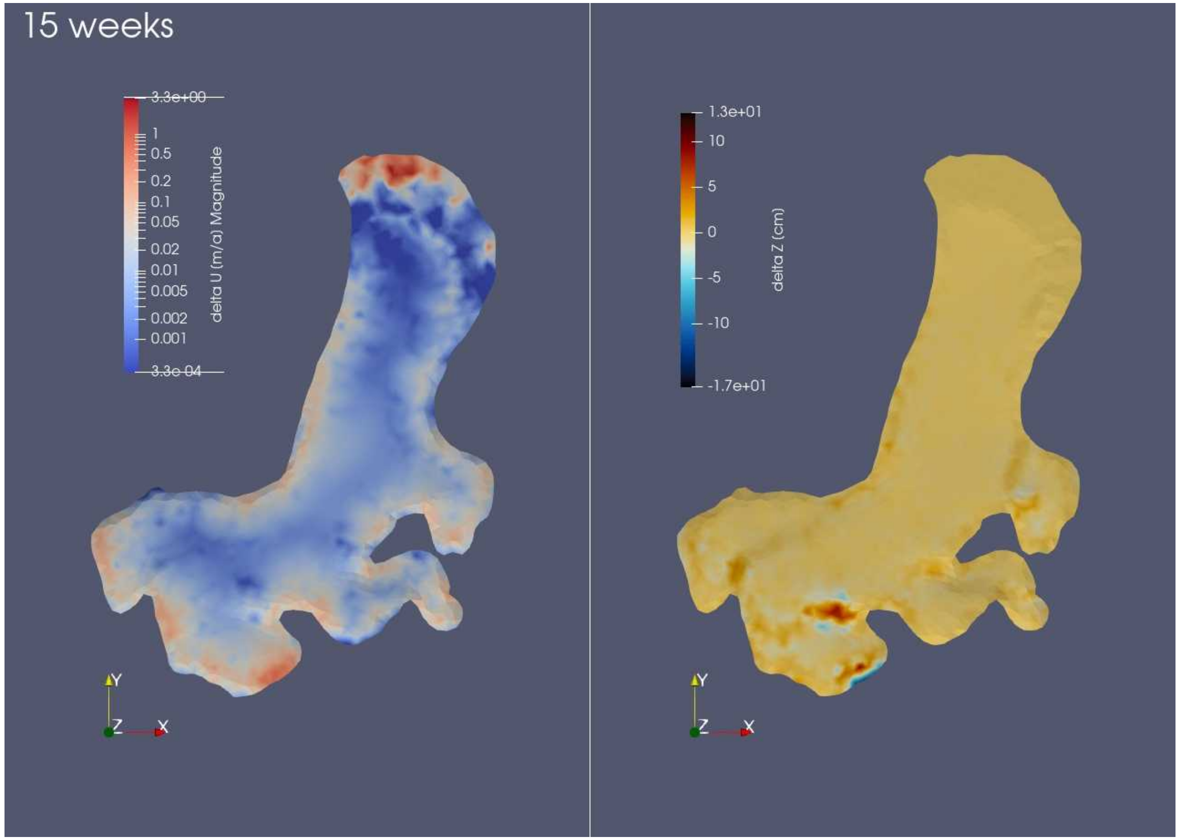
Prognostic relaxation run



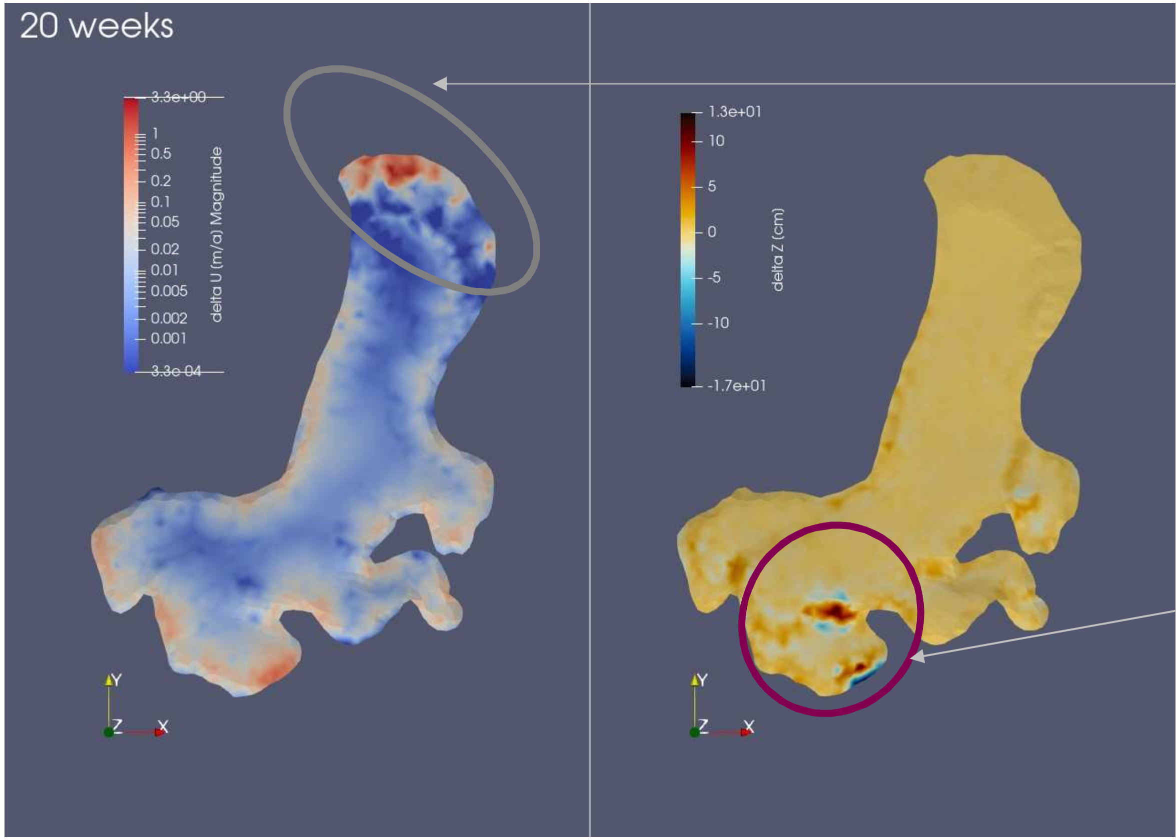
Prognostic relaxation run



Prognostic relaxation run



Prognostic relaxation run



Issues with artificial SMB in unglaciated area

(Over-)compensation of wrong geometry

End of session

Things to try, if time permits

- You could take the relaxed surface and re-run the emergence velocity on it
    - Or simply include emergence velocity computation in the relaxation run and watch its change
- You could re-compute a steady state age-distribution on the relaxed geometry and investigate, whether things changed
- You could use the equilibrium SMB (=negative emergence velocity) of the relaxed geometry and do a prognostic run with it
    - You tough might have to fix the issue with the artificial accumulation over the deglaciated area at the front (perhaps use a function that cuts off)

- Thanks to **Jack Kohler** (NPI) for providing the bedrock and surface DEM