Sensitivity of centennial mass loss projections of the Amundsen basin to the friction law

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Commonly-used friction laws

Several friction laws have been developed:

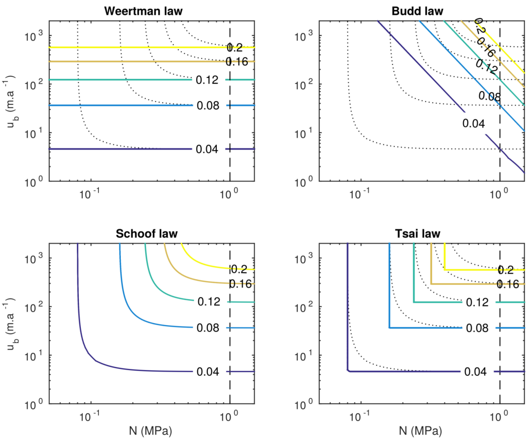
$$\tau_b = C_W u_b{}^m \quad \text{Weertman}$$

$$\tau_b = C_B u_b{}^m N \quad \text{Budd}$$

$$\tau_b = \frac{C_S u_b{}^m}{\left(1 + \left(\frac{C_S}{C_{max} N}\right)^{\frac{1}{m}} u_b\right)^m} \quad \text{Schoof}$$

$$\tau_b = min\left[C_W u_b{}^m, f_C N\right] \quad \text{Tsai}$$

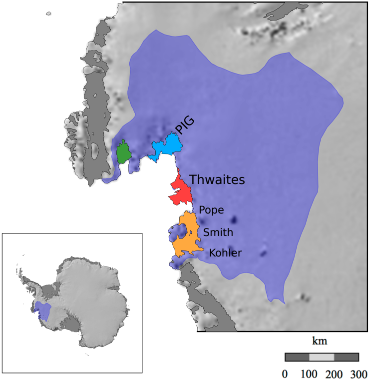
m = 1/3 **$C_W = C_B = C_S$** **$C_{max} = f_c$**



Brondex *et al.* (2017)

Goals of the study

- **Amundsen basin** (West Antarctica):
  - → ~ 1.2 m SLE (Rignot, 2008)
  - → Total ice discharge ↗ by 77% since 1973 (Mouginot *et al.*, 2014)
- How to implement the Schoof law for a real case application ?
- How sensitive are the mass loss projections at a 100-year time horizon to the choice of the friction law ?

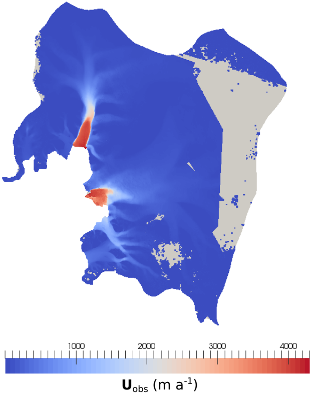


1/ Construction of 3 initial states using inverse methods

2/ 100 yr schematic perturbation experiments with a **Weertman law** (linear and non-linear), a non-linear **Budd law** and a **Schoof law** (for 2 values of C_{max})

Datasets and parameterisations

- Surface velocities (Rignot *et al.*, 2011)
- Surface elevation (Fretwell *et al.*, 2013)
- Bed elevation (Fretwell *et al.*, 2013 & Millan *et al.*, 2017)
- SMB 1979-2015 (MAR Agosta, personal communication)
- Temperature field (Van Liefferinge *and* Pattyn, 2013)
- Sub-ice-shelf melting parameterisation (Pollard *and* DeConto, 2012)
- Perfect hydrological connectivity to the ocean $\rightarrow \mathbf{N} = \rho_\mathrm{i} \mathbf{g} \mathbf{H} - \rho_\mathrm{w} \mathbf{g}(\mathbf{z}_\mathrm{sl} - \mathbf{z}_\mathrm{b})$



τ_b ? η ?

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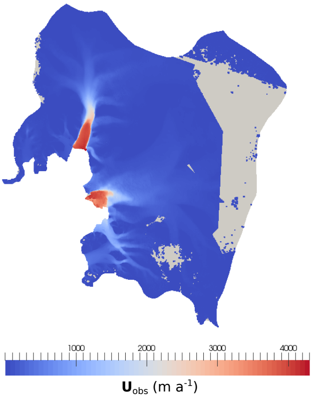

τ_b ? η ? $\rightarrow$ **Inverse methods !**

Inverse methods and initialisation strategies

2 unknowns: τ_b & η

Inverse methods → Optimization of τ_b and η so that modeled velocites fit observed velocities

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Construction of 3 initial states:

I_{SV} $I_{R\gamma,100}$ $I_{R\gamma,1}$

↓

Assumption: $\eta = f(T)$

Inversion of τ_b only !

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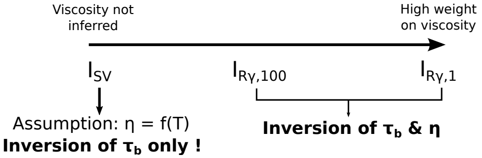

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Viscosity not inferred → High weight on viscosity

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$I_{R\gamma,100}$, $I_{R\gamma,1}$: **Inversion of τ_b & η**

A friction law needs to be prescribed for the inversion algorithm !

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Schoof law : $$\tau_b = \frac{C_s u_b^m}{\left(1 + \left(\frac{C_s}{C_{max} N}\right)^{1/m} u_b\right)^m}$$

- $m = 1/3$ → ice rheology
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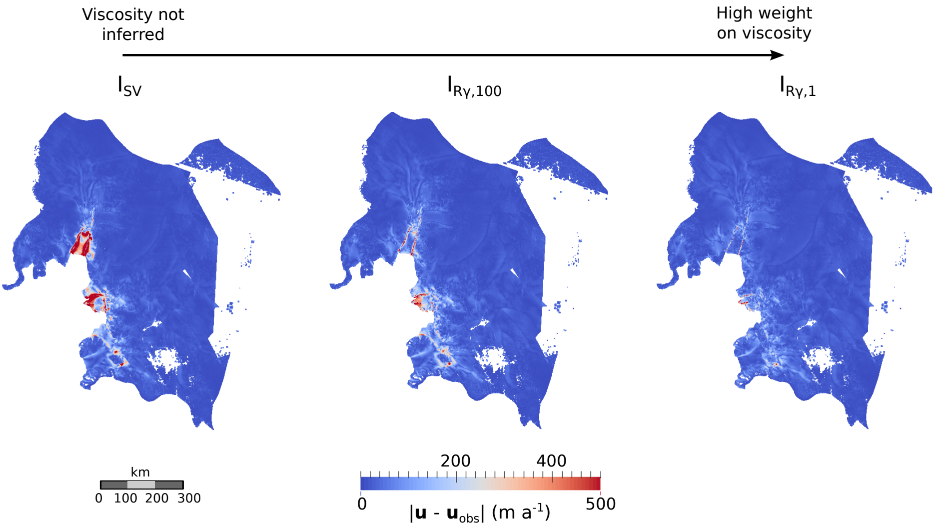
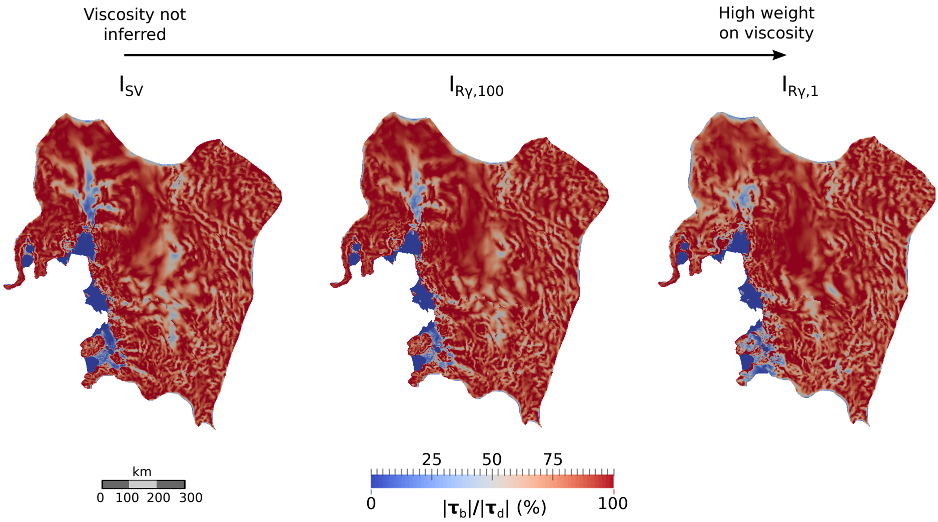
Inversion performed with **linear Weertman law:** $\tau_b = C_{wl}\, u_b$

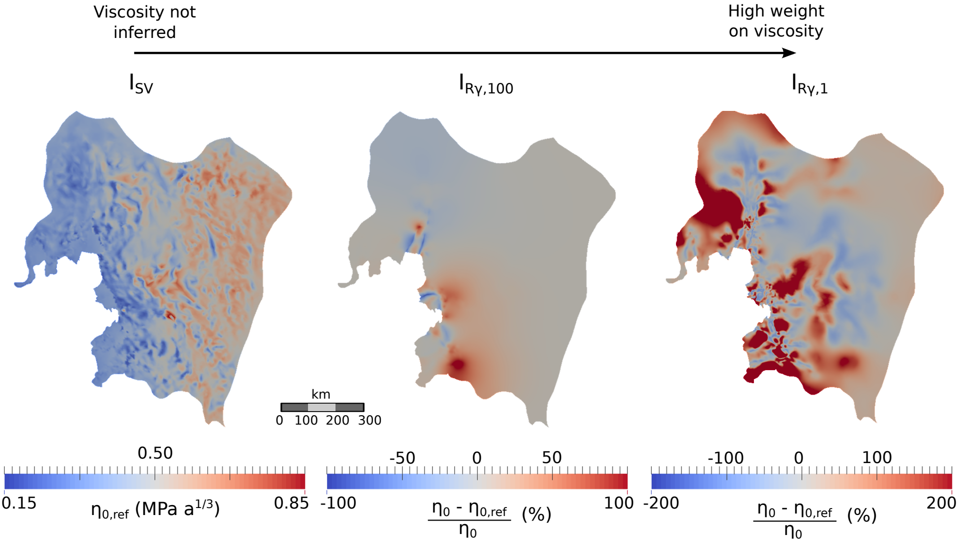
Results of initialisations: error on modeled velocities



Results of initialisations: basal shear stress



Results of initialisations: viscosity



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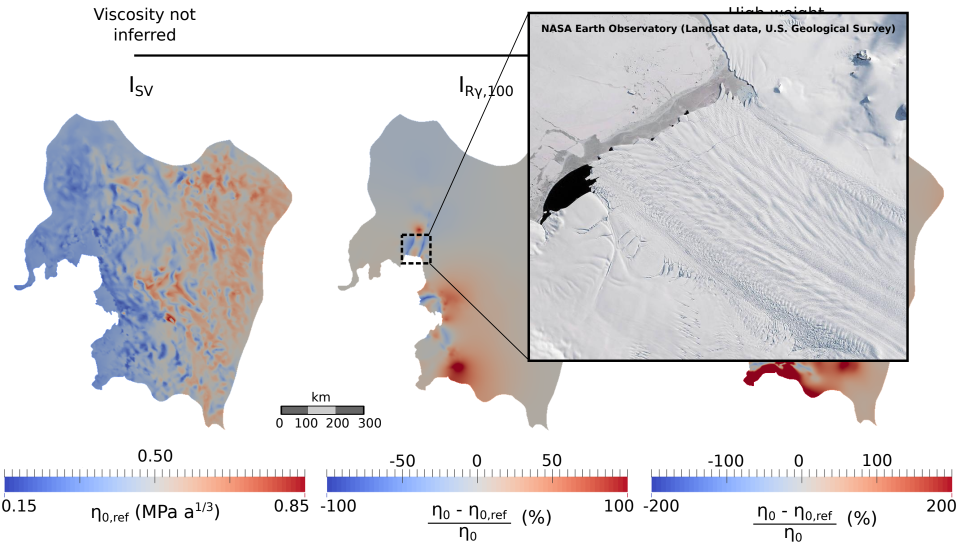
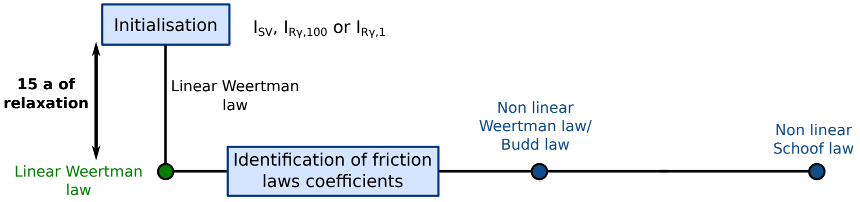


Experimental procedure



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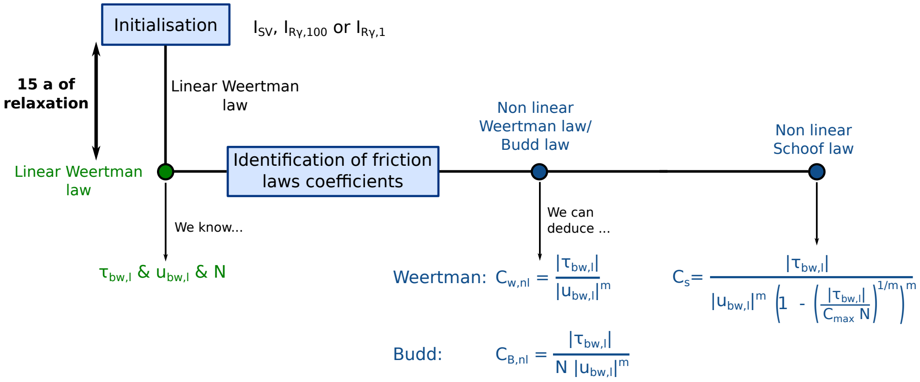

Experimental procedure

Initialisation — I_{SV}, $I_{R\gamma,100}$ or $I_{R\gamma,1}$

15 a of relaxation

Linear Weertman law

Linear Weertman law

Identification of friction laws coefficients

Non linear Weertman law/ Budd law

Non linear Schoof law

We know...

$\tau_{bw,l}$ & $u_{bw,l}$ & N

We can deduce ...

Weertman: $C_{w,nl} = \dfrac{|\tau_{bw,l}|}{|u_{bw,l}|^m}$

Budd: $C_{B,nl} = \dfrac{|\tau_{bw,l}|}{N\,|u_{bw,l}|^m}$

$$C_s = \frac{|\tau_{bw,l}|}{|u_{bw,l}|^m \left(1 - \left(\frac{|\tau_{bw,l}|}{C_{max}\,N}\right)^{1/m}\right)^m}$$

C_s is defined only if $\dfrac{|\tau_{bw,l}|}{N} < C_{max}$

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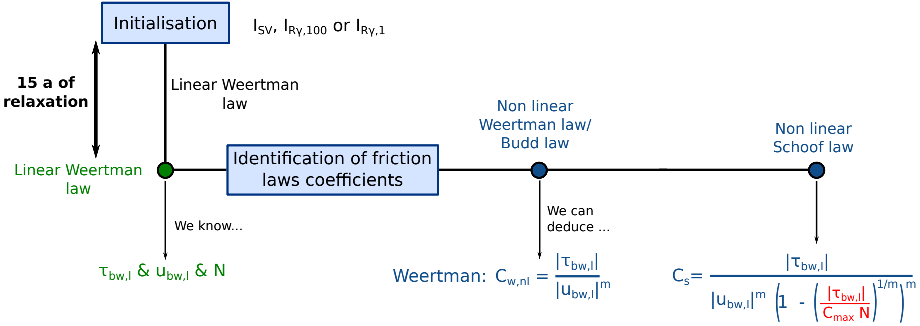

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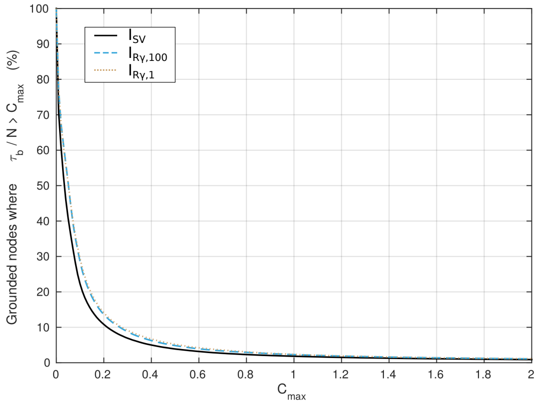
Obtention of two C_S fields



- Till deformation $\rightarrow 0.17 \leq C_{max} \leq 0.84$ (Cuffey *and* Paterson, 2010)

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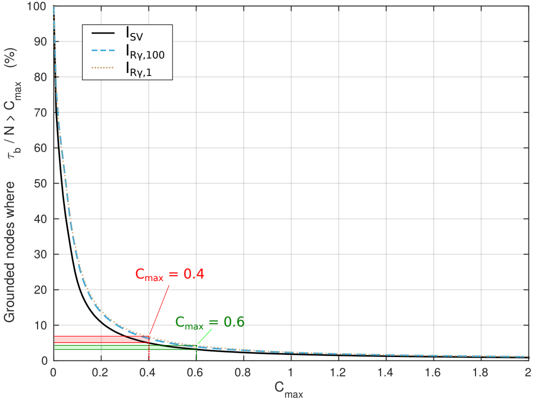
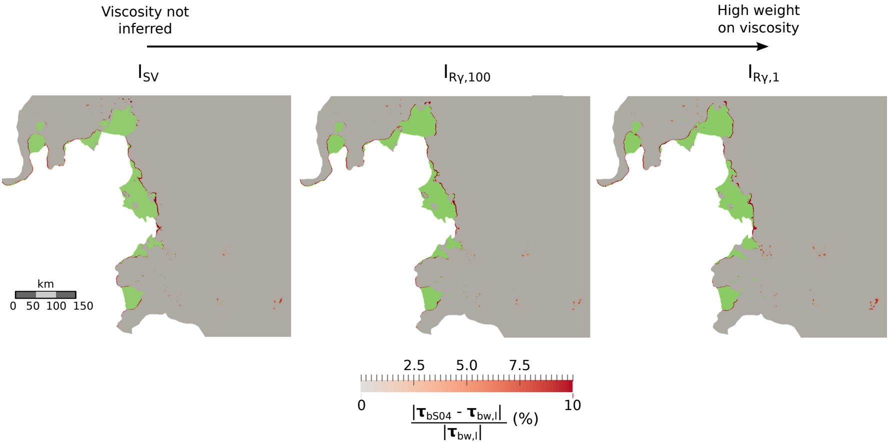


- Till deformation $\rightarrow 0.17 \leq C_{max} \leq 0.84$ (Cuffey *and* Paterson, 2010)
- We test $C_{max} = 0.4$ and $C_{max} = 0.6$

Schoof law: error on recalculated basal shear stress

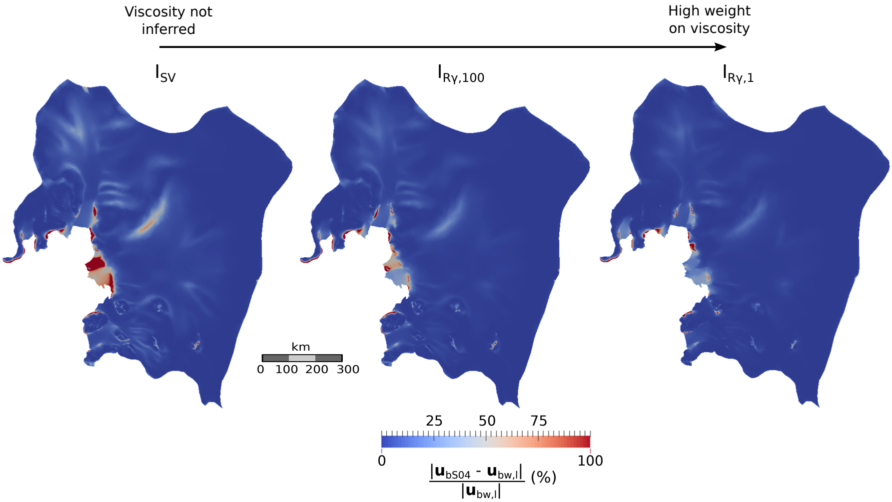
- τ_b distribution calculated with the Schoof law assuming $C_{max} = 0.4$



→ Slight differences on τ_b at nodes where C_S needs to be interpolated !

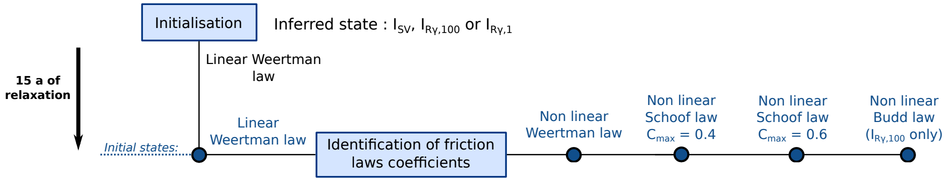
Schoof law: error on recalculated flow field

- u_b distribution calculated with the Schoof law assuming $C_{max} = 0.4$



→ In addition to numerical errors, differences on u_b due to differences on τ_b !

Experimental procedure



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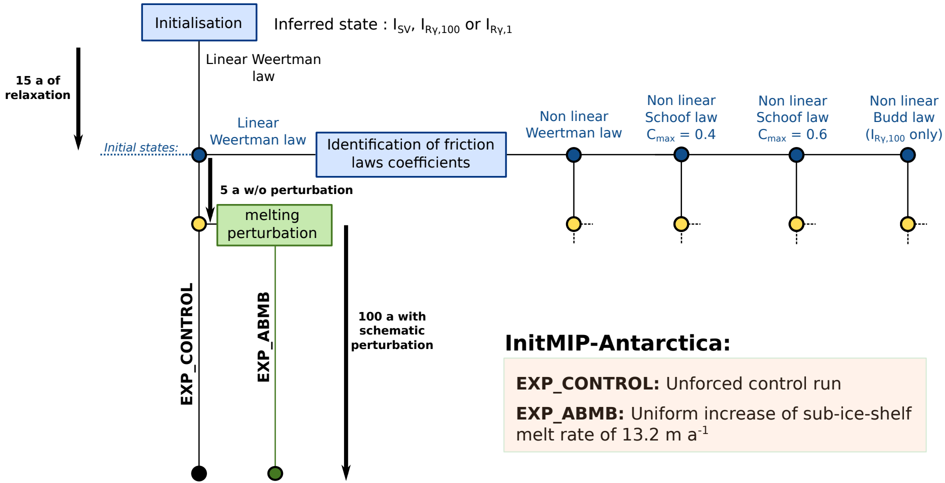
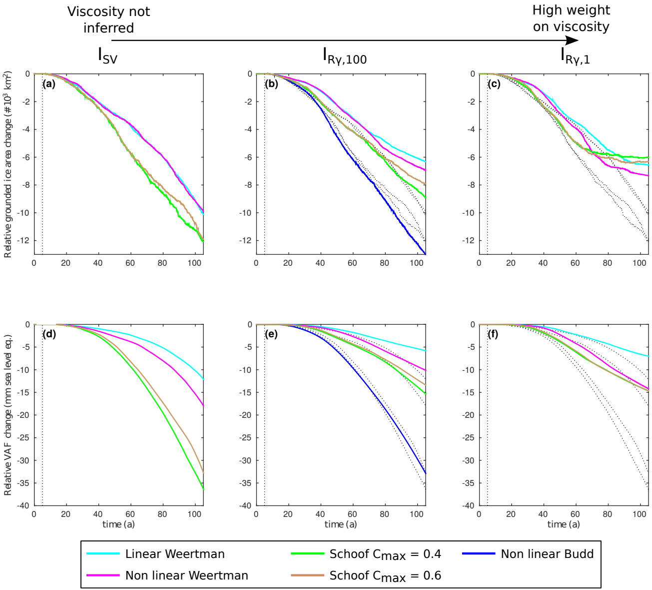


InitMIP-Antarctica:

EXP_CONTROL: Unforced control run

EXP_ABMB: Uniform increase of sub-ice-shelf melt rate of 13.2 m a^{-1}

Results of EXP_ABMB relative to EXP_CONTROL



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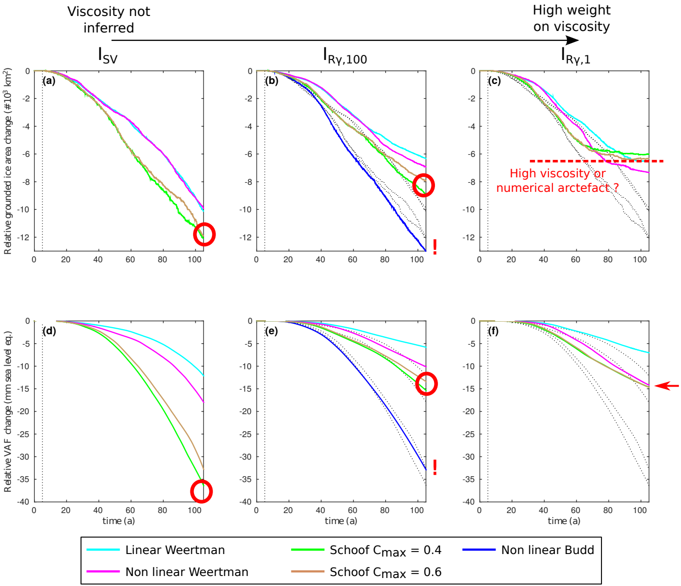

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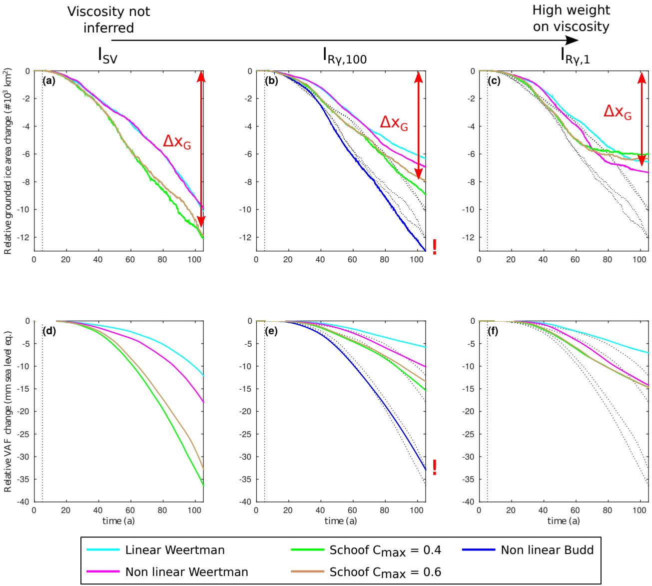

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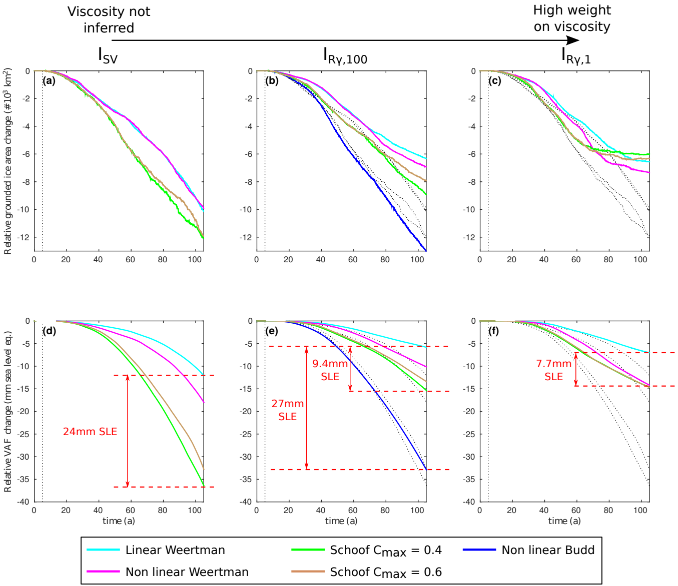

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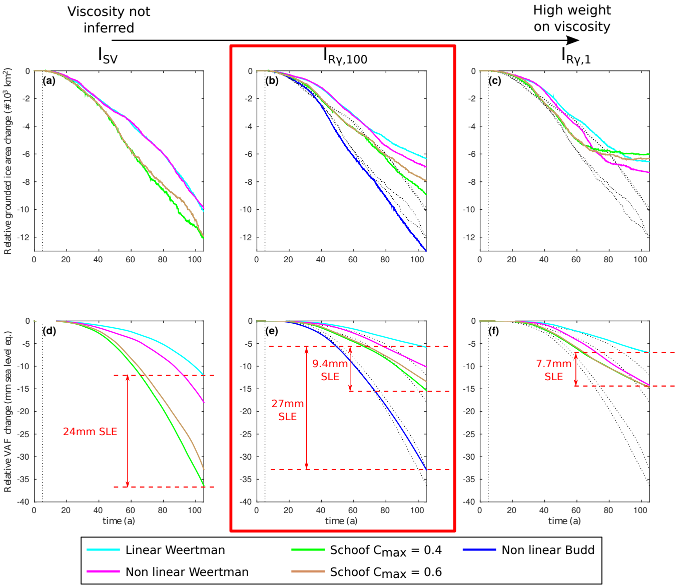
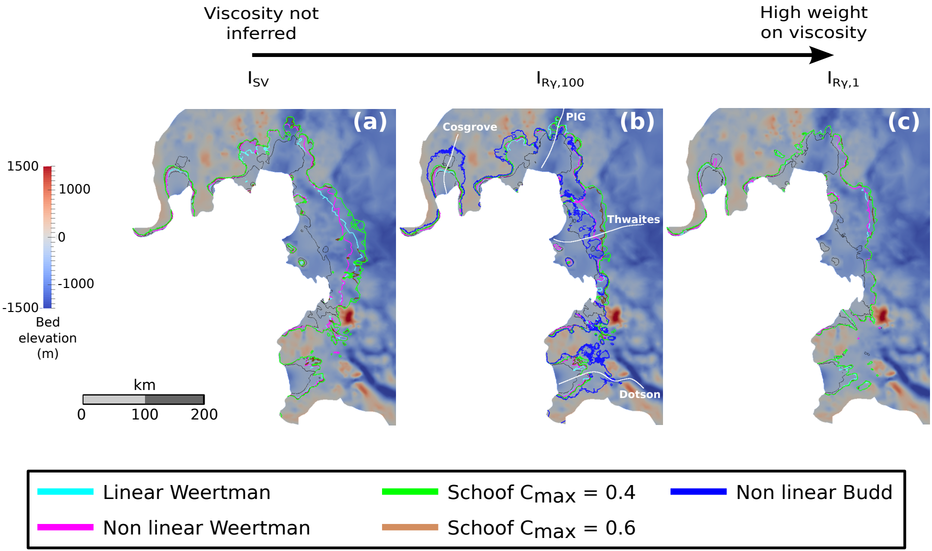


Results of EXP_ABMB: final GL positions



Conclusion

1/ The implementation of a Schoof law requires an **assumption on the value of C_{max}**
→ differences on recalculated τ_b !

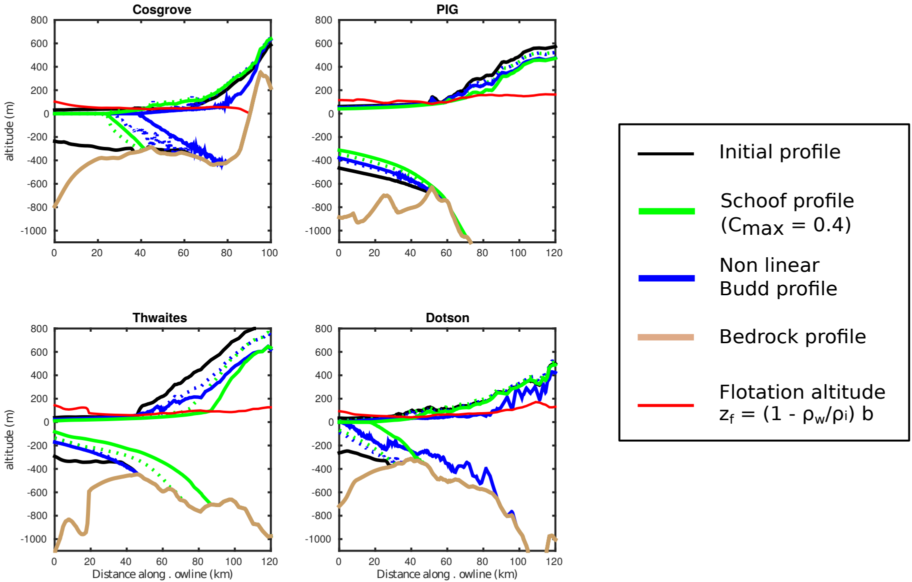
2/ The **Weertman law significantly underestimates the contribution of the Amundsen basin to SLR** relative to the Schoof law

3/ The projections of future SLR obtained with the **Budd law are dramatically higher** than the ones produced with the other laws

4/ Because it depends on N over the whole domain, **the Budd law produces different GL retreat patterns** than the other laws

- Independently of the chosen friction law, the GL dynamics is sensitive to the initialisation strategy
- The sensitivity of the GL dynamics to the friction law decreases when more weight is put on viscosity during initialisation
  → For the most realistic initial state $I_{R\gamma,100}$ this sensitivity remains significant

Results of EXP_ABMB for $l_{R\gamma,100}$: Budd and Schoof ice sheet profiles



Ice thickness variation rate