Accounting for transient water pressure in friction law

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1957, JoG

ON THE SLIDING OF GLACIERS

By J. Weertman
(Naval Research Laboratory, Washington, D.C.)

Abstract. A model is proposed to explain the sliding of any glacier whose bottom surface is at the pressure melting point. Two mechanisms are considered. One is pressure melting and the other is creep rate enhancement through stress concentrations. Neither of the mechanisms operating alone is sufficient to explain sliding. If both mechanisms operate together appreciable sliding can occur.

Résumé. On propose un modèle pour expliquer le glissement d'un glacier dont le fond se maintient au point de fusion. On considère deux mécanismes : le fusion de pression et l'augmentation de la vitesse de déformation causée par les concentrations de tension. Ni l'un ni l'autre en agissant seul ne suffit à expliquer le glissement. Mais ensemble ils occasionneraient un glissement assez important.

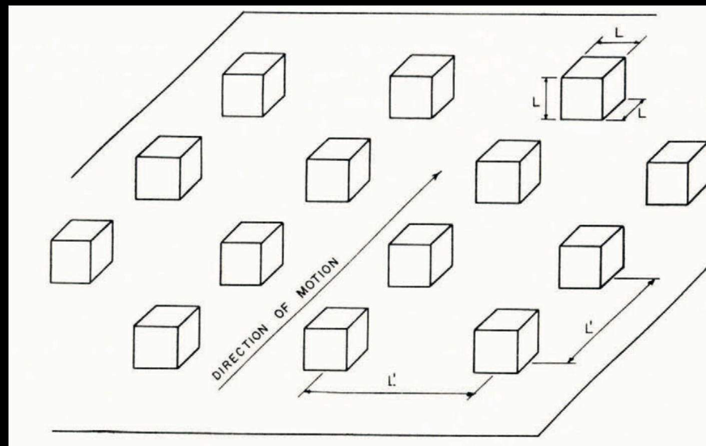


Fig. 1. Idealized glacier bed

2 mechanisms:

- pressure melting/refreezing
- creep rate enhancement

1958, CRAS (in French)

GLACIOLOGIE. — *Contribution à la théorie du frottement du glacier sur son lit.* Note (*) M. **Louis Lliboutry**, présentée par M. Léon Moret.

L'introduction d'un troisième mécanisme de franchissement des protubérances permet d'expliquer l'indépendance du frottement dynamique vis-à-vis de la vitesse, la valeur plus élevée du frottement statique (et donc le mouvement par saccades), l'accélération du glacier aux époques chaudes, et enfin l'usure caractéristique qui conduit à des roches moutonnées.

3rd mechanisms:

opening of **cavities** on the lee side of bed bumps

Lliboutry will publish more than 20 papers on that specific subject!

This one in 1968 :

Journal of Glaciology, Vol. 7, No. 49, 1968

GENERAL THEORY OF SUBGLACIAL CAVITATION AND SLIDING OF TEMPERATE GLACIERS

By L. LLIBOUTRY
(Laboratoire de Glaciologie du C.N.R.S., Grenoble, Isère, France)

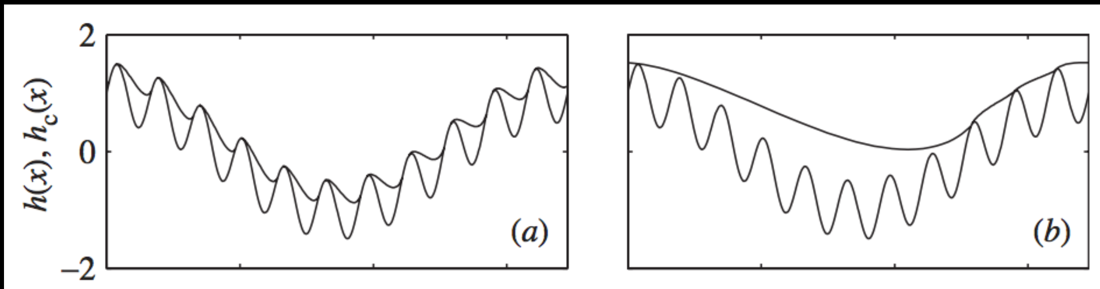
followed by one from Weertman in 1972:

REVIEWS OF GEOPHYSICS AND SPACE PHYSICS, VOL. 10, No. 1, PP. 287–333, FEBRUARY 1972

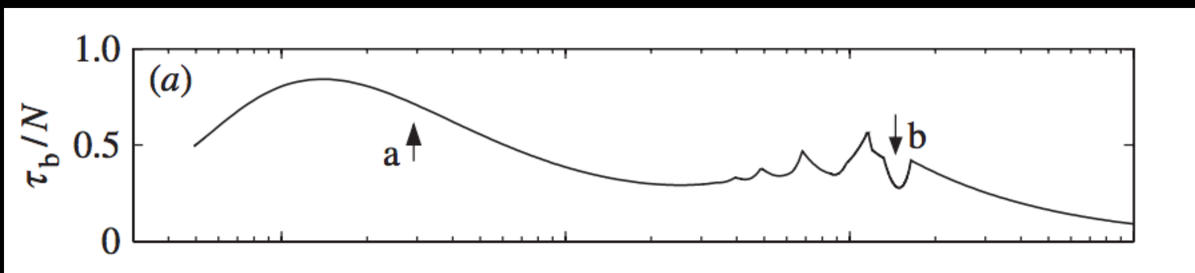
General Theory of Water Flow at the Base of a Glacier or Ice Sheet

J. WEERTMAN
Scott Polar Research Institute
Cambridge, England CB2 1ER

Schoof 2005



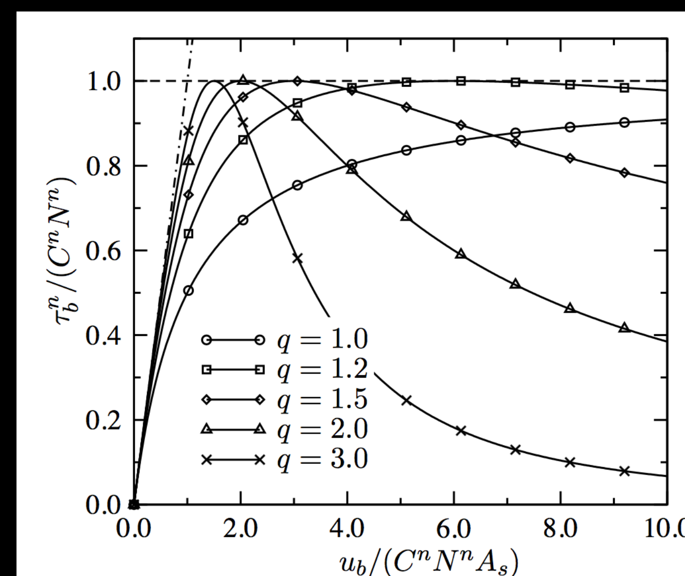
$$\frac{\tau_b}{N} = C\left(\frac{\Lambda}{\Lambda + \Lambda_0}\right)^{1/n}, \quad \Lambda = \frac{u_b}{N^n}$$



Gagliardini et al., 2007

$$\frac{\tau_b}{CN} = \left(\frac{\chi}{1+\alpha\chi^q}\right)^{1/n}$$

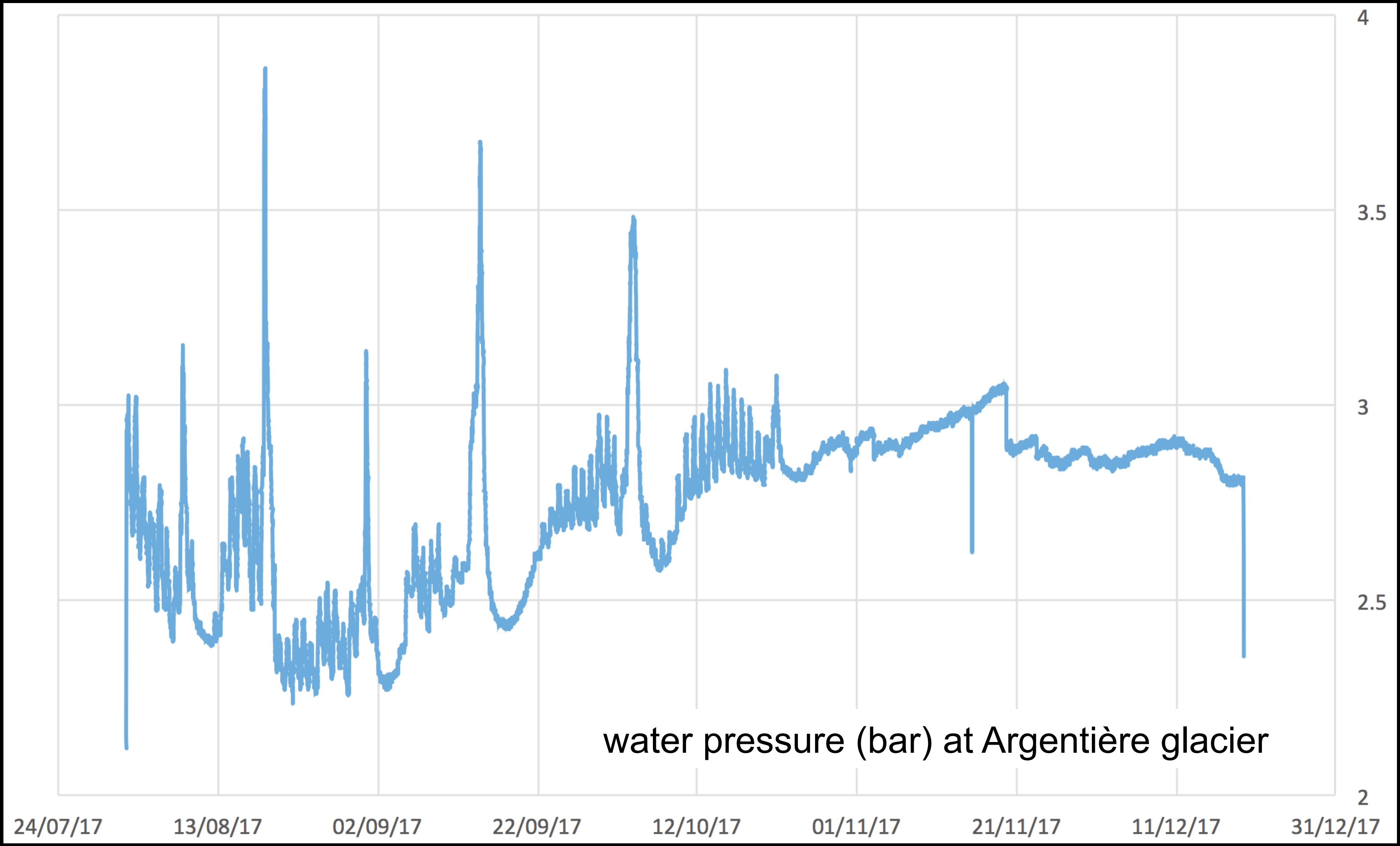
$$\chi = \frac{u_b}{A_s C^n N^n} \; ; \; \alpha = \frac{(q-1)^{(}q-1)^{q}}{q}$$



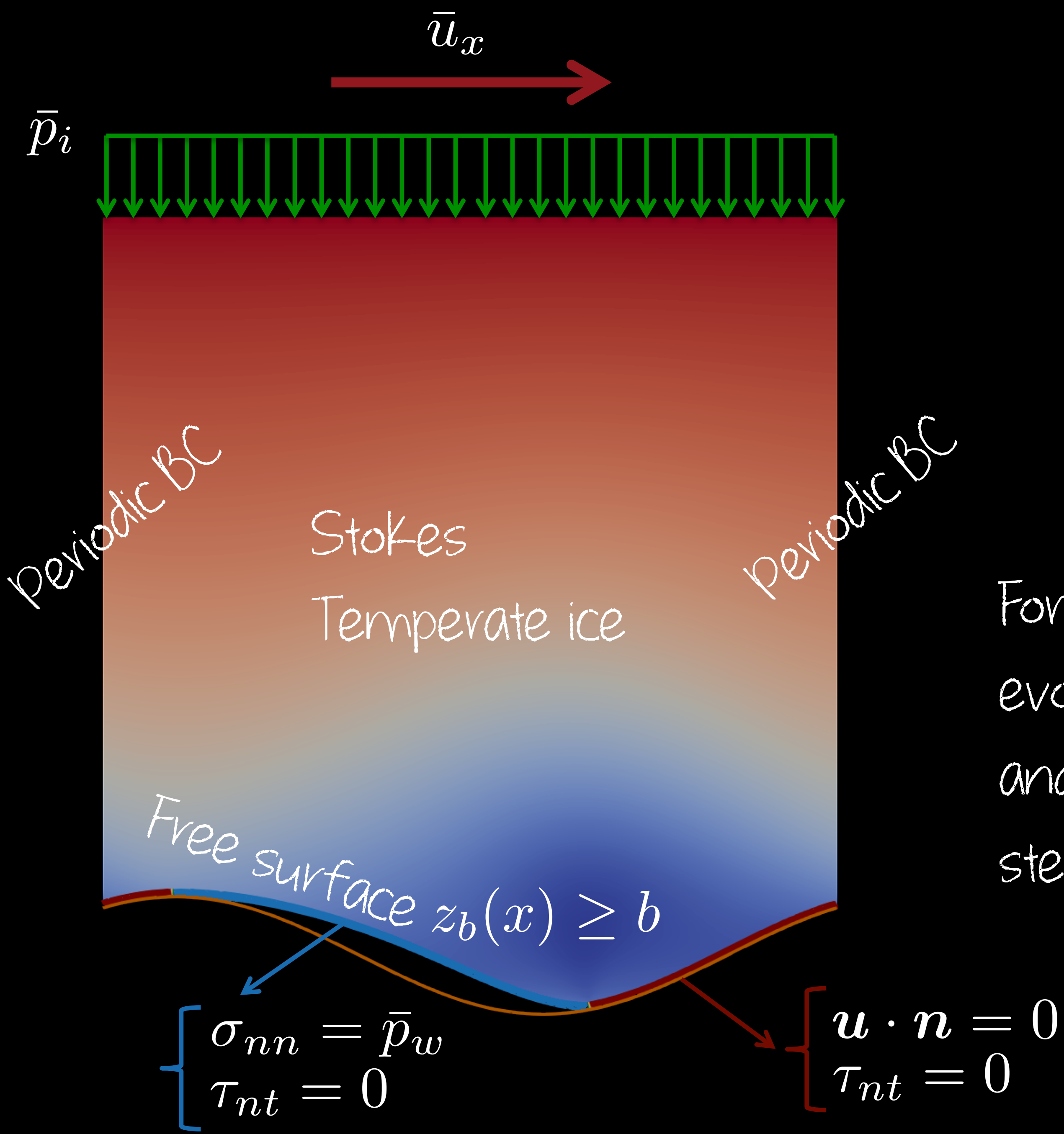
All these formulations assume **steady** water pressure

How are they modified if water pressure at the base is **unsteady**?

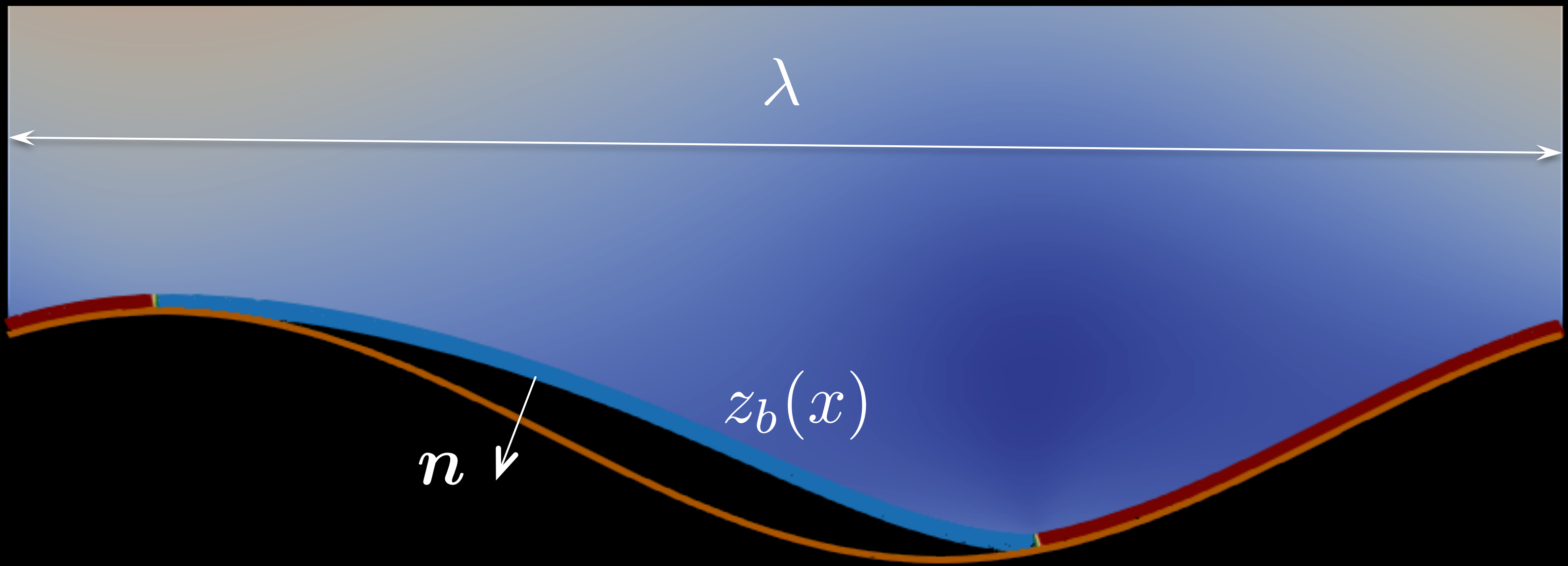
and water pressure is not steady...



@Piard 2018



For given $\bar{p}_i$ and $\bar{u}_x$
evolve $\bar{p}_w / \bar{p}_i$
and look for
steady state



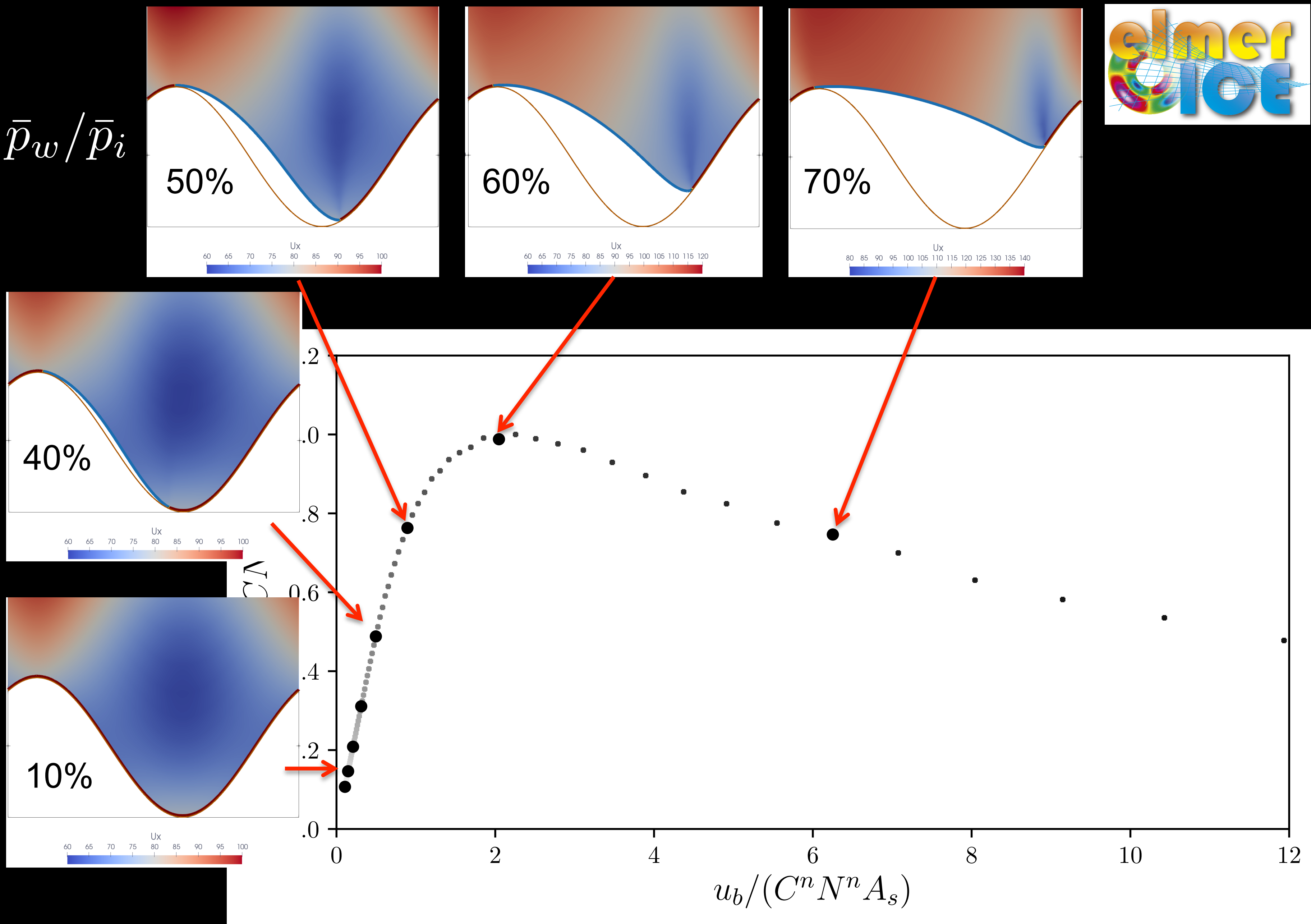
$$\tau_b = \frac{1}{\lambda}\int_0^\lambda \sigma_{nn} n_x ds = \frac{1}{\lambda}\int_0^\lambda \sigma_{nn}\frac{\partial z_b}{\partial x}dx$$

$$u_b = \frac{1}{\lambda}\int_0^\lambda u dx$$

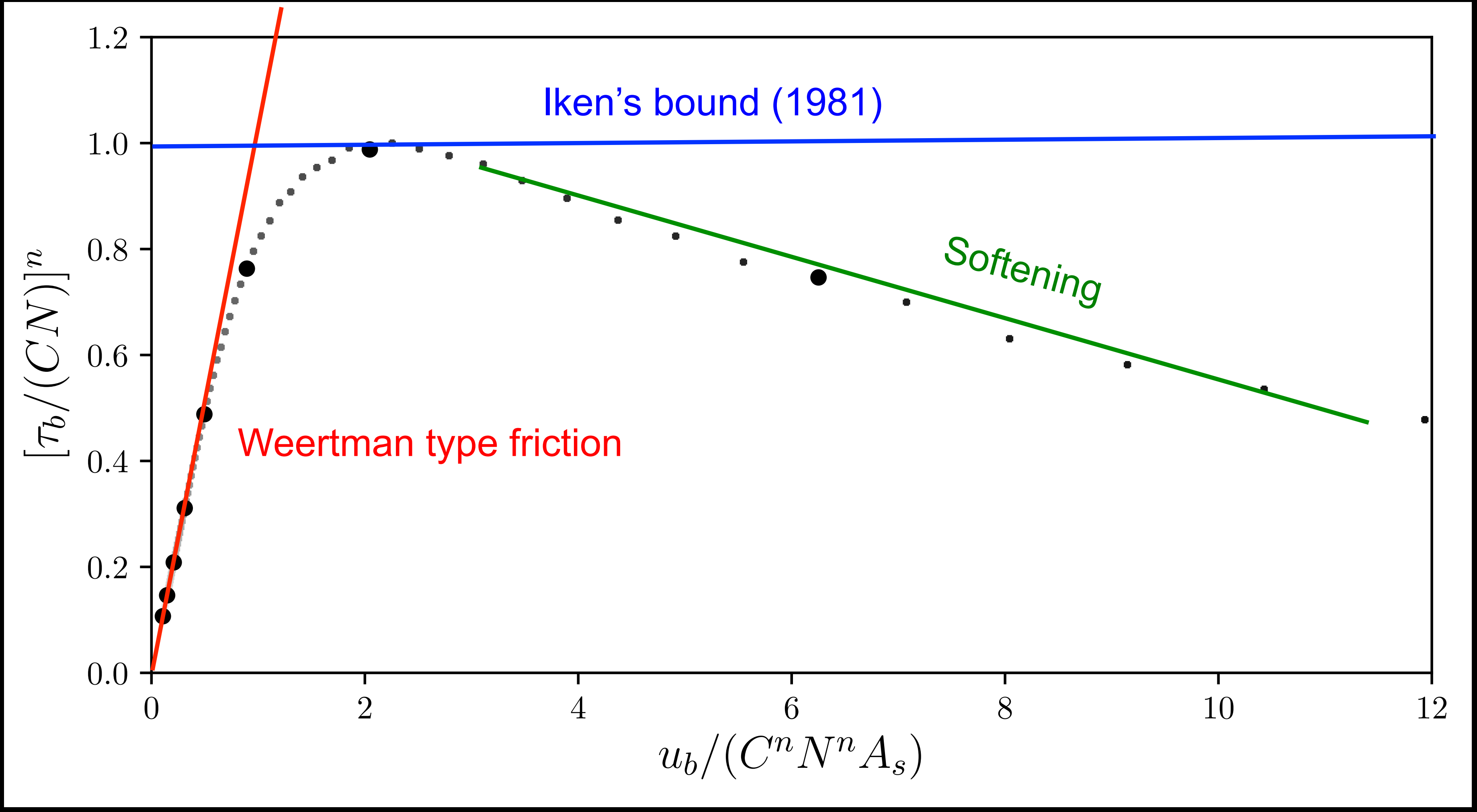
$$\rightarrow \tau_b = f(u_b, N)$$

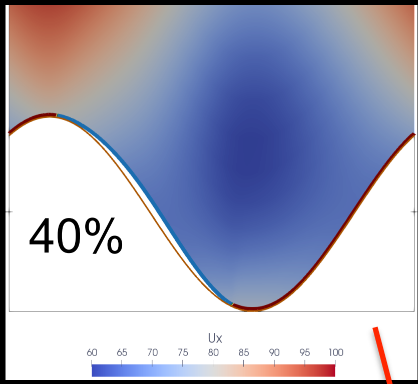
$$N = p_i - \bar{p}_w$$

$$p_i = \frac{1}{\lambda}\int_0^\lambda \sigma_{nn} n_y ds = -\frac{1}{\lambda}\int_0^\lambda \sigma_{nn} dx \approx \bar{p}_i$$

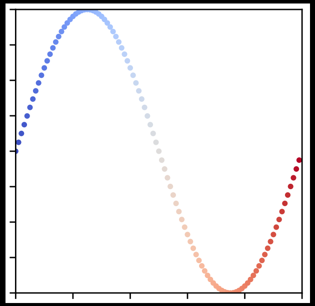


$\bar{p}_w/\bar{p}_i$
50%
Ux
60 65 70 75 80 85 90 95 100
60%
Ux
60 65 70 75 80 85 90 95 100 105 110 115 120
70%
Ux
80 85 90 95 100 105 110 115 120 125 130 135 140
40%
Ux
60 65 70 75 80 85 90 95 100
10%
Ux
60 65 70 75 80 85 90 95 100
.2
.0
.8
0.6
.4
.2
.0
0 2 4 6 8 10 12
$u_b/(C^n N^n A_s)$

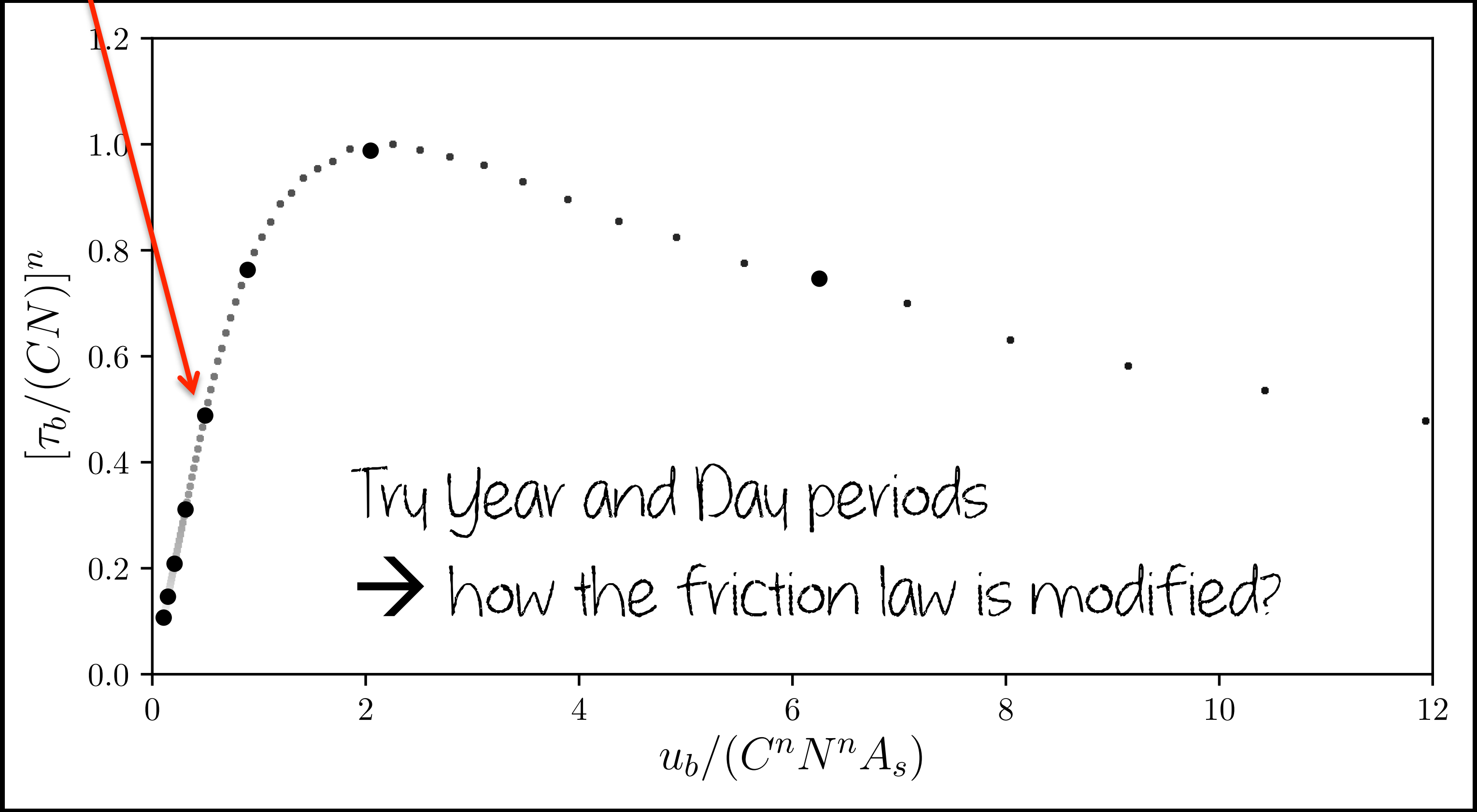

Iken's bound (1981)
Softening
Weertman type friction
$[\tau_b/(CN)]^n$
$u_b/(C^n N^n A_s)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
2
4
6
8
10
12



$$p_w(t) = \bar{p}_w + \Delta p_w \sin(\omega t)$$

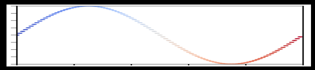


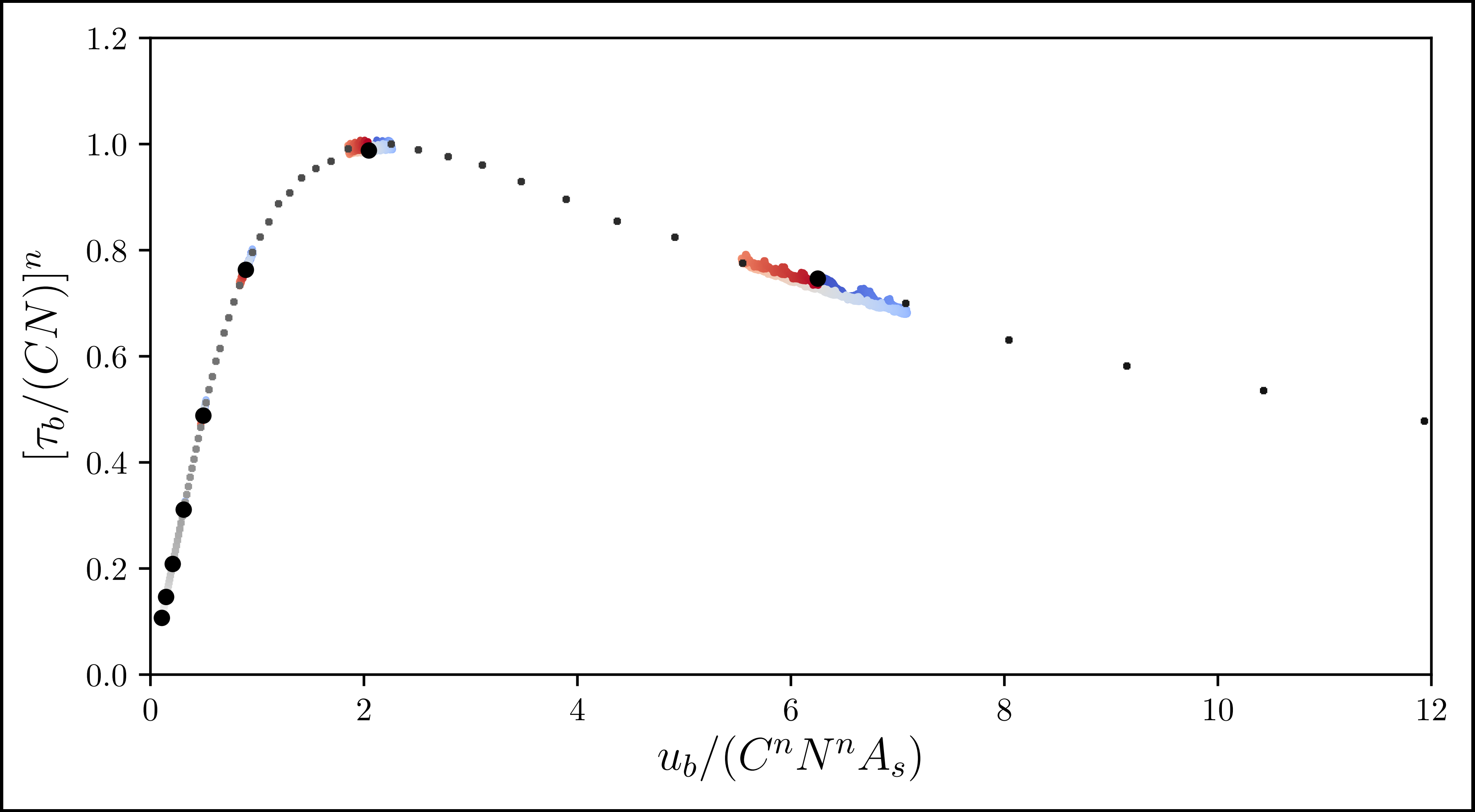
$$\Delta p_w / \bar{p}_i = 2\%$$



One **year** period

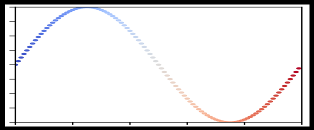
$\Delta p_w / \bar{p}_i = 1\%$

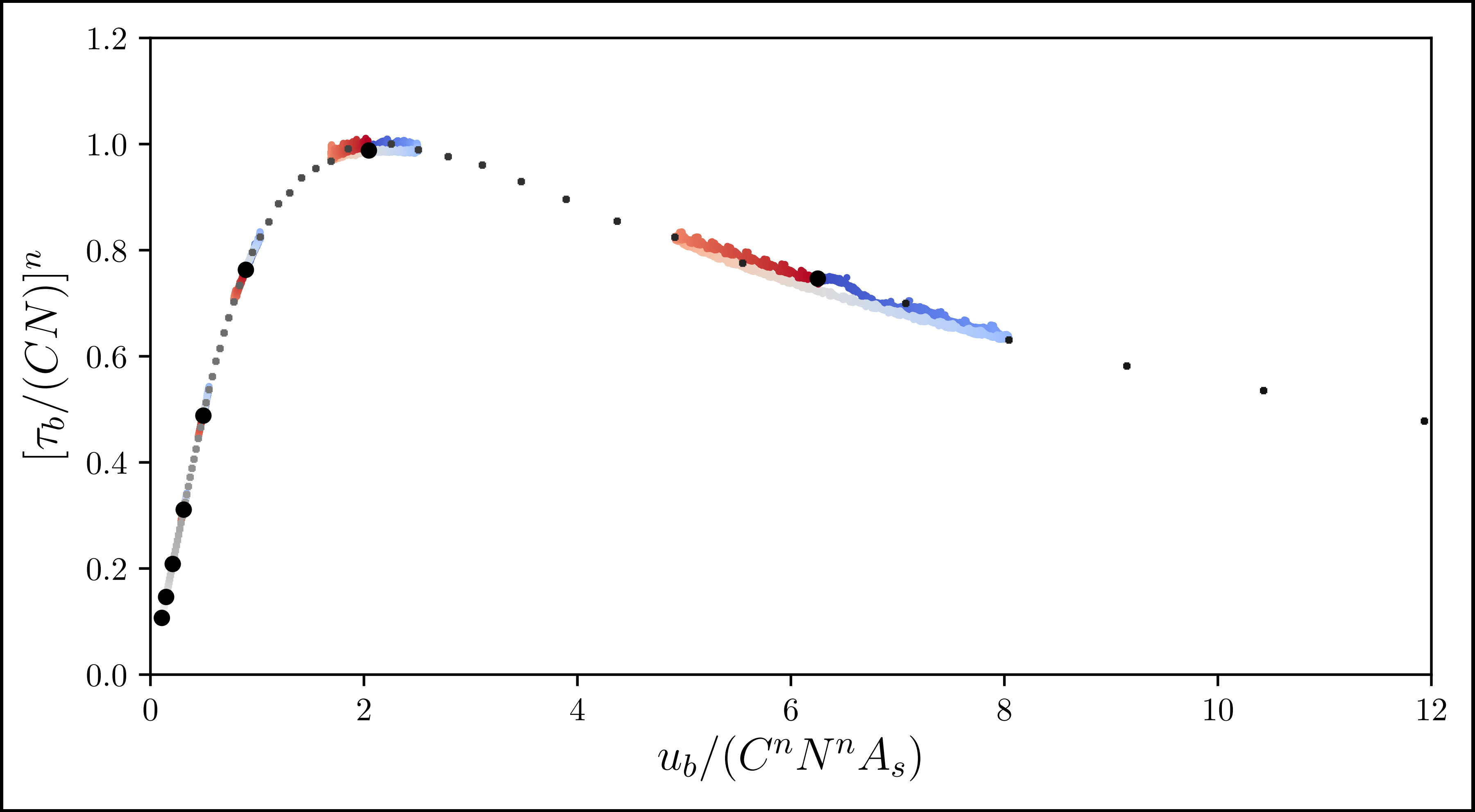




One **year** period

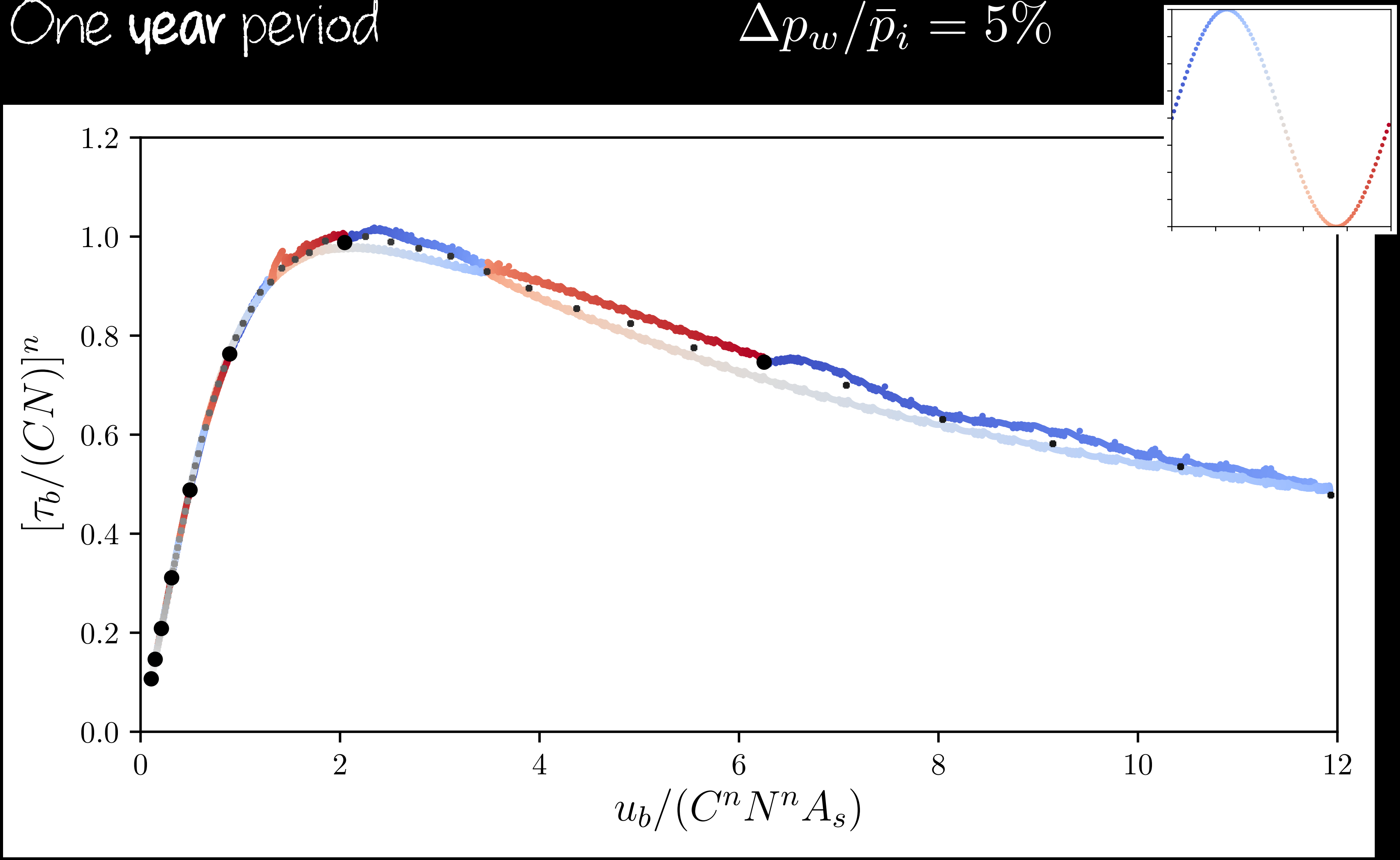
$\Delta p_w / \bar{p}_i = 2\%$





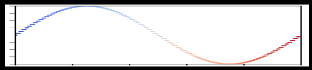
One **year** period

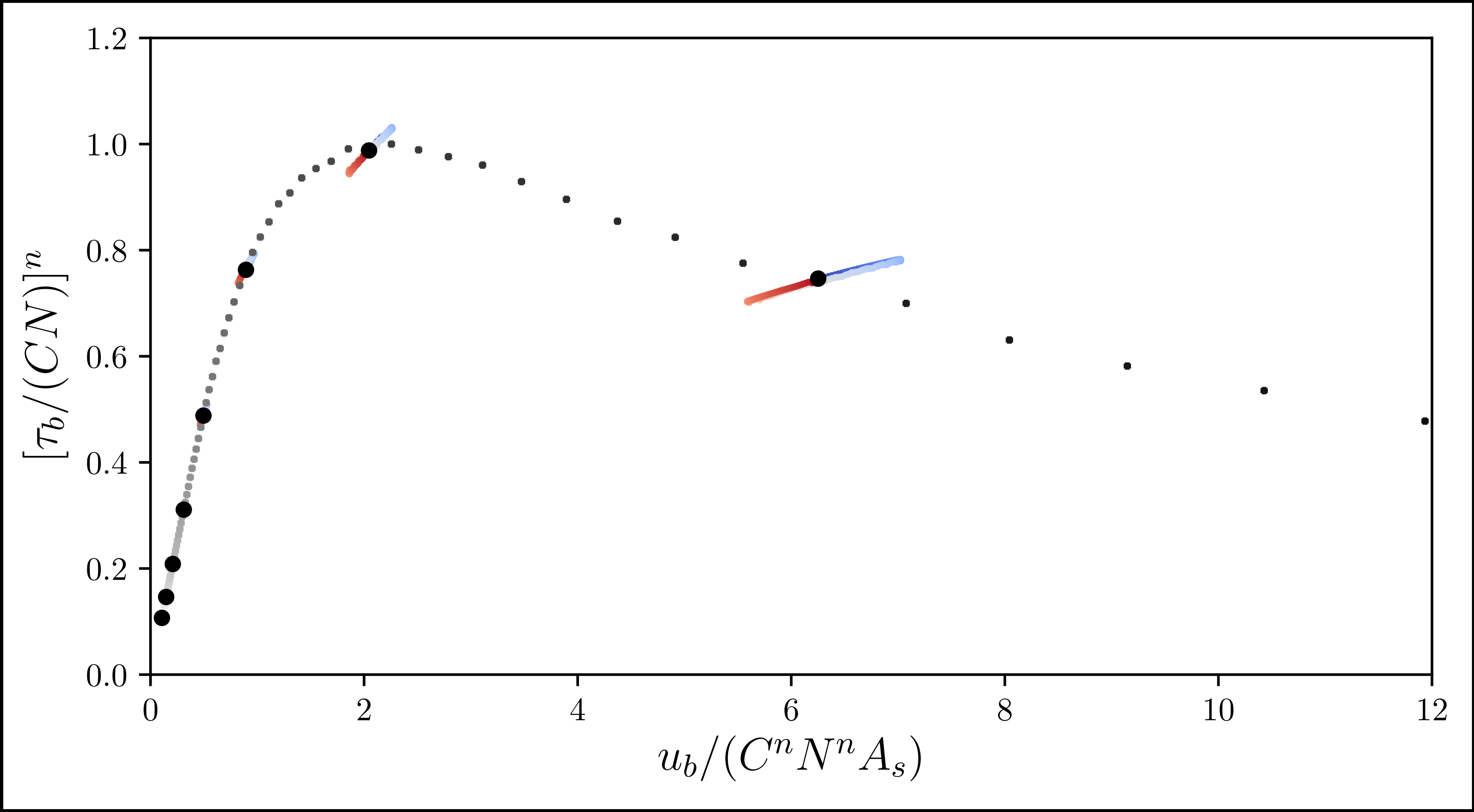
$\Delta p_w / \bar{p}_i = 5\%$



One **day** period

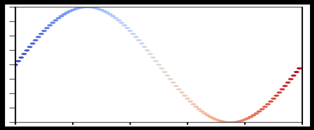
$\Delta p_w / \bar{p}_i = 1\%$

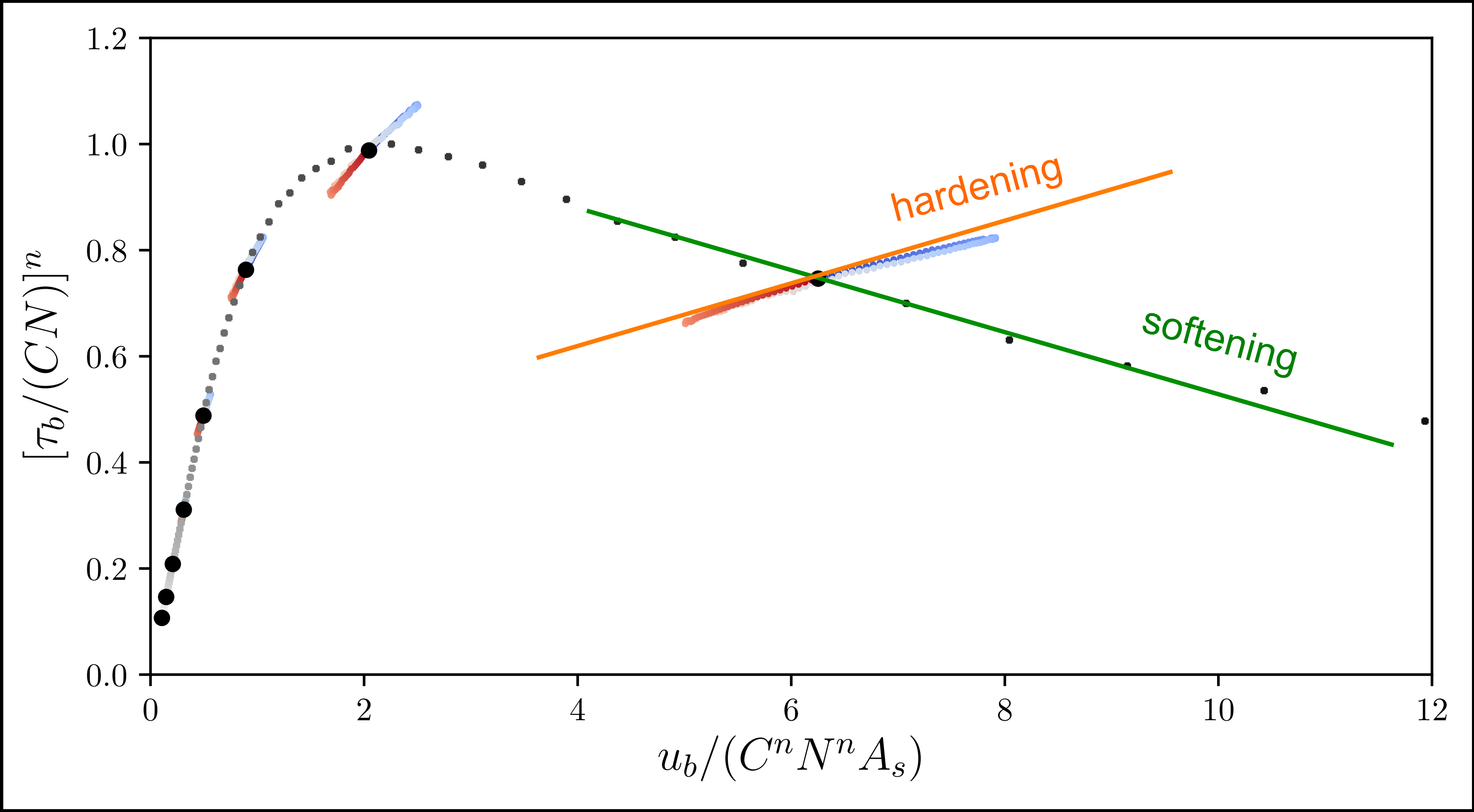




One **day** period

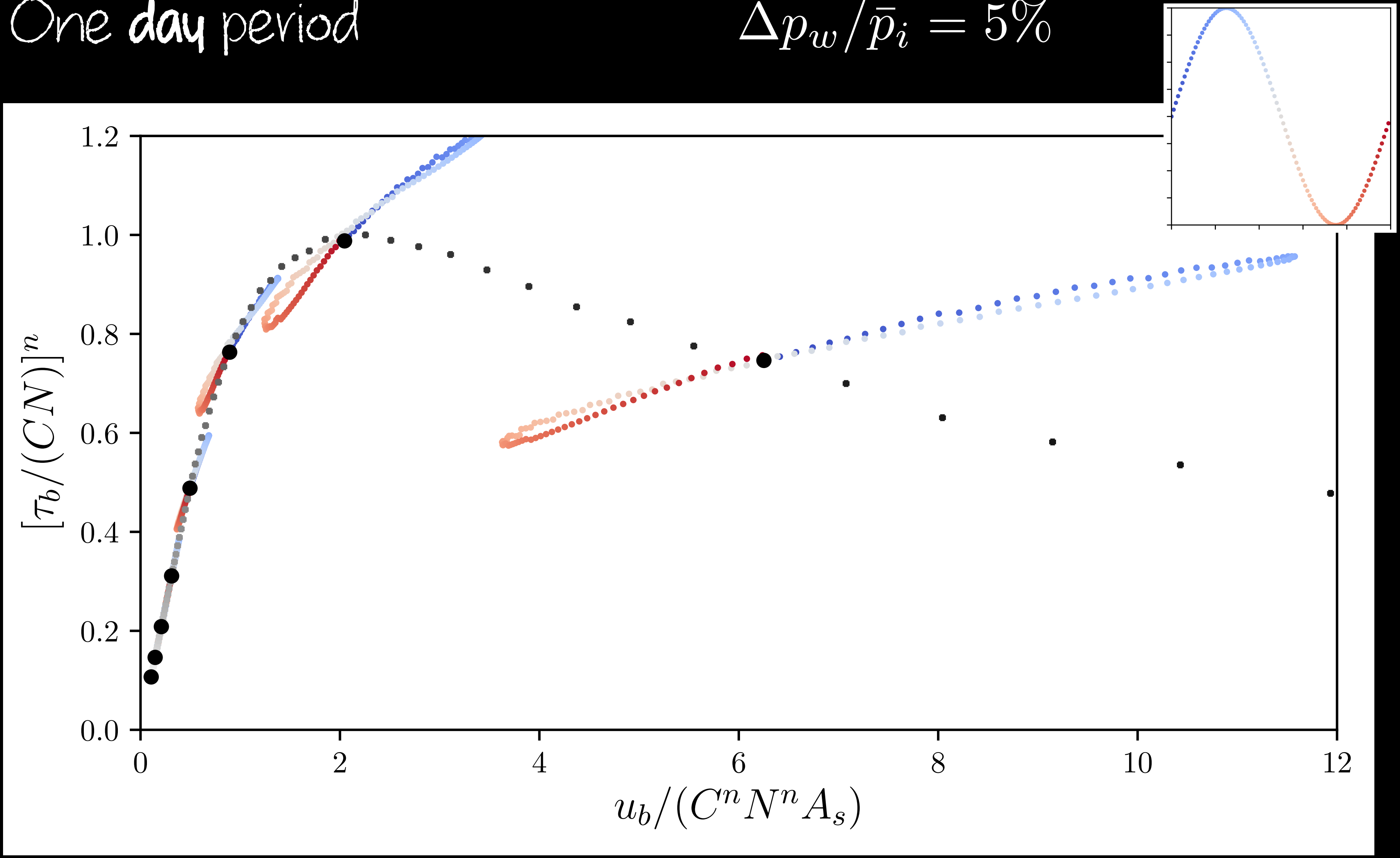
$\Delta p_w / \bar{p}_i = 2\%$





One **day** period

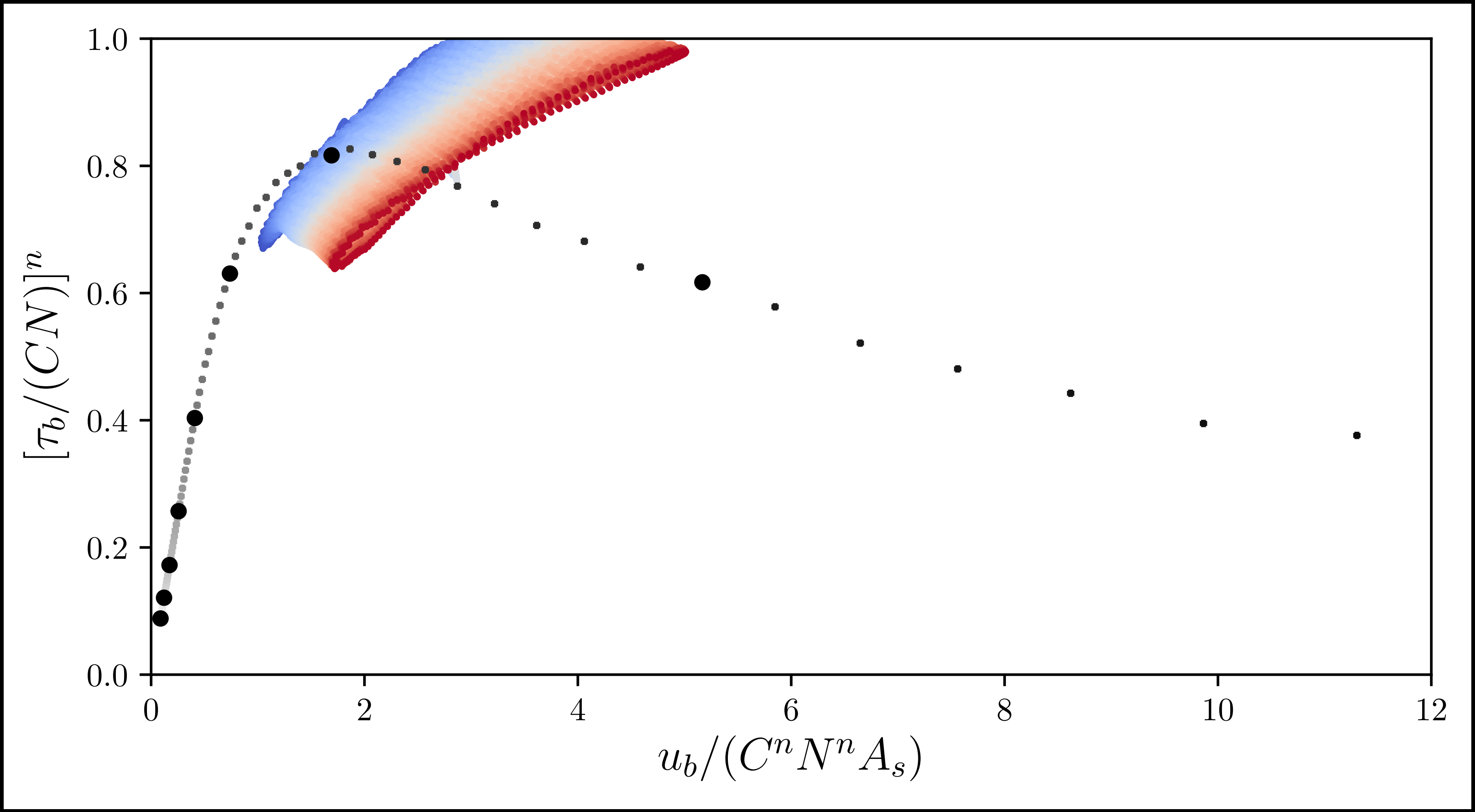
$\Delta p_w / \bar{p}_i = 5\%$

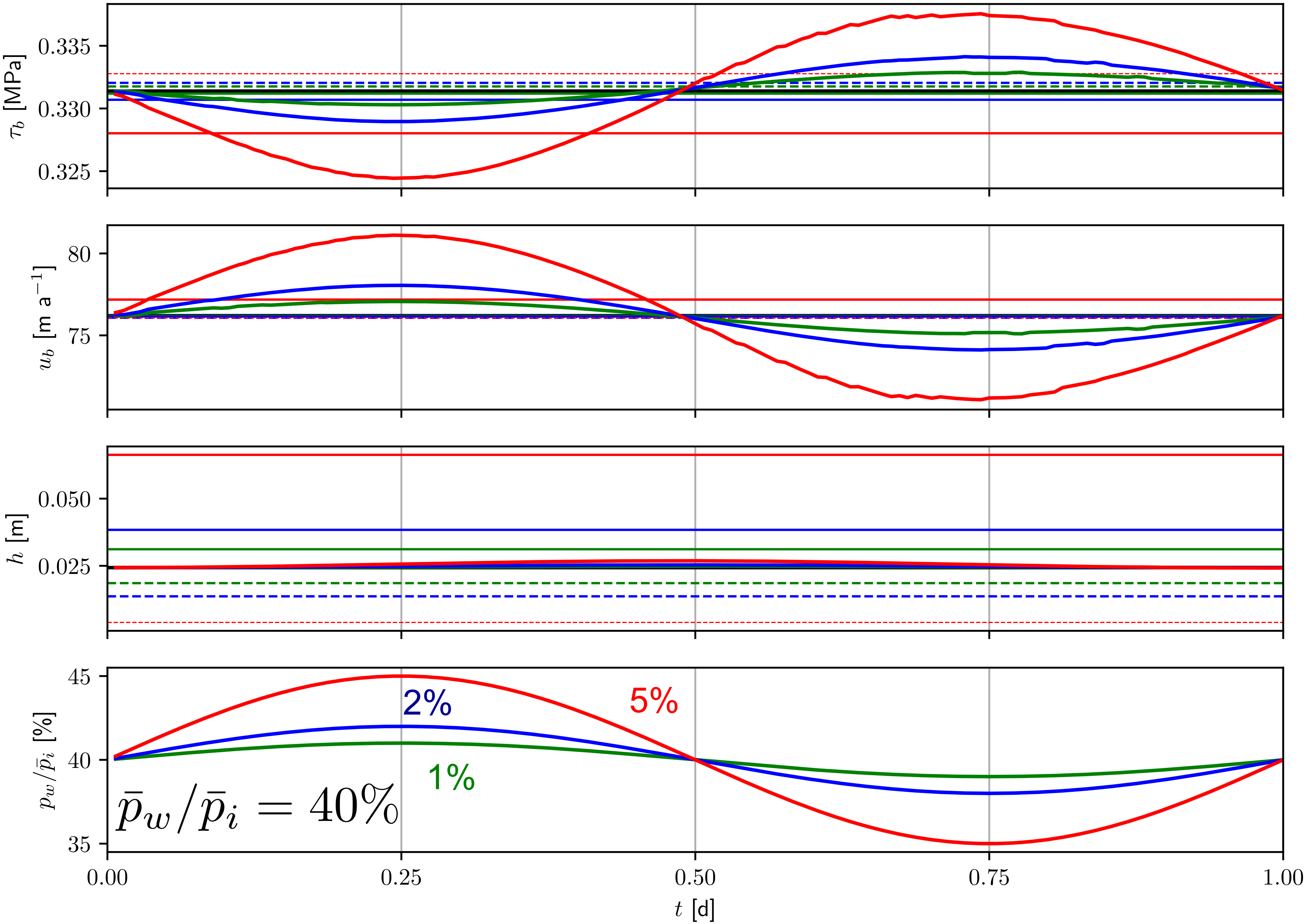


No time for the cavity to adapt to the pressure changes !

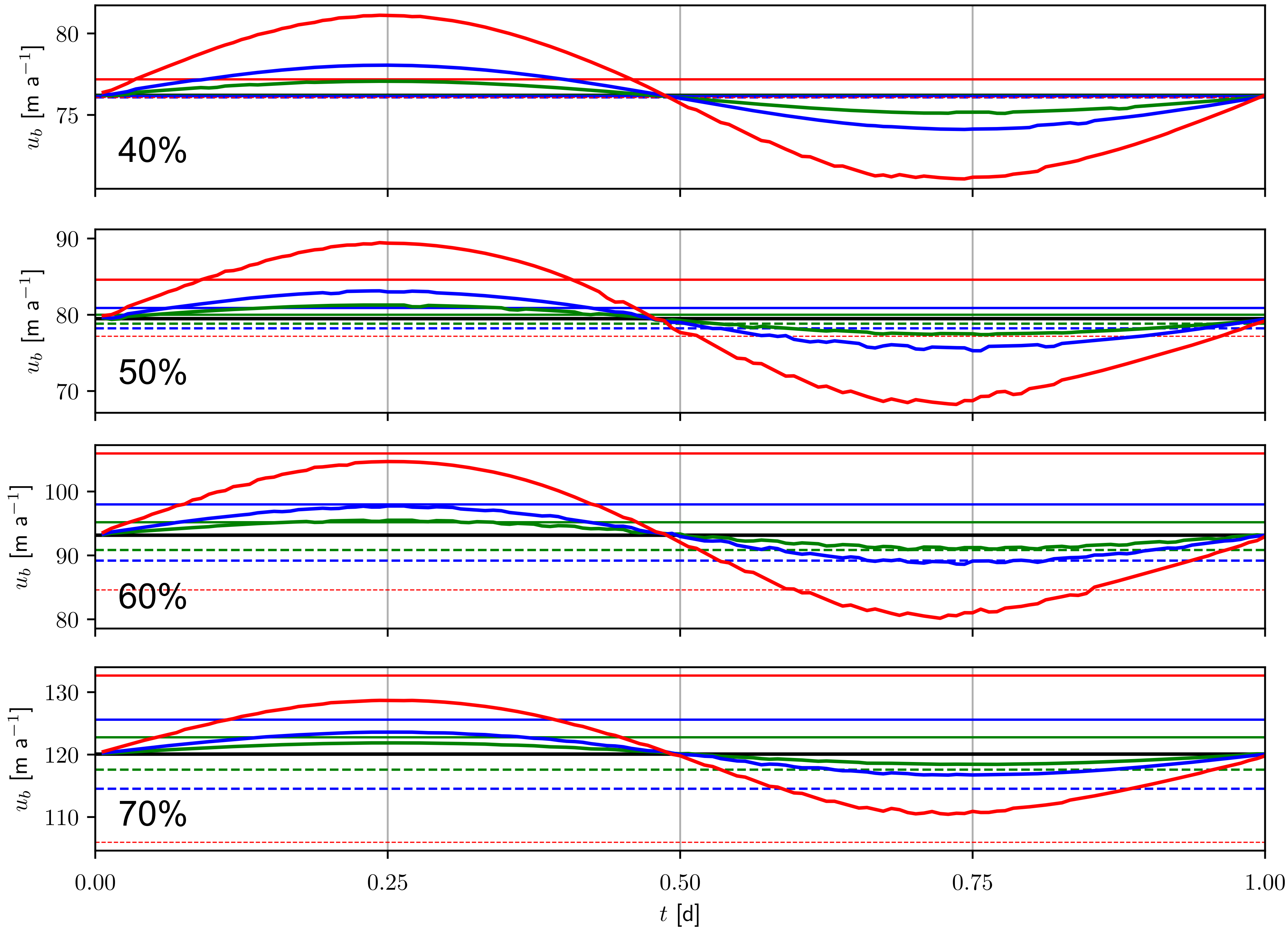
Year + day period

$\Delta p_w / \bar{p}_i = 5\%$





τ_b [MPa]
0.335
0.330
0.325
u_b [m a^{-1}]
80
75
h [m]
0.050
0.025
$p_w/\bar{p}_i$ [%]
45
40
35
2%
5%
1%
$\bar{p}_w/\bar{p}_i = 40\%$
0.00
0.25
0.50
0.75
1.00
t [d]



40%
50%
60%
70%
u_b [m a^{-1}]
80
75
90
70
100
130
120
110
0.00
0.25
0.50
0.75
1.00
t [d]

Conclusions

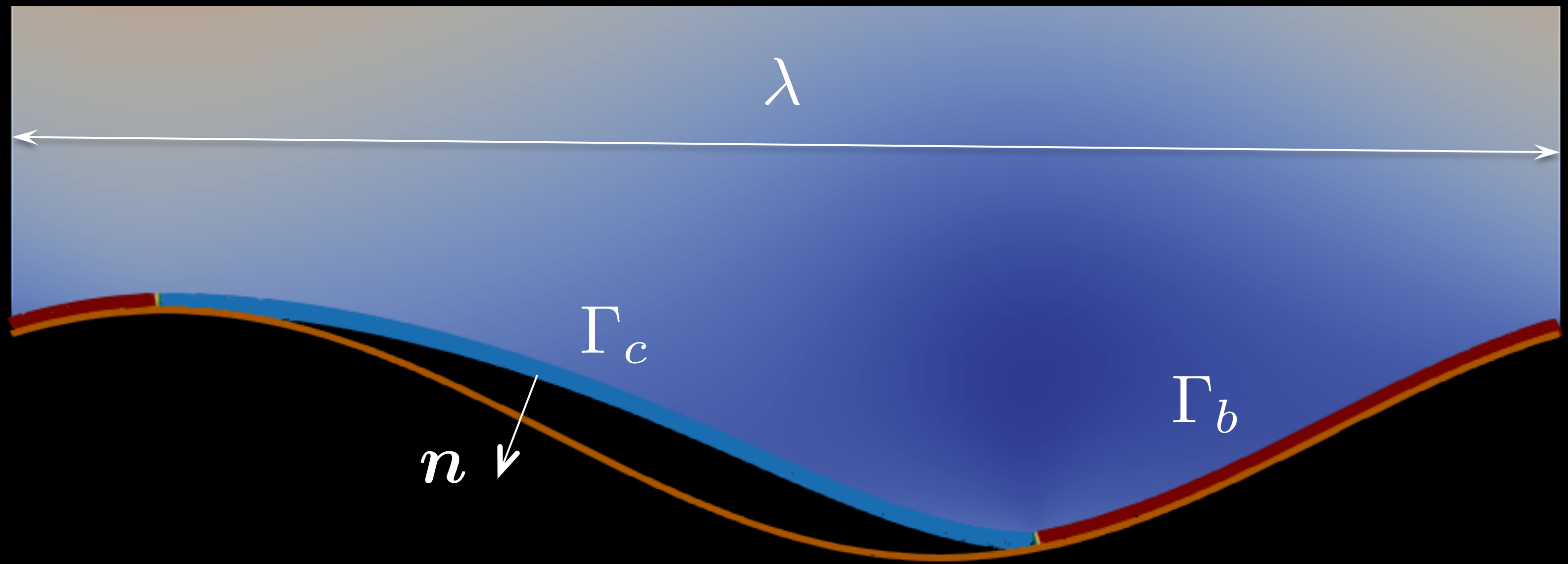
- Daily variations in water pressure not accounted for by such law
- The "softening" obtained for steady pressure might not exist in reality...
- What about Iken's bound?

. More work to be done

- how to incorporate this short term pressure variations in a friction law?
- effect of 3d geometry?

Looking for

- a Master student (spring 2019)
- a PhD student (start Sept 2019)



Over Γ_c: $\sigma_{nn} = p_w$

$$p_i = cte = -\frac{1}{\lambda}\int_{\Gamma_c} \sigma_{nn} dx - \frac{1}{\lambda}\int_{\Gamma_b} \sigma_{nn} dx$$

→ if $p_w \nearrow$, then $\sigma_{nn} = p_w$ on $\Gamma_c \nearrow$, then σ_{nn} on $\Gamma_b \searrow$

→ $$\tau_b = \frac{1}{\lambda}\int_{\Gamma_c} \sigma_{nn} \frac{\partial z_b}{\partial x} dx + \frac{1}{\lambda}\int_{\Gamma_b} \sigma_{nn} \frac{\partial z_b}{\partial x} dx$$?