Elmer/Ice advanced Workshop

22-24 November 2017

Marine ice-sheets and the Grounding line problem

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Grounding Line

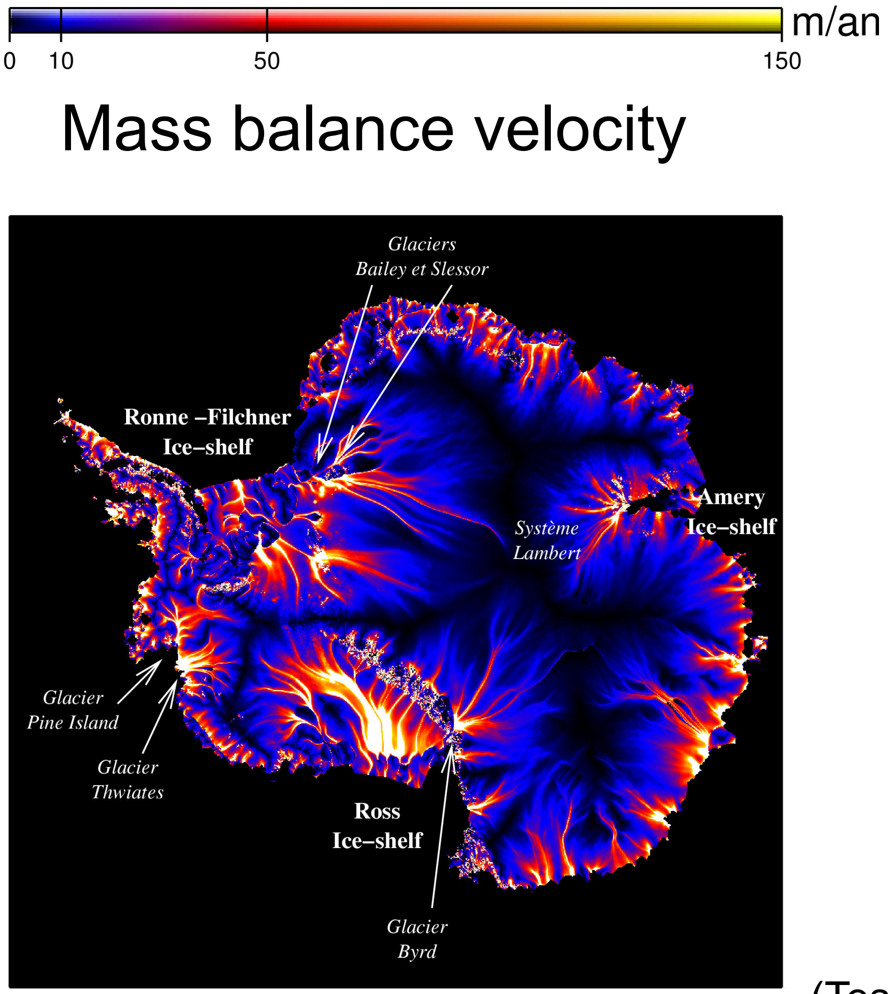
- ✓ **The Physics**
  - - Dynamics of ice-sheets
  - - The transition zone
  - - Results from grounding line models

- ✓ **Equations to be solved**
  - - The Schoof equation
  - - Solution of a contact problem

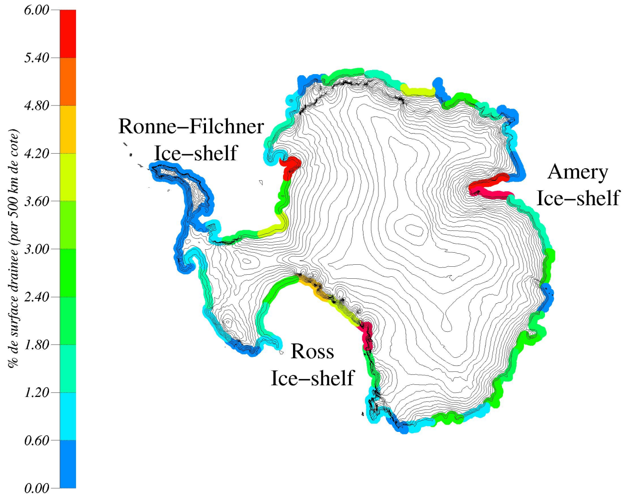
- ✓ **Implementation in Elmer/Ice (Stokes – see SSA presentation also)**
  - - The basal boundary
  - - How to evaluate the contact?
  - - Mesh size issue
  - - Interpolation of the friction

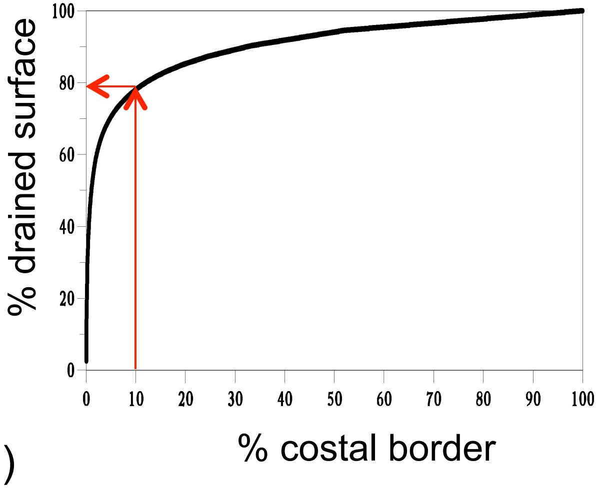
- ✓ **Example**
  - - MISMIP test

Importance of ice-stream



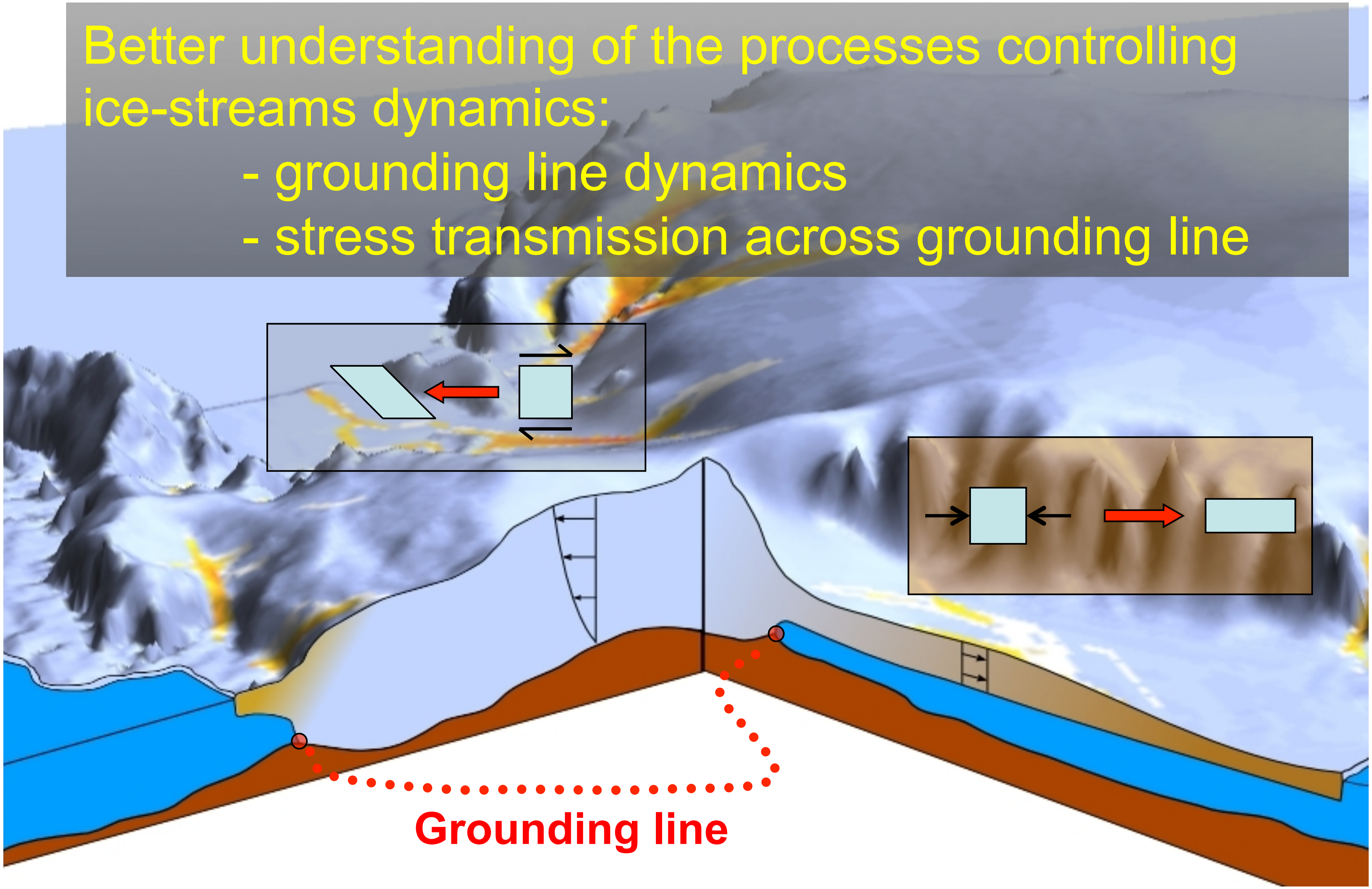
Mass balance velocity





(Testut, 2000)

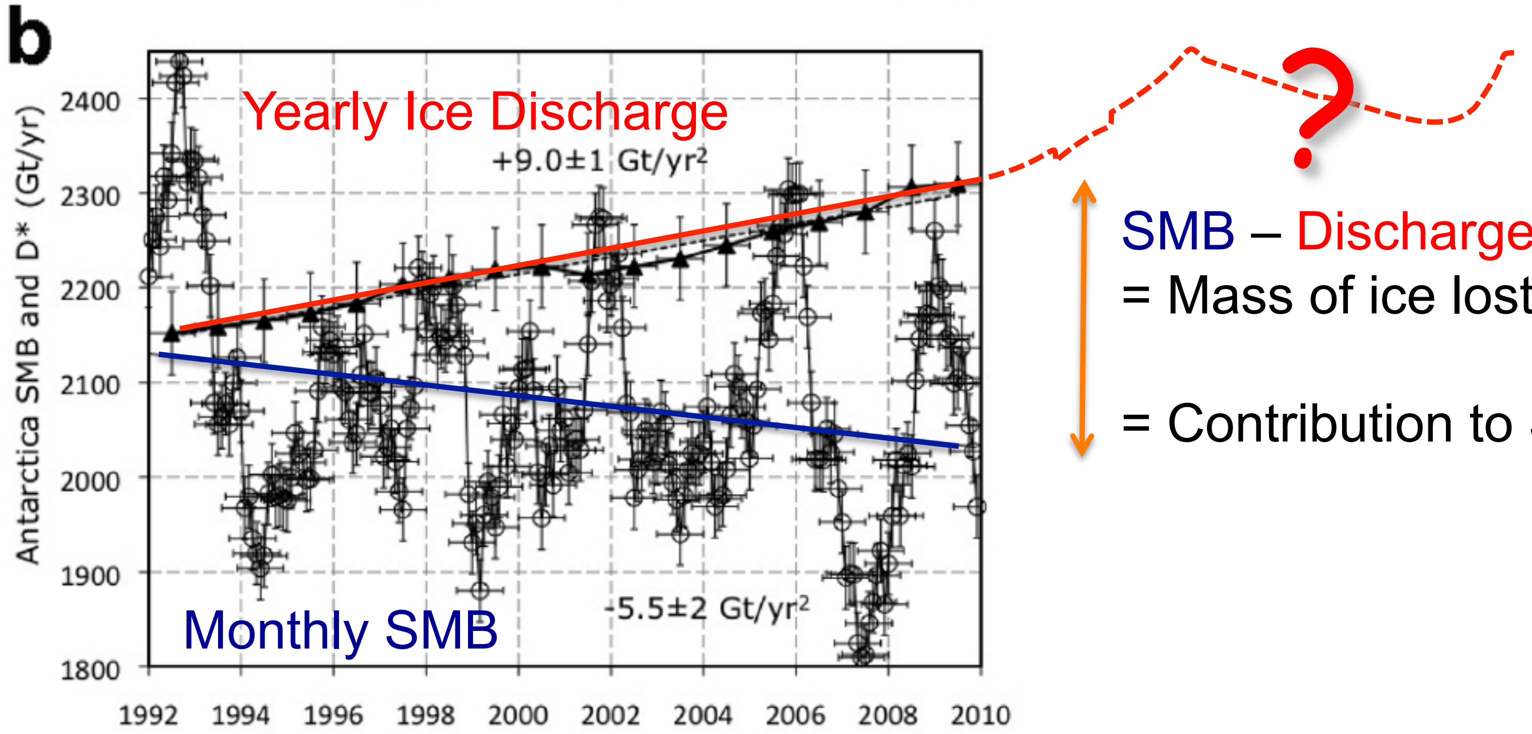
Transition zone



Better understanding of the processes controlling ice-streams dynamics:

- grounding line dynamics
- stress transmission across grounding line

Ice Discharge



SMB – Discharge
= Mass of ice lost

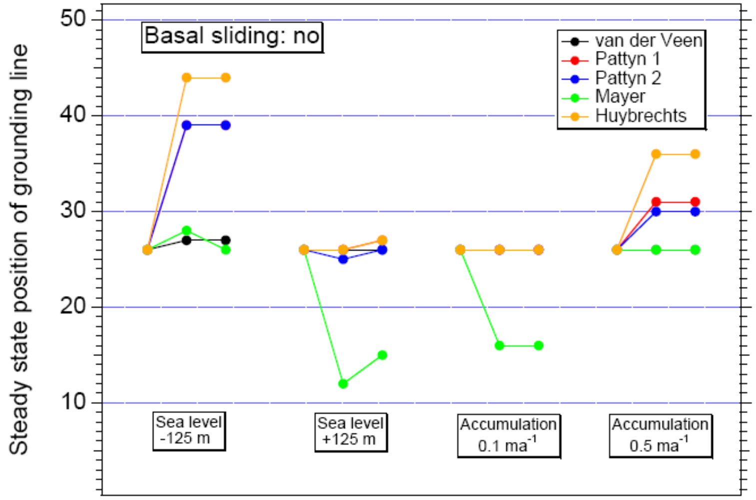
= Contribution to SLR

[Rignot et al., 2011]

What will be the future contribution of Ice Discharge for the next centuries?

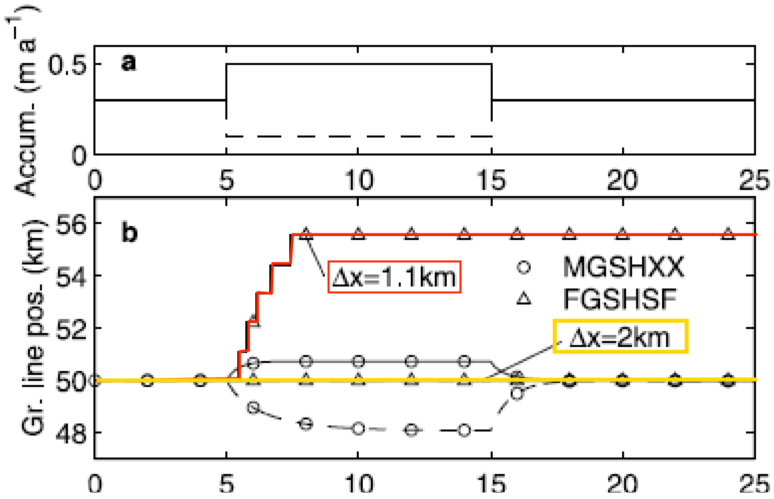
➡ **Need accurate description of the Grounding Line dynamics**

EISMINT Results



- **No consensus** on the results
- **No consensus** on how the GL should be modelled
- It is **unclear** wheter these results are indicative of neutral equilibrium

[Huybrechts, 1998]



Influnce of the horizontal grid size

[Vieli and Payne, 2005]

Poor ability of the model to capture the GL dynamic until recently

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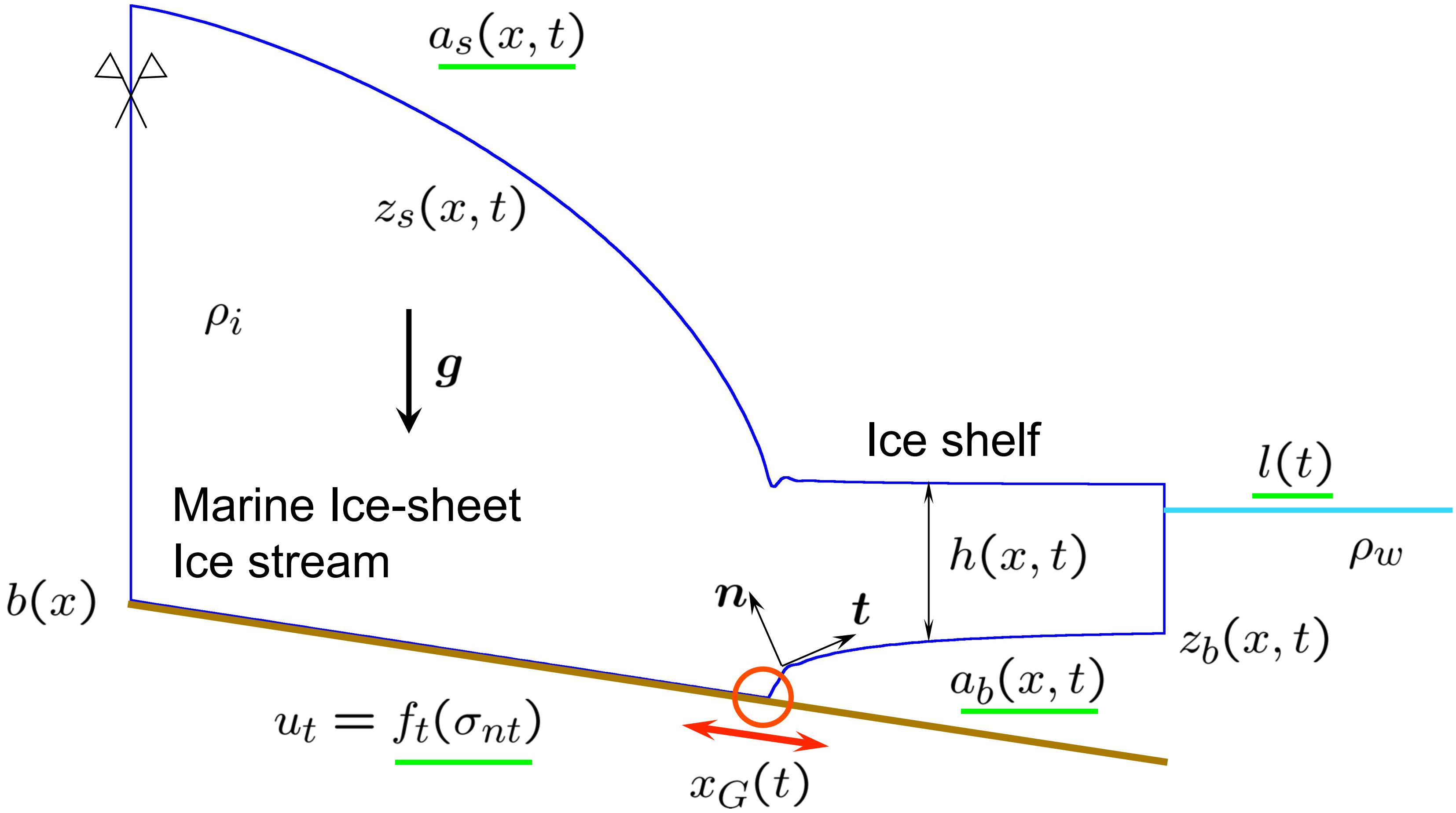
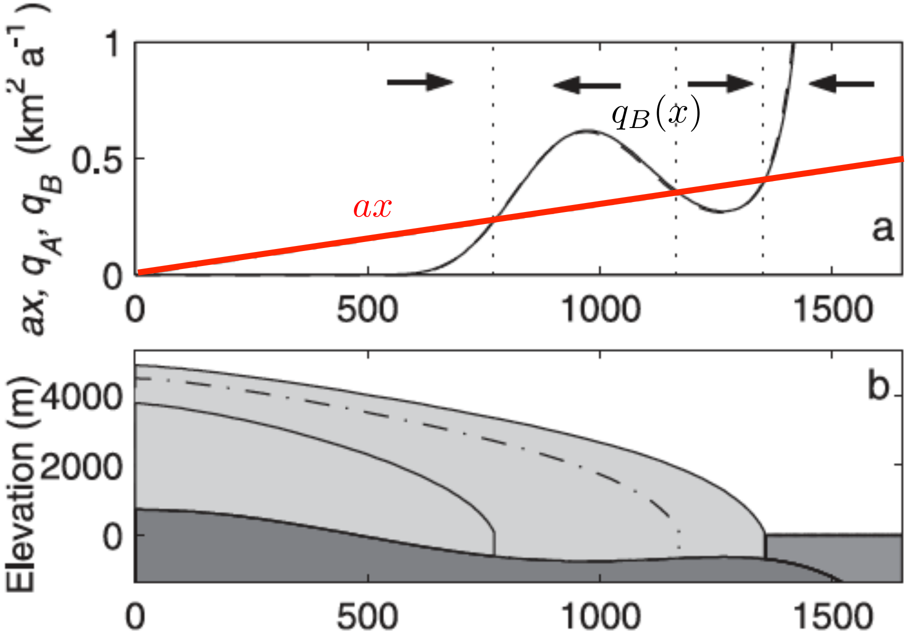
- ✓ **Example**
  - - MISMIP test

Notation / Concept



How the grounding line evolves for different scenarii ?

Schoof's solution (2007) – MISI in 2D

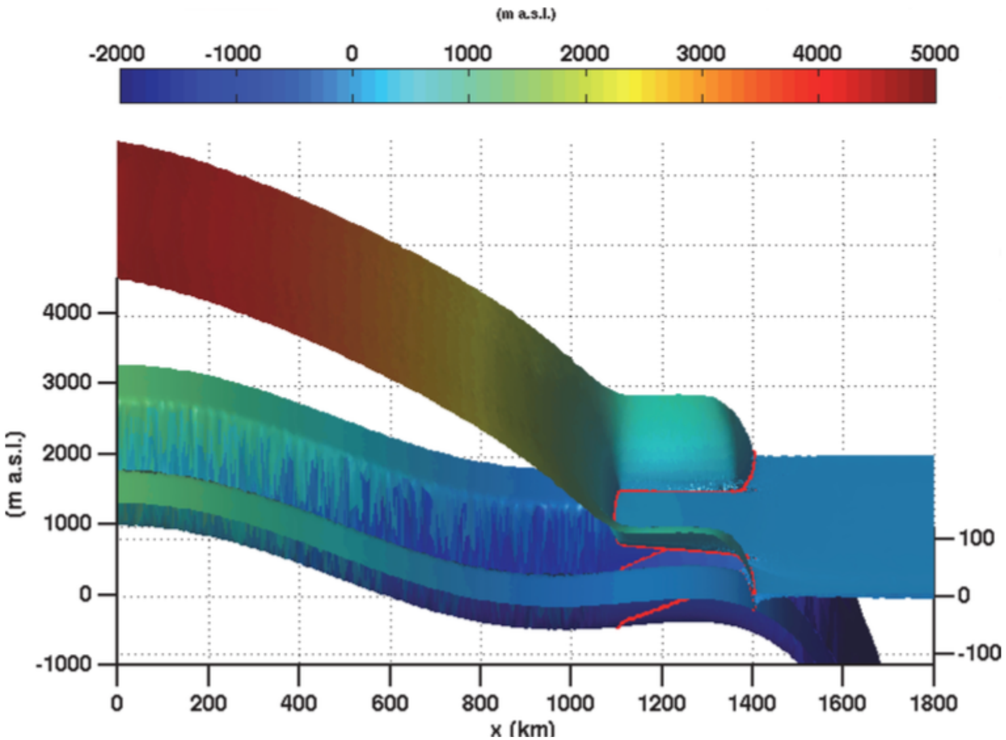


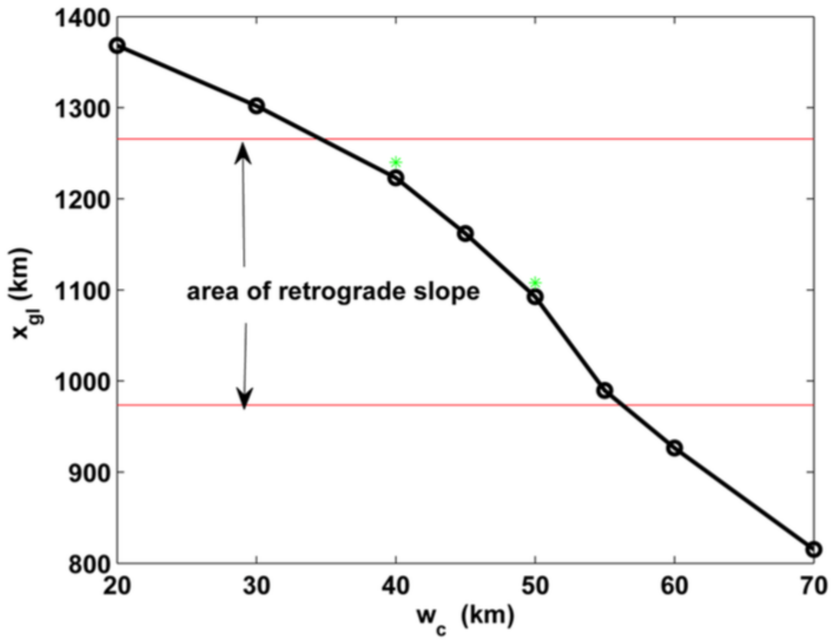
[Schoof, 2007, 2011]

$$a x_g = q_B(x_g) = \left(\frac{\bar{A}(\rho_i g)^{n+1}(1-\rho_i/\rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}}$$

- Confirms that there is no stable position of the GL on an upsloping bed
- For a given surface mass balance, gives the steady GL position (in 2D)

MISI in 3D

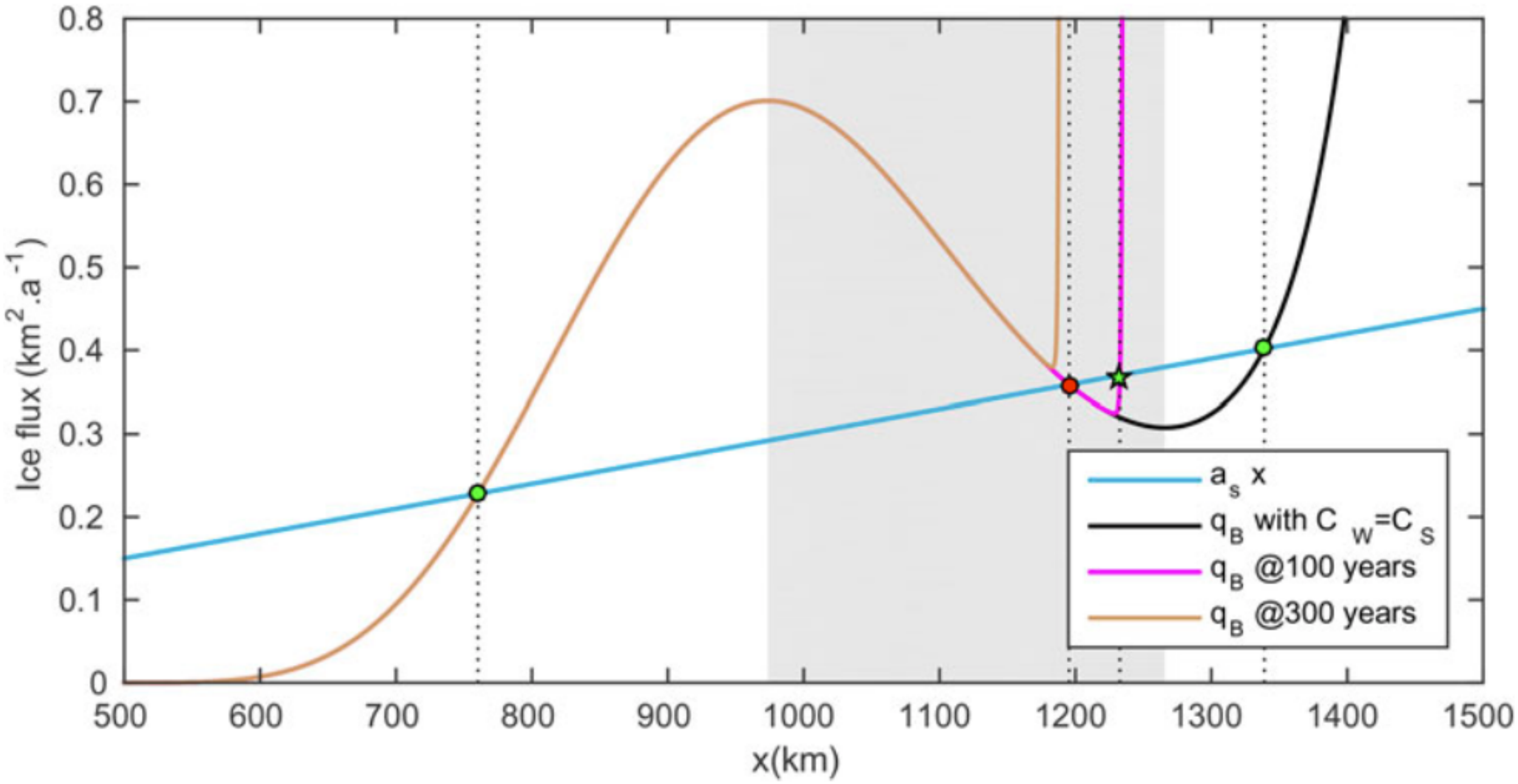




[Gudmundsson et al., 2012]

Marine ice sheets are not unconditionally unstable in two horizontal dimensions

MISI in 2D – non uniform friction parameter



[Brondex et al., 2017]

Stable positions can be found in the MISI for non uniform basal friction parameter

Equations to be solved

Ice Flow

$$\begin{cases} \boldsymbol{D} = A\tau_e^{n-1}\boldsymbol{S} \\ \operatorname{div}\boldsymbol{\sigma} + \rho\boldsymbol{g} = 0 \\ \operatorname{div}\boldsymbol{u} = 0 \end{cases}$$

Top free surface

$$\frac{\partial z_s}{\partial t} + u_x\frac{\partial z_s}{\partial x} + u_y\frac{\partial z_s}{\partial y} - u_z = a_s$$

Bottom free surface

$$\begin{cases} \dfrac{\partial z_b}{\partial t} + u_x\dfrac{\partial z_b}{\partial x} + u_y\dfrac{\partial z_b}{\partial y} - u_z = a_b \\ \text{with} \quad z_b \geq b \end{cases}$$

Ice - Bed contact

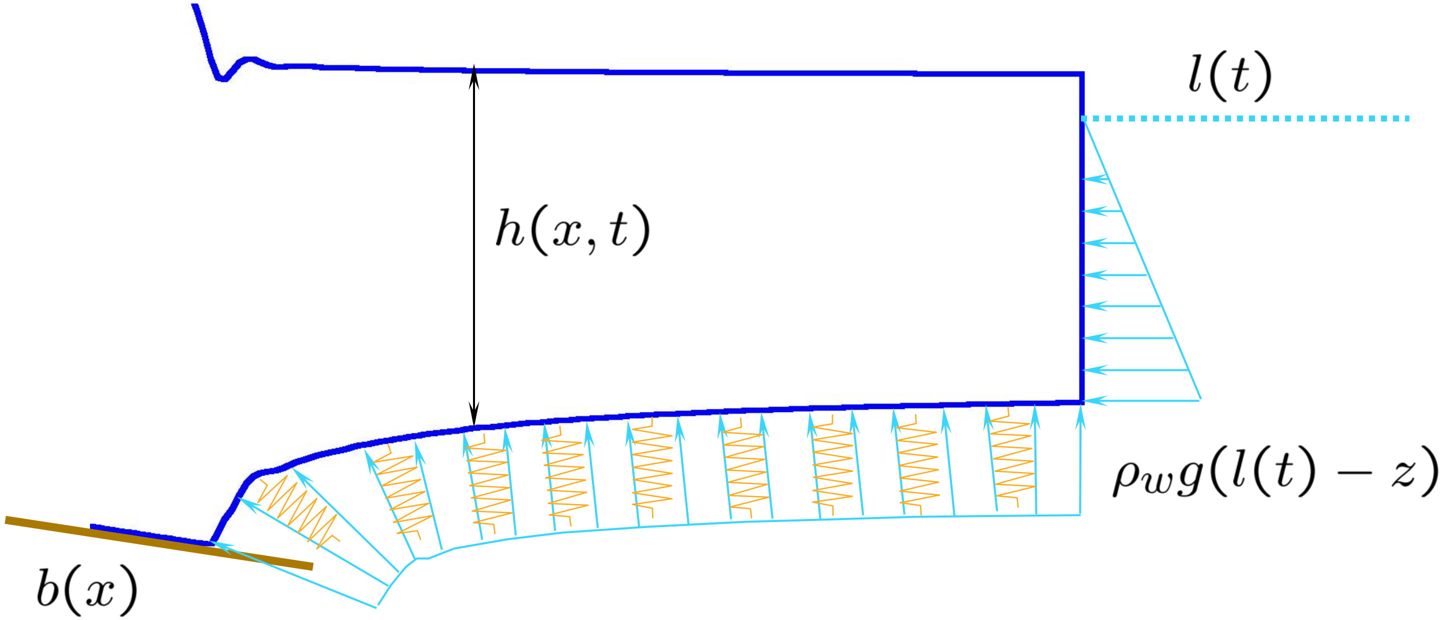
$z_b = b$ and $-\sigma_{nn} > p_w$ → $\boldsymbol{u}.\boldsymbol{n} = 0$, $u_t = f_t(\sigma_{nt})$

Ice - Sea

$$\left.\begin{array}{l} z_b = b \quad \text{and} \quad -\sigma_{nn} \leq p_w \\ \text{or} \quad z_b > b \end{array}\right\}$$

→ Buoyancy condition

Buoyancy BC



BC Stokes: if $z_b(x,t) > b(x)$

$$\sigma_{nn}(x) = \rho_w g(l(t) - z_b(x,t)) \quad \text{and} \quad \sigma_{nt}(x) = 0$$

$$z_b(x,t) = z_b(x, t - \mathrm{d}t) + u_n \sqrt{1 + (\mathrm{d}z_b/\,\mathrm{d}x)^2}\,\mathrm{d}t$$

$$\Rightarrow \sigma_{nn}(x) = \underline{\rho_w g(l(t) - z_b(x, t - \mathrm{d}t))} - \underline{\rho_w g \sqrt{1 + (\mathrm{d}z_b/\,\mathrm{d}x)^2}\,\mathrm{d}t.u_n}$$

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✓ **Example**
- MISMIP test

The basal boundary

Condition applied on the basal boundary depend if
- the ice is in contact with the bedrock
- or the ice is in contact with the sea

The limit between grounded and floating parts (the GL) is unknown and solution of the contact problem

Add a `Mask` variable (only on the basal surface) which tells if grounded, floating or at the GL

Mask = 1 if grounded
Mask = -1 if floating
Mask = 0 if at the GL

The basal boundary

In Elmer, the use of a **conditional Dirichlet** condition allows to deal with this evolving limit.

Example in the SIF:

```
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable Mask
   Real MATC "tx + 0.5"
```

Mask = -1 → the Dirichlet BC is not applied

Mask = 1 or 0 → the Dirichlet BC $u_n = 0$ is applied

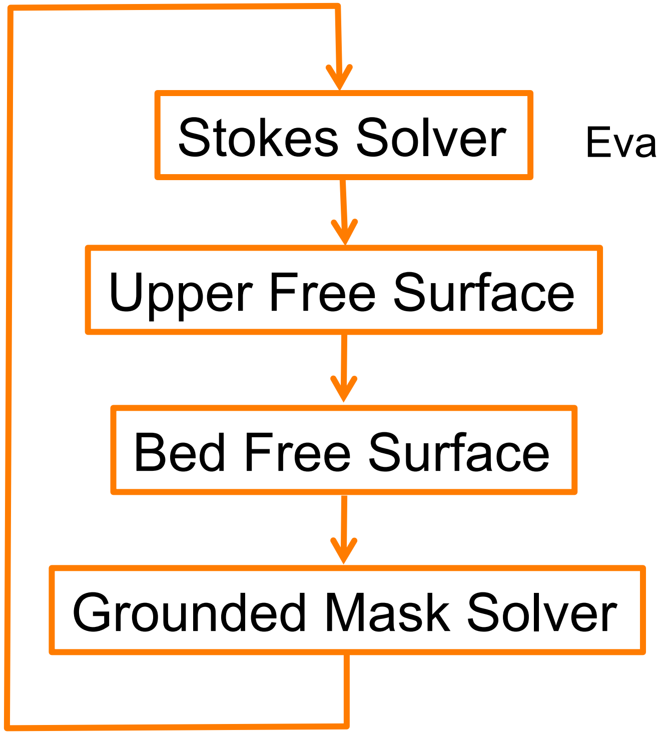
The contact problem

$z_b = b$ and $-\sigma_{nn} > p_w$ ⇒ Ice - Bed condition

$z_b = b$ and $-\sigma_{nn} \leq p_w$

or $z_b > b$

⇒ Ice - Sea condition



Stokes Solver

Upper Free Surface

Bed Free Surface

Grounded Mask Solver

Evaluate the condition $-\sigma_{nn} > p_w$ after the non-linear system has started converged

Evaluate the condition $z_b > b$

The contact problem

The condition $-\sigma_{nn} > p_w$ is in fact evaluated using nodal force (and not stress)

- the force exerted by the ice on the bed is given by the residual of the Stoles solution

In the Stokes solver

```
Exported Variable 1 = Flow Solution Loads[Stress Vector:2 CEQ Residual:1]
Calculate Loads = Logical True
```

- the nodal water force is the integrated water pressure with respect to the surface element

add a new solver to integrate the water pressure

Water force

```
SUBROUTINE GetHydrostaticLoads( Model,Solver,dt,TransientSimulation )

VariableValues = 0.0_dp

DO t = 1, Solver % NumberOfActiveElements
    Element => GetActiveElement(t)
    IF (ParEnv % myPe .NE. Element % partIndex) CYCLE
    n = GetElementNOFNodes()

    BC => GetBC( Element )
    pwt(1:n) =  -1.0 * ListGetReal(BC, 'External Pressure', n, &
                    Element % NodeIndexes , GotIt)
    CALL GetElementNodes( Nodes )
    IP = GaussPoints( Element )
    DO p = 1, IP % n

      stat = ElementInfo( Element, Nodes, IP % U(p), IP % V(p), &
      IP % W(p), detJ, Basis, dBasisdx, ddBasisddx, .FALSE.)
      s = detJ * IP % S(p)

      Normal = NormalVector( Element, Nodes, IP % U(p), IP % V(p), .TRUE.)
      pwi = SUM(pwt(1:n)*Basis(1:n))
      PwVector(1:DIM) = pwi * Normal(1:DIM)

      DO i = 1, n
            Nn = Permutation(Element % NodeIndexes(i))
            DO j = 1, DIM
              VariableValues(DIM*(Nn-1)+j) = VariableValues(DIM*(Nn-1)+j) + PwVector(j) *
s * Basis(i)
            END DO
      END DO
    END DO
  END DO
  IF ( ParEnv % PEs>1 ) CALL ParallelSumVector( Solver % Matrix, VariableValues )
!------------------------------------------------------------------------------
END SUBROUTINE GetHydrostaticLoads
!------------------------------------------------------------------------------
```

The bed boundary condition

```
Boundary Condition 1
Target Boundaries = 1
  Body Id = 3

  Normal-Tangential Velocity = Logical True
  Flow Force BC = Logical True
!
! Bedrock conditions
!
  Slip Coefficient 2 = Variable Coordinate 1
    Real Procedure ”ElmerIceUSF" "SlidCoef_Contact”
    Sliding Law = String "Weertman"
    Weertman Friction Coefficient = Real $C
    Weertman Exponent = Real $(1.0/n)
    Weertman Linear Velocity = Real 1.0

  Grounding line Definition = String “Discontinuous”

  Velocity 1 = Real 0.0
  Velocity 1 Condition = Variable GroundedMask
    Real MATC "tx + 0.5"
!
! Shelf conditions
!
  External Pressure = Variable Coordinate 2
     Real Procedure ”ElmerIceUSF" "SeaPressure"

  Slip Coefficient 1 = Variable Coordinate 2
     Real Procedure ”ElmerIceUSF" "SeaSpring”
End
```

The variable `GroundedMask` is updated in this User Function `SlidCoef_Contact`

`Here shown for Weertman, work also for other friction laws`

See note after

Will only apply if the Dirichlet condition `Velocity 1 = 0` is not applied

The user function `SlidCoef_Contact`

Test the contact condition:

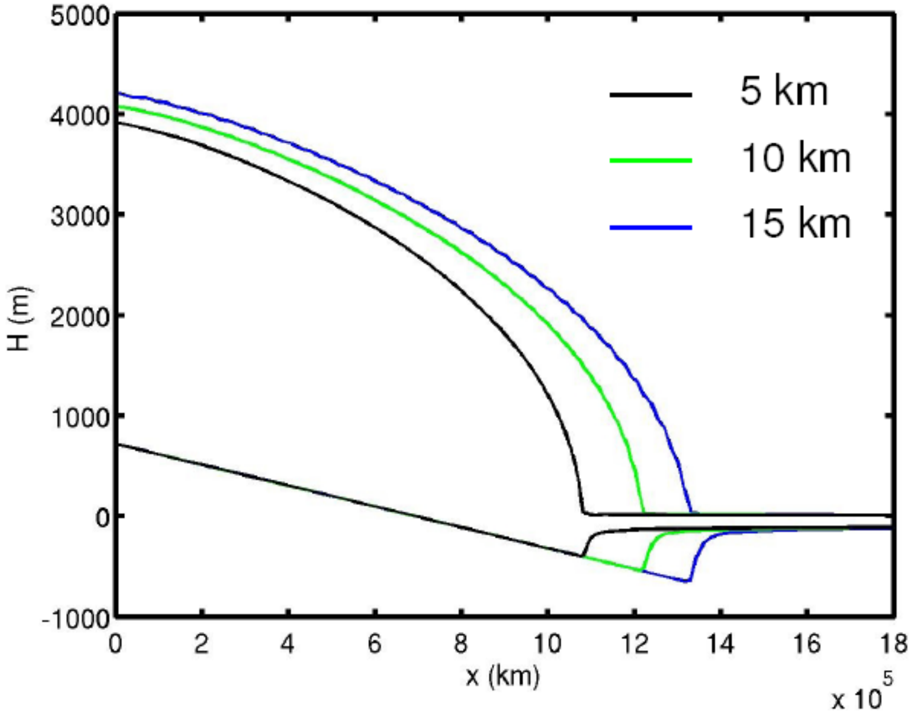
```
Normal = NormalValues(DIM*(NormalPerm(jj)-1)+1 : DIM*NormalPerm(jj))
Fwater = Hydro(DIM*(HydroPerm(jj)-1)+1 : DIM*HydroPerm(jj))
Fbase = ResidValues((DIM+1)*(ResidPerm(jj)-1)+1 : (DIM+1)*ResidPerm(jj)-1)


comp = ABS( SUM( Fwater * Normal ) ) - ABS( SUM( Fbase * Normal ) )

IF (comp >= 0.0_dp)  GroundedMask(Nn) = -1.0_dp
```

and return the sliding coefficient:

- appropriate if grounded
- 0 if floating

```
cond = GroundedMask(GroundedMaskPerm(nodenumber))
IF (cond > -0.5_dp) THEN
          Bdrag = Sliding_weertman(Model, nodenumber, y)
ELSE
          Bdrag = 0.0_dp
END IF
```

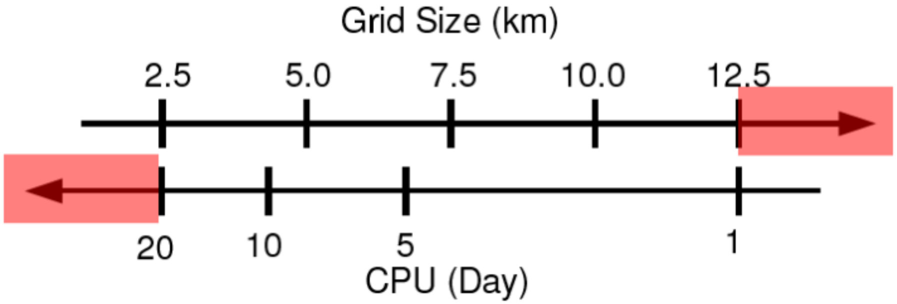
Sensitivity to the grid size



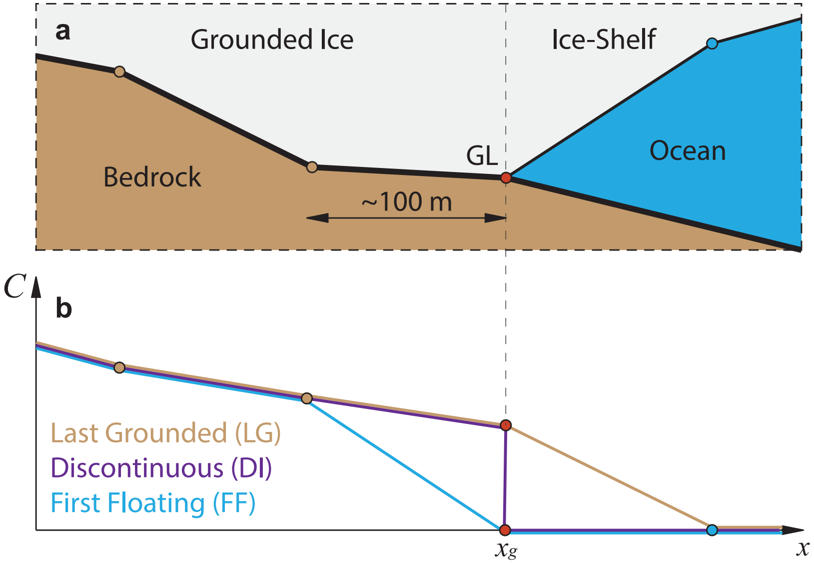
Steady for different horizontal grid sizes

Strong influence of the grid size !

Problem : CPU cost !



Interpolation of the friction

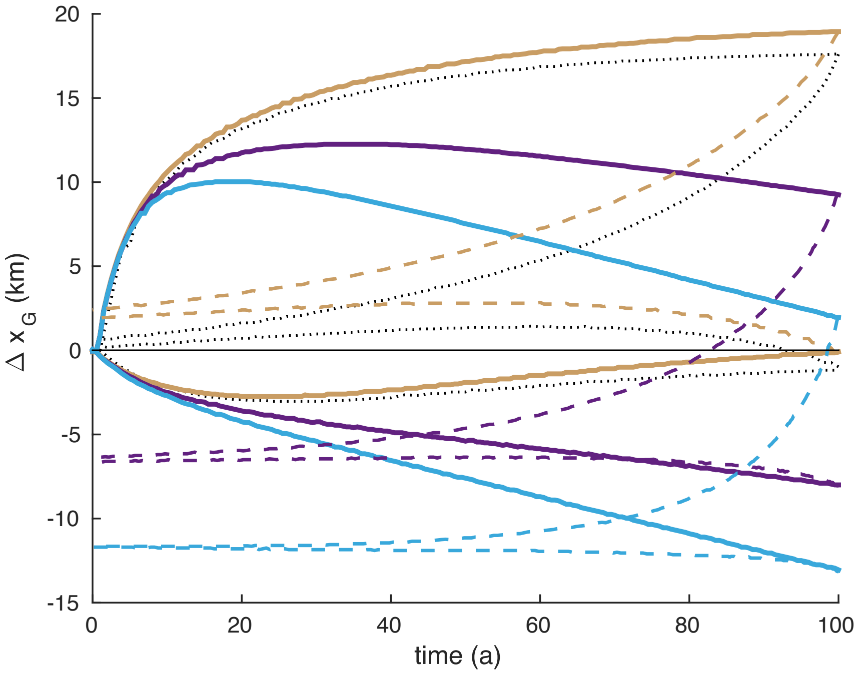


Use Discontinous!

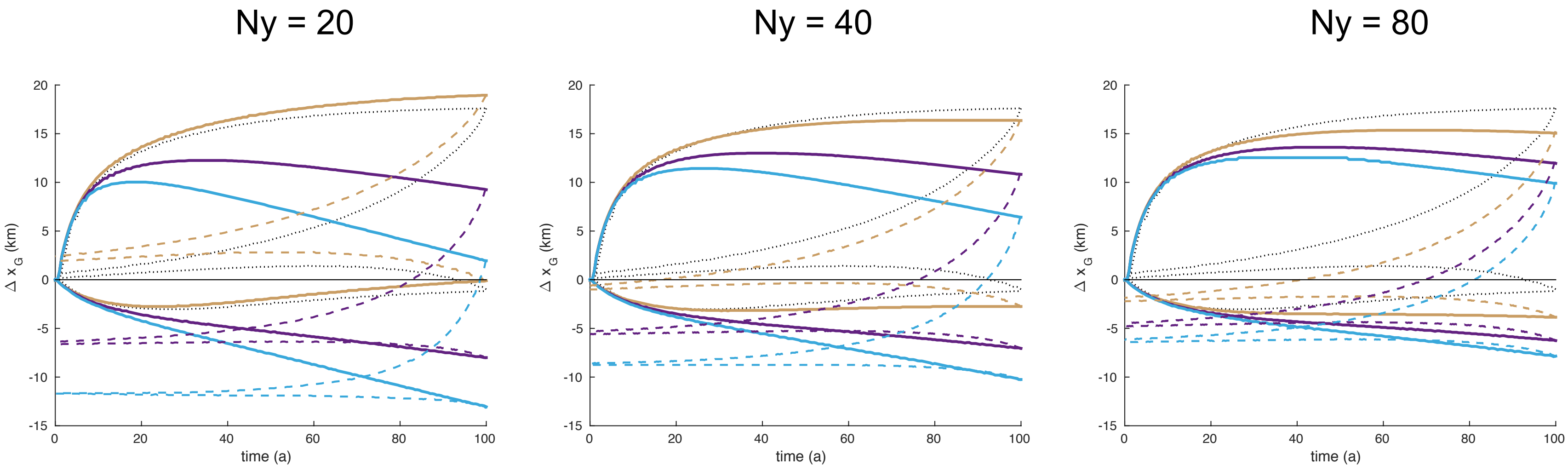
Or better: use a water pressure dependant friction law (see presentation on friction law)

[Gagliardini et al., 2016]

MISMIP3d – Ny = 20

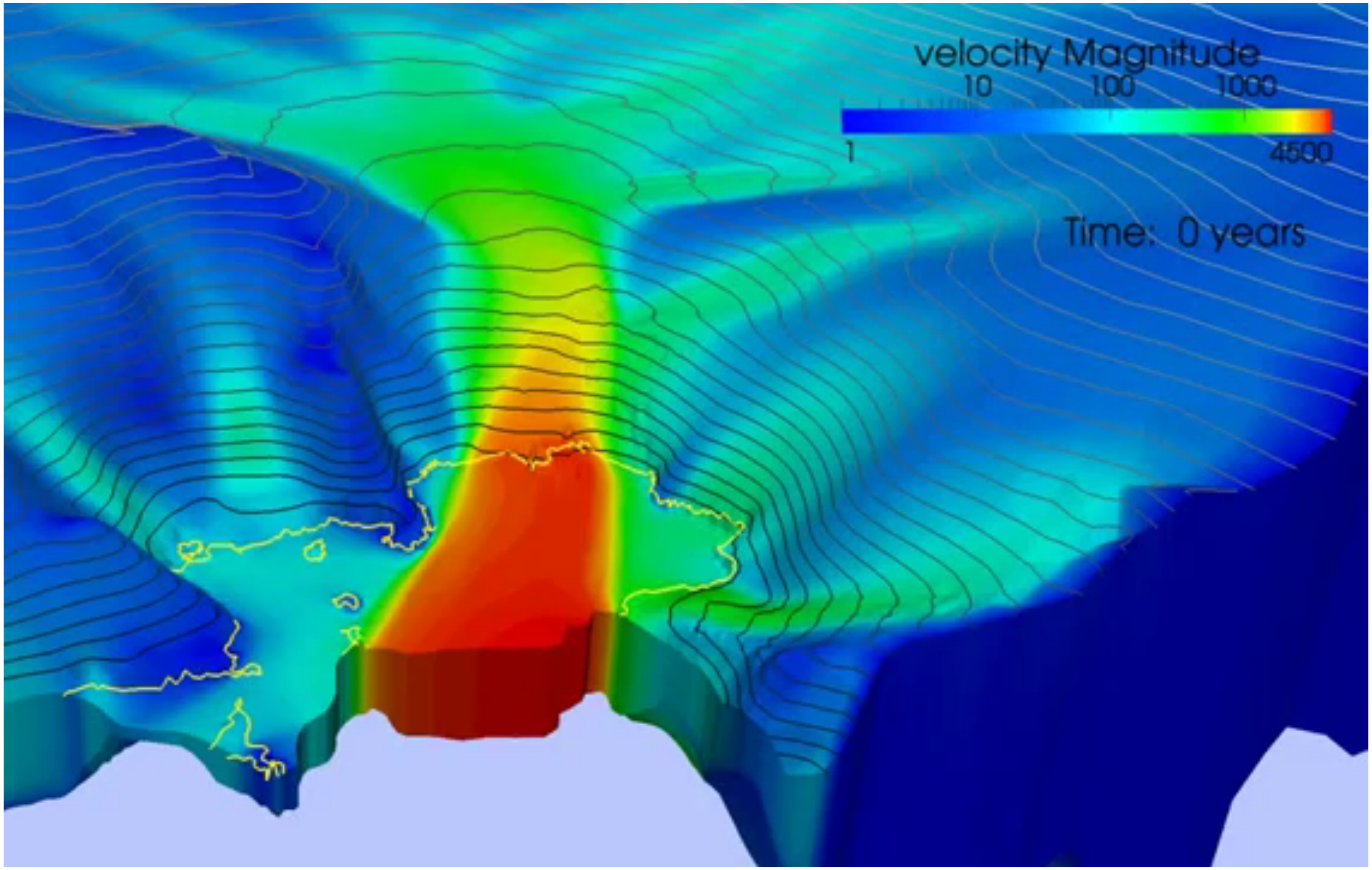


Interpolation of the friction



[Gagliardini et al., 2016]

PIG example (Favier et al., 2014)



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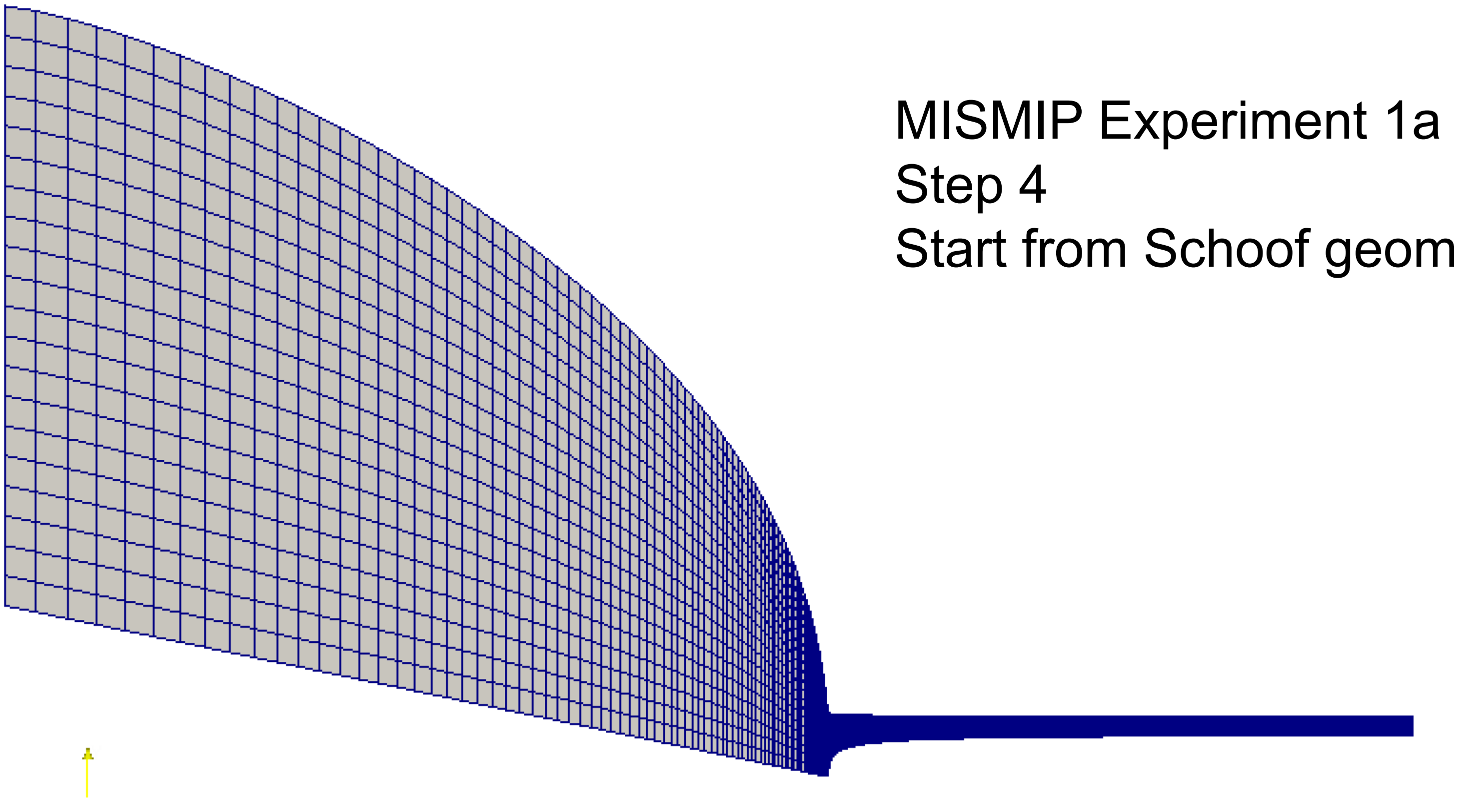
Example GL_MISMIP



MISMIP Experiment 1a
Step 4
Start from Schoof geometry

http://elmerice.elmerfem.org/wiki/doku.php?id=problems:groundingline

[ELMER_TRUNK]/elmerice/Tests/GL_MISMIP

References

Durand G., O. Gagliardini, B. de Fleurian, T. Zwinger and E. Le Meur. 2009. Marine Ice-Sheet Dynamics: Hysteresis and Neutral Equilibrium, J. of Geophys. Res., Earth Surface, 114, F03009, doi:10.1029/2008JF001170.

Durand G., O. Gagliardini, T. Zwinger, E. Le Meur and R.C.A. Hindmarsh, 2009. Full-Stokes modeling of marine ice-sheets: influence of the grid size., Annals of Glaciology, 50(52), p. 109-114.

Gagliardini O., G. Durand, T. Zwinger, R. Hindmarsh and E. Le Meur, 2010. Coupling of ice-shelf melting and buttressing is a key process in ice-sheets dynamics, Geophys. Res. Lett., 37, L14501, doi:10.1029/2010GL043334.

Durand G., O. Gagliardini, L. Favier, T. Zwinger and E. le Meur, 2011. Impact of bedrock description on modeling ice sheet dynamic, Geophys. Res. Lett., 38, L20501, doi:10.1029/2011GL048892.

Favier L., O. Gagliardini, G. Durand, and T. Zwinger, 2012. A three-dimensional full Stokes model of the grounding line dynamics: effect of a pinning point beneath the ice shelf, The Cryosphere, 6, 101-112, doi:10.5194/tc-6-101-2012.

Pattyn, F., C. Schoof, L. Perichon, R.C.A. Hindmarsh, E. Bueler, B. de Fleurian, G. Durand, O. Gagliardini, R. Gladstone, D. Goldberg, G.H. Gudmundsson, V. Lee, F.M. Nick, A.J. Payne, D. Pollard, O. Rybak, F. Saito and A. Vieli, 2012. Results of the Marine Ice Sheet Model Intercomparison Project, MISMIP, The Cryosphere, 6, 573-588, doi:10.5194/tc-6-573-2012

Gudmundsson, G. H., J. Krug, G. Durand, L. Favier and O. Gagliardini, 2012. The stability of grounding lines on retrograde slopes, The Cryosphere, 6, 1497-1505, doi:10.5194/tc-6-1497-2012.

Drouet, A. S., D. Docquier, G. Durand, R. Hindmarsh, F. Pattyn, O. Gagliardini, and T. Zwinger, 2013. Grounding line transient response in marine ice sheet models, The Cryosphere, 7, 395-406, doi:10.5194/tc-7-395-2013.

Pattyn, F, L. Perichon, G. Durand, L. Favier, O. Gagliardini, R. C. A. Hindmarsh, T. Zwinger, T. Albrecht, S. Cornford, D. Docquier, J. J. Fürst, D. Golberg, G. H. Gudmundsson, A. Humbert, M. Hütten, P. Huybrechts, G. Jouvet, T. Kleiner, E. Larour, D. Martin, M. Morlighem, A. J. Payne, D. Pollard, M. Rückamp, O. Rybak, H. Seroussi, M. Thoma and N. Wilkens, 2013. Grounding-line migration in plan-view marine ice-sheet models: results of the ice2sea MISMIP3d intercomparison, J. Glaciol., 59, doi:10.3189/2013JoG12J129.

Favier, L., G. Durand, S. L. Cornford, G. H. Gudmundsson, O. Gagliardini, F. Giller-Chaulet, T. Zwinger, A. J. Payne and A. M. Le Brocq, 2014. Retreat of Pine Island Glacier controlled by marine ice-sheet instability, Nature Climate Change, doi:10.1038/nclimate2094.

References

Krug, J., J. Weiss, O. Gagliardini and G. Durand, 2014. Combining damage and fracture mechanics to model calving, The Cryosphere, 8, 2101-2117, doi:10.5194/tc-8-2101-2014.

Todd, J., and P. Christophersen, 2014. Are seasonal calving dynamics forced by buttressing from ice mélange or undercutting by melting? Outcomes from full-Stokes simulations of Store Glacier, West Greenland , The Cryosphere, 8, 2353-2365, doi:10.5194/tc-8-2353-2014.

Krug, J., G. Durand, O. Gagliardini and J. Weiss, 2015. Modelling the impact of submarine frontal melting and ice mélange on glacier dynamics, The Cryosphere, 9, 989-1003, doi:10.5194/tc-9-989-2015.

Gagliardini O., J. Brondex, F. Gillet-Chaulet, L. Tavard, V. Peyaud and G. Durand, 2016. Brief communication: Impact of mesh resolution for MISMIP and MISMIP3d experiments using Elmer/Ice, The Cryosphere, 10, 307-312, doi:10.5194/tc-10-307-2016.

Gladstone, R.M., R.C. Warner, B.K. Galton-Fenzi, O. Gagliardini, T. Zwinger and R. Greve, 2017. Marine ice sheet model performance depends on basal sliding physics and sub-shelf melting, The Cryosphere, 11, 319-329, doi:10.5194/tc-11-319-2017.

~ 15 papers published so far using the contact problem implemented for the Stokes equations to solve the GL dynamics.