Elmer/Ice advanced Workshop

22-24 November 2017

Basal Conditions
(Friction laws & Hydrology)

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Basal Conditions

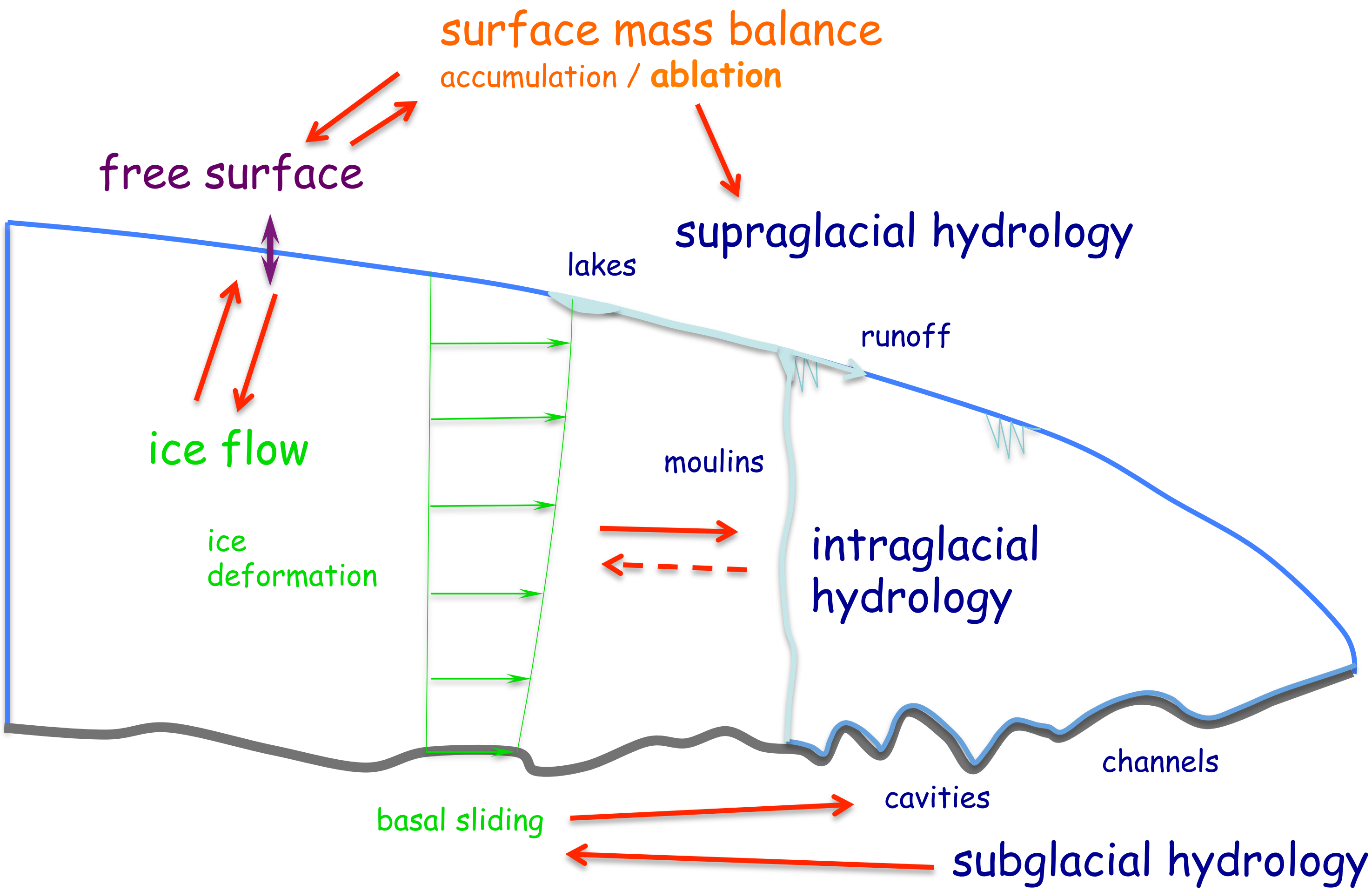
✓ **The Physics**
- Sliding at the base of glacier
- The role of basal water
- Different drainage systems

✓ **Friction laws and Hydrology**
- Linear friction law
- Weertman type friction law
- Water-pressure dependant friction laws
- Double continuum hydrology model
- GlaDS model

✓ **Implementation in Elmer/Ice**
- Various friction laws

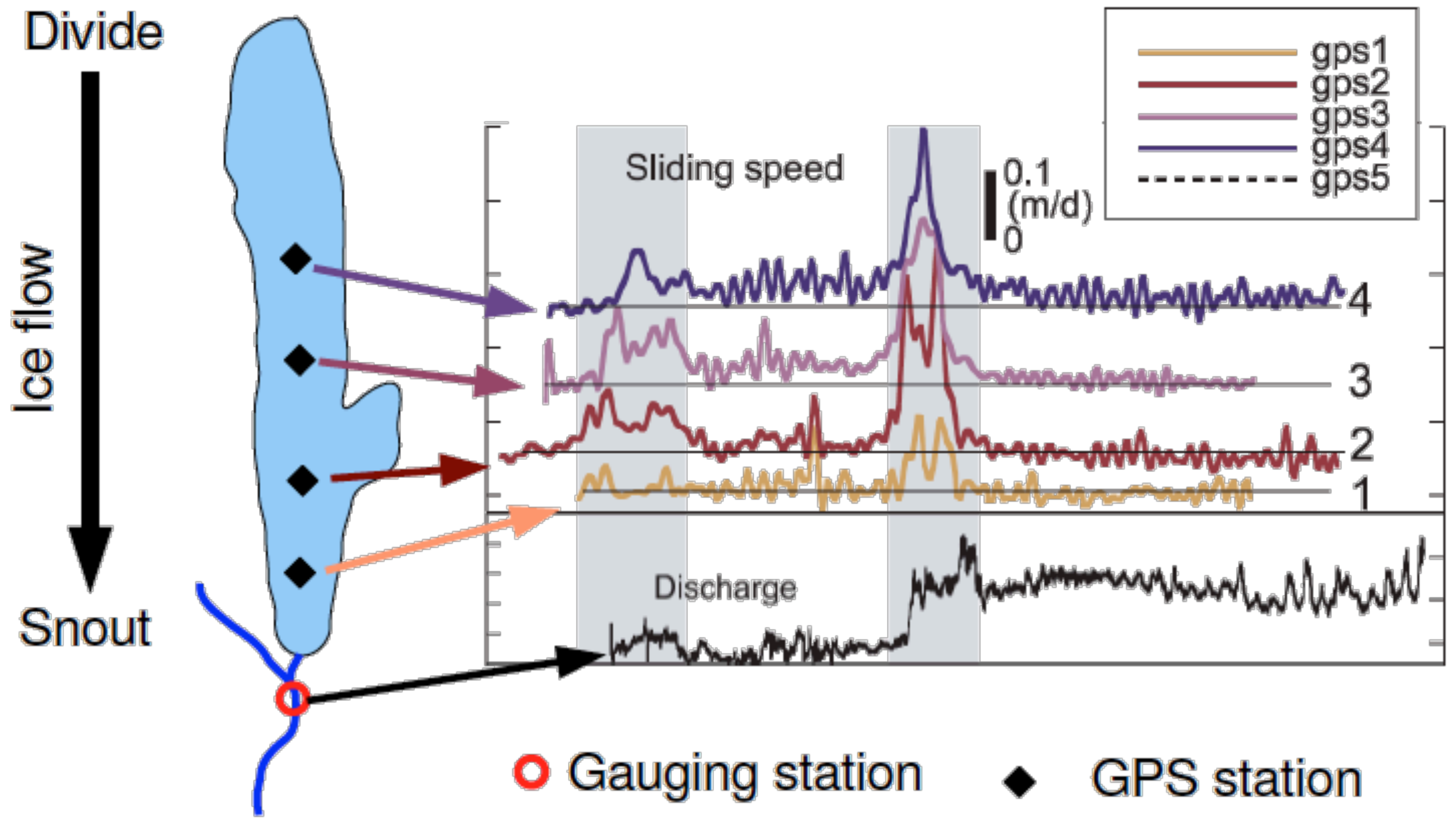
✓ **Examples**

Coupling water / friction and more...



Relationship between velocity and water

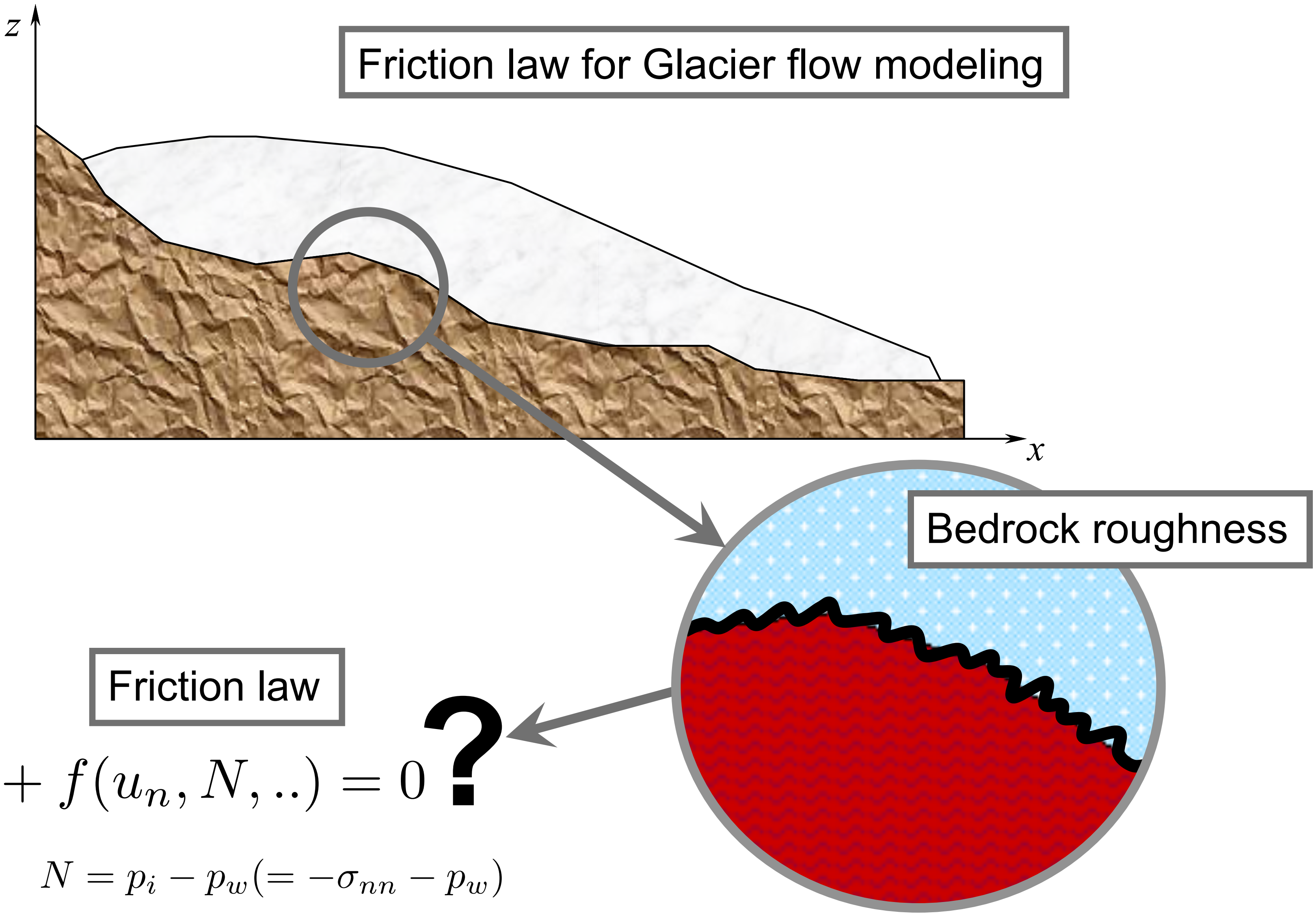
Velocity and discharge measurements on Bench glacier (Alaska)



@deFleurian

Figure adapted from [Anderson et al., 2004]

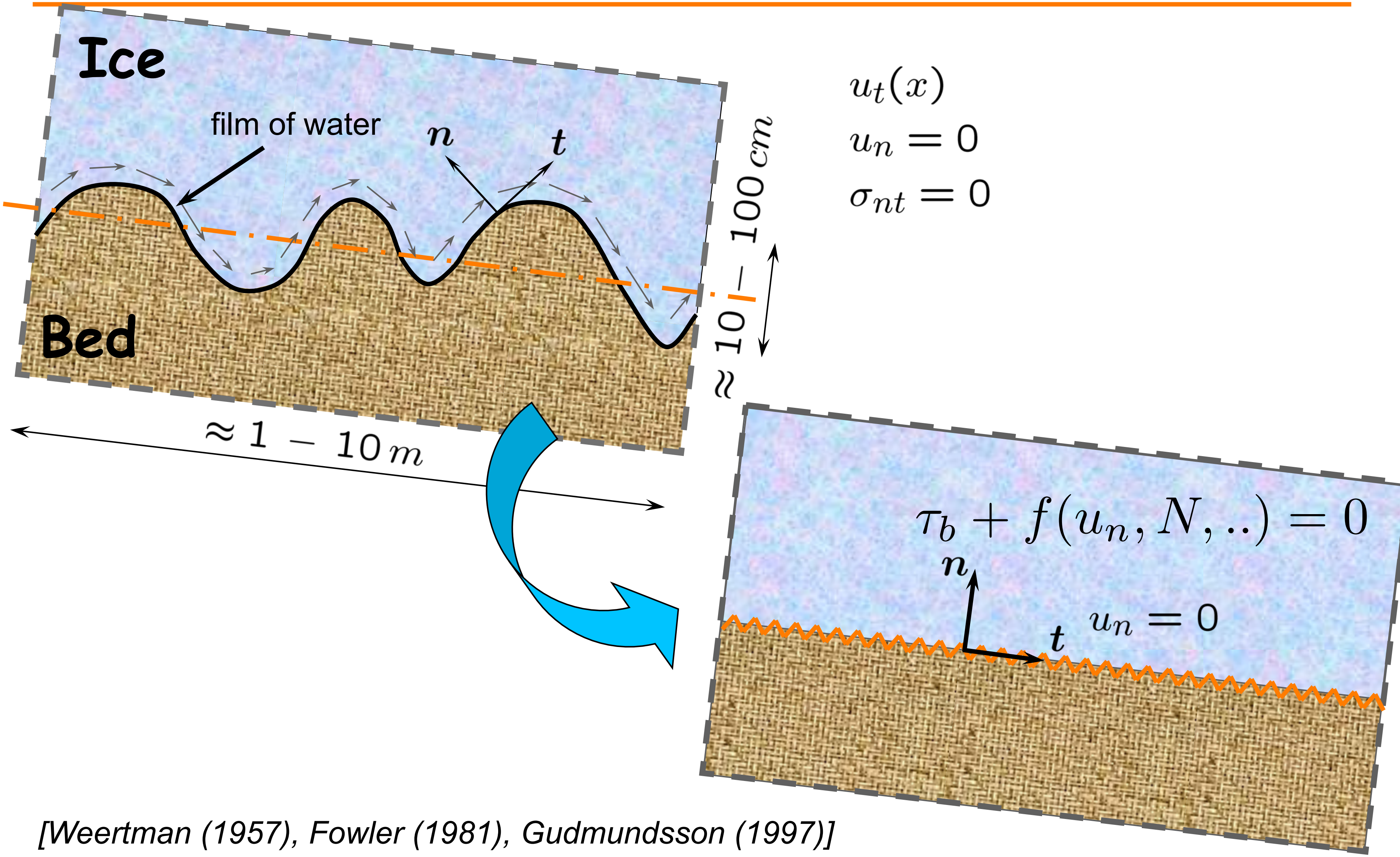
Scale of interest



$$\tau_b + f(u_n, N, ..) = 0$$

$$N = p_i - p_w (= -\sigma_{nn} - p_w)$$

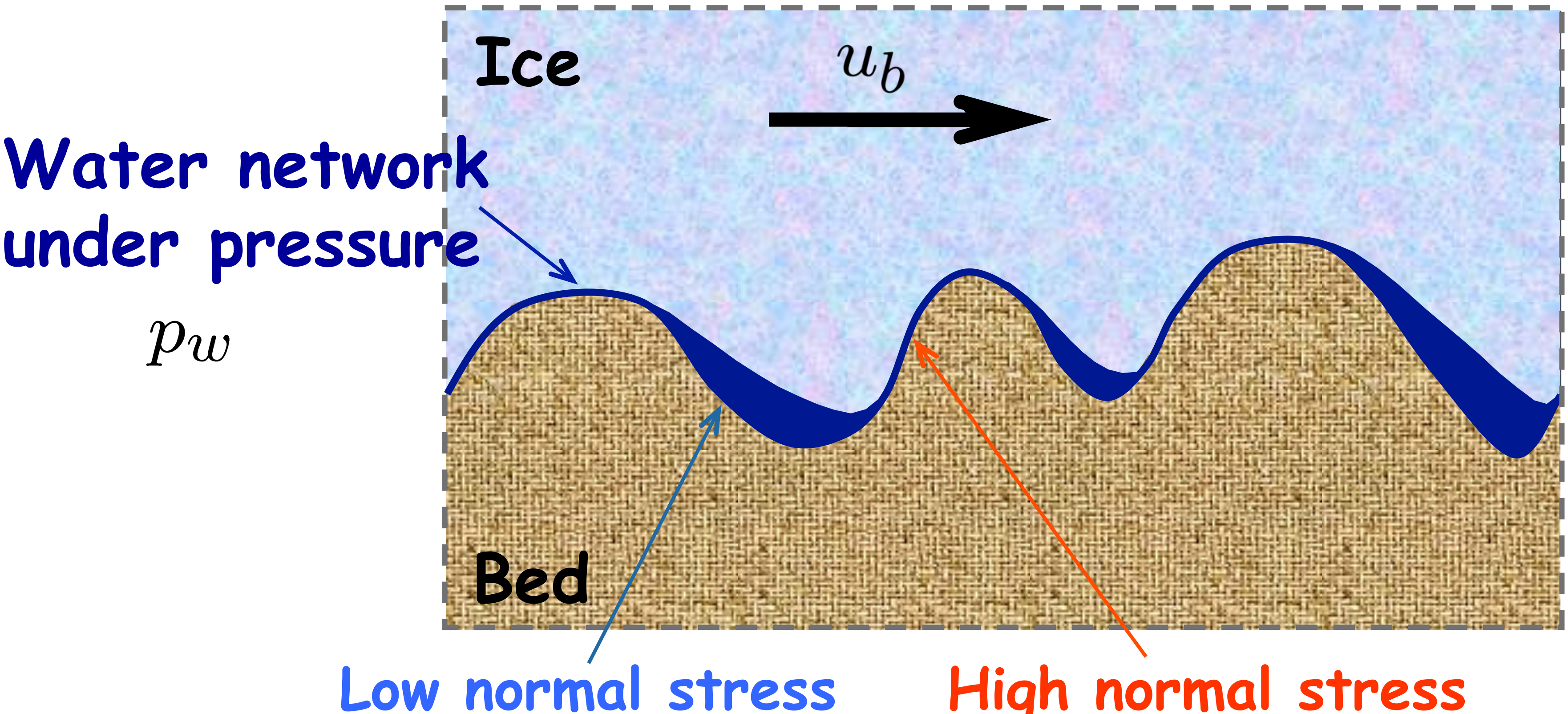
Concept of friction law



[Weertman (1957), Fowler (1981), Gudmundsson (1997)]

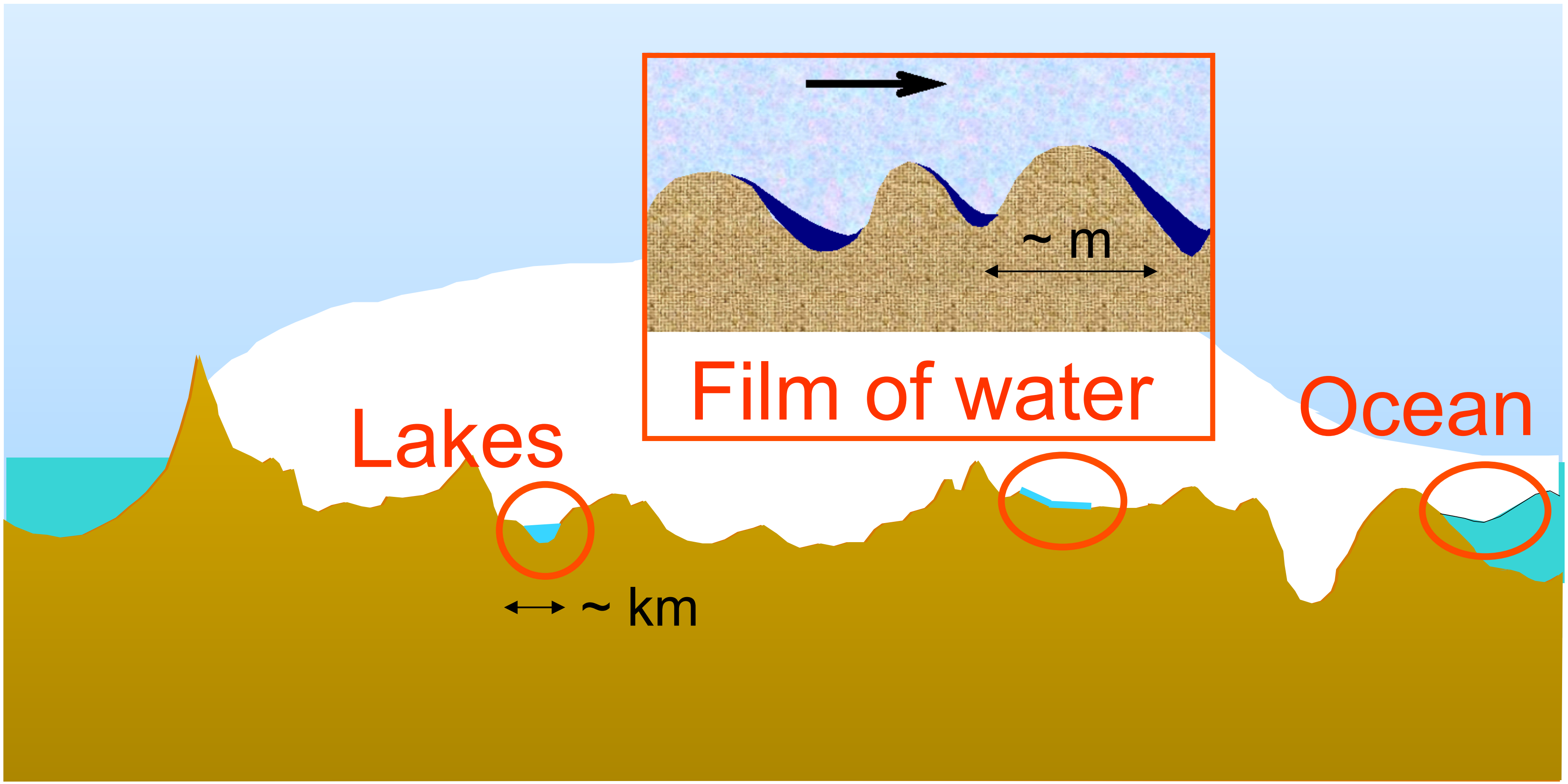
How water enhances glacier sliding

If water pressure and/or velocity increase



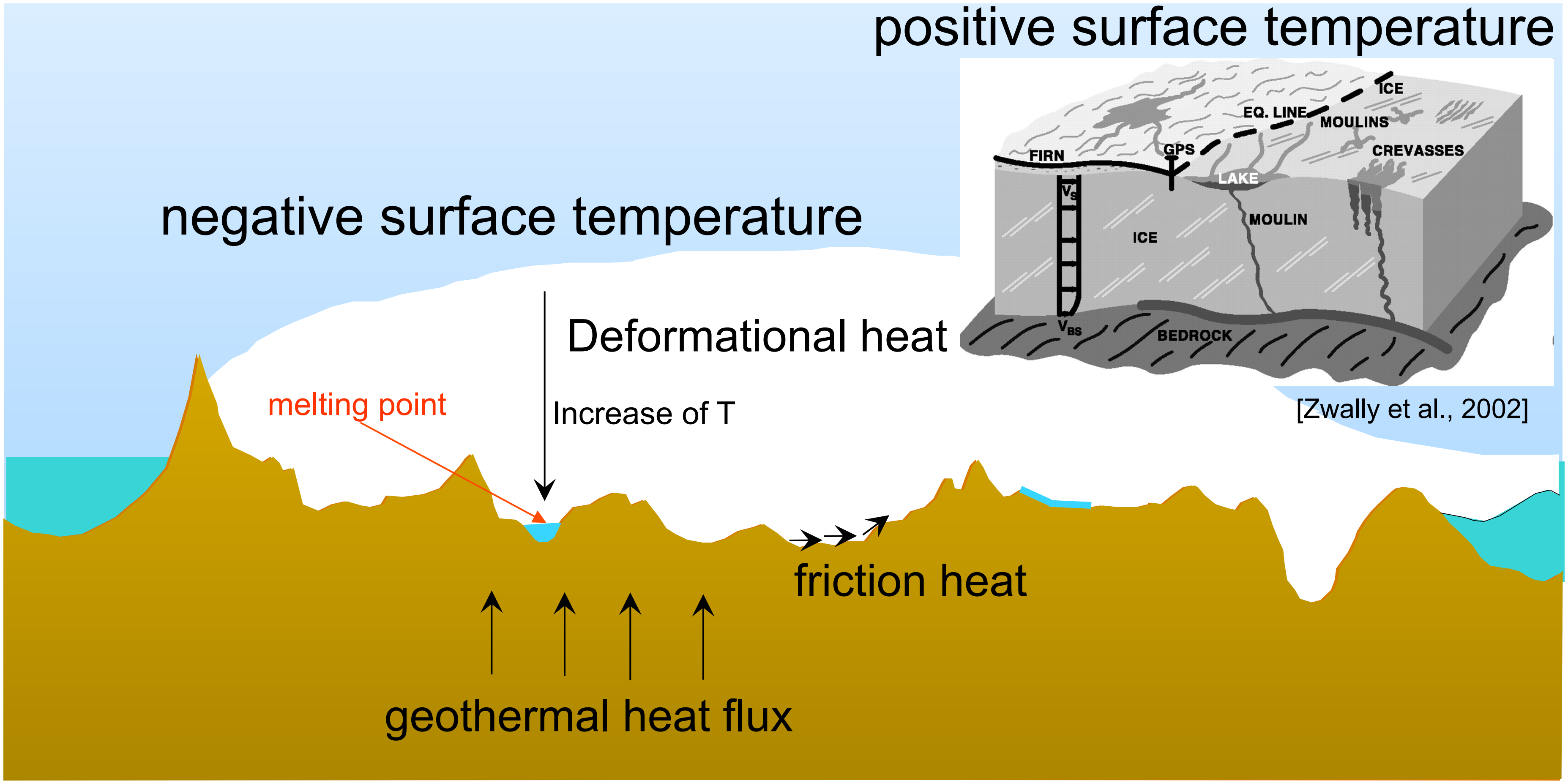
How does it affect the form of the sliding law ?

Water at the base of glaciers



Effective pressure: $N = -\sigma_{nn} - p_w$

Why is there (liquid) water?



[Zwally et al., 2002]

Two types of drainage systems

- Inefficient drainage systems

$$\left.\begin{array}{r}\text{low conductivities}\\ \text{high water pressure}\\ \text{distributed systems}\end{array}\right\}$$

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Two types of drainage systems

- Inefficient drainage systems

low conductivities
high water pressure
distributed systems

- Sediment layer

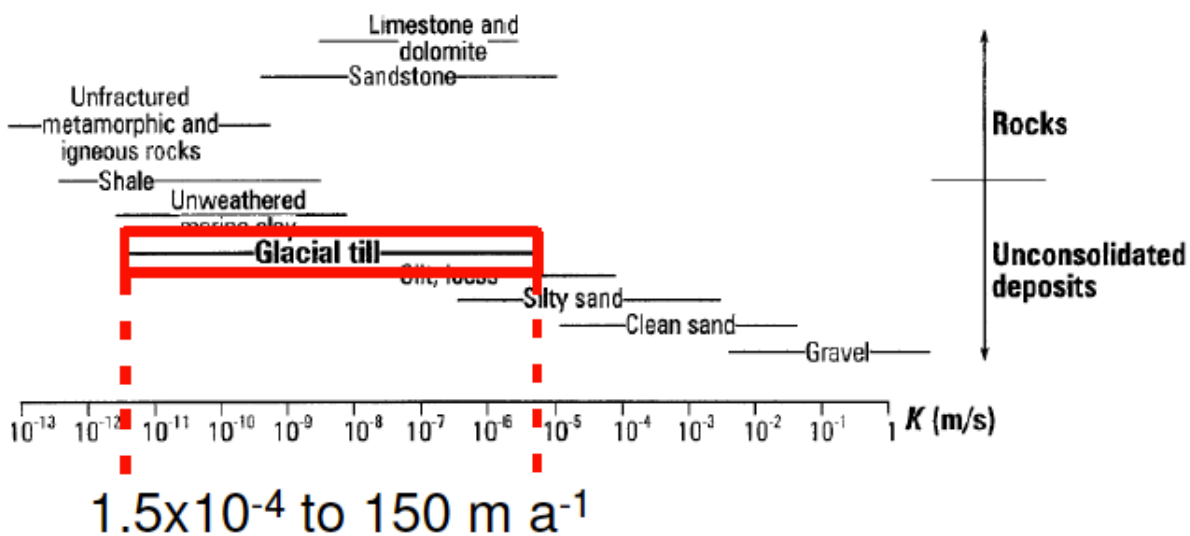

Figure from [Freeze and Cherry, 1979]

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Two types of drainage systems

- Inefficient drainage systems

low conductivities
high water pressure
distributed systems

- Sediment layer
- Linked cavities

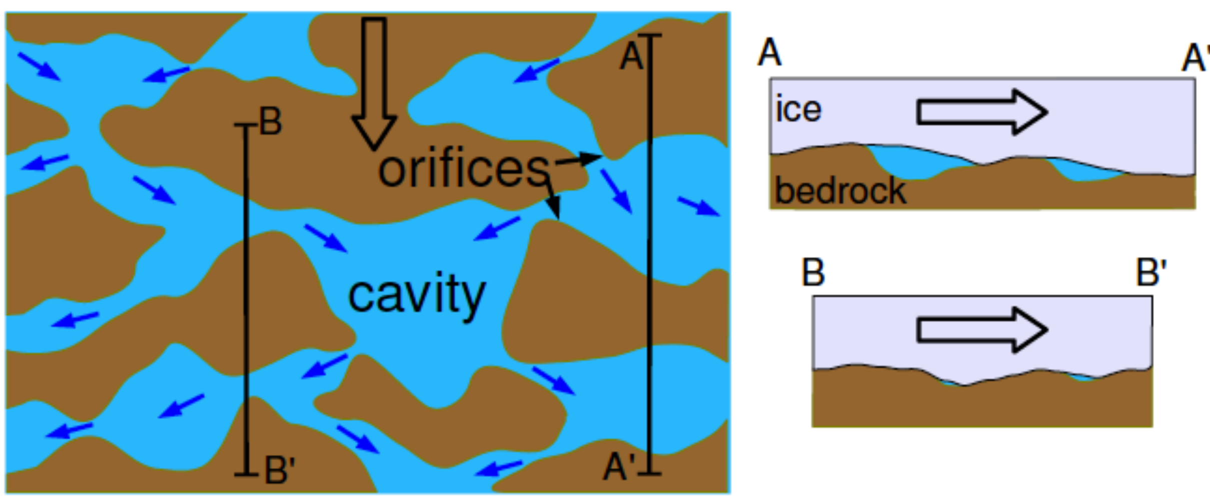

Figure from [Kamb, 1987]

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Two types of drainage systems

- Inefficient drainage systems

low conductivities
high water pressure
distributed systems

- Sediment layer
- Linked cavities
- Water film

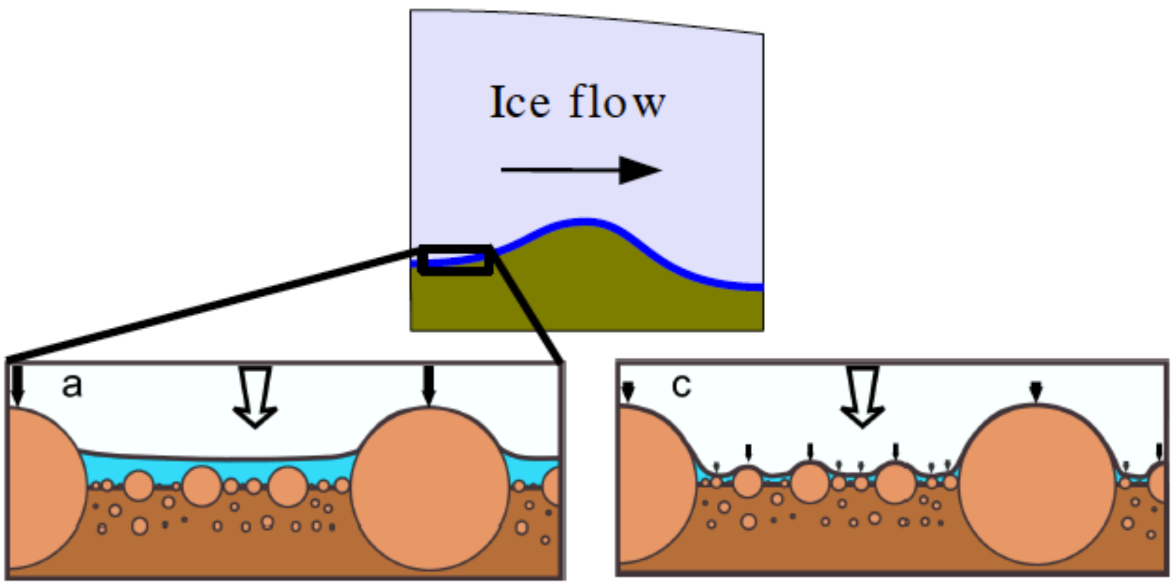

Figure from [Creyts and Schoof, 2009]

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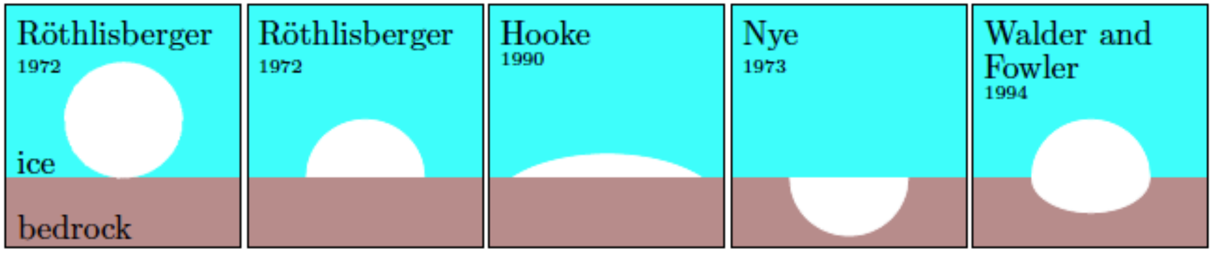
Two types of drainage systems

- Inefficient drainage systems
- Efficient drainage systems

high conductivities
low water pressure
localized systems
} ▸ Channels



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high conductivities
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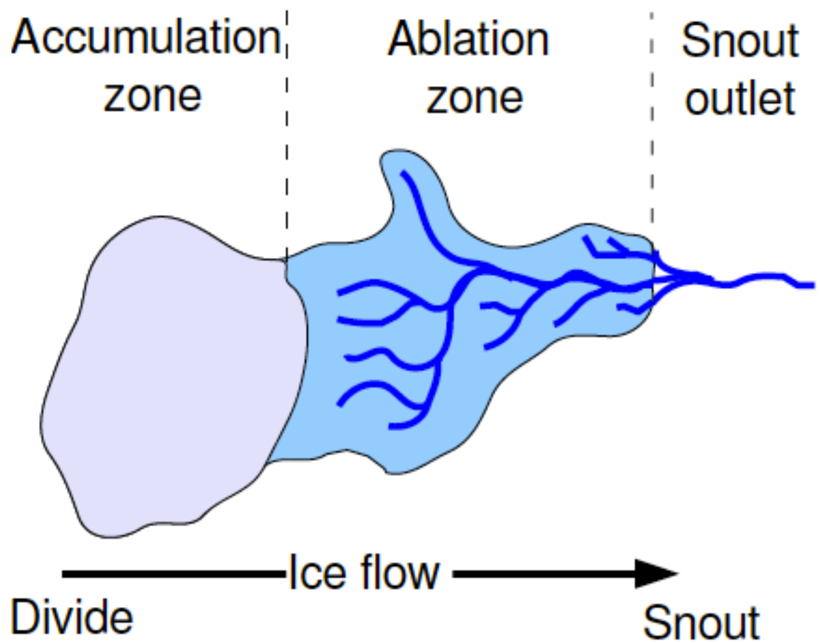

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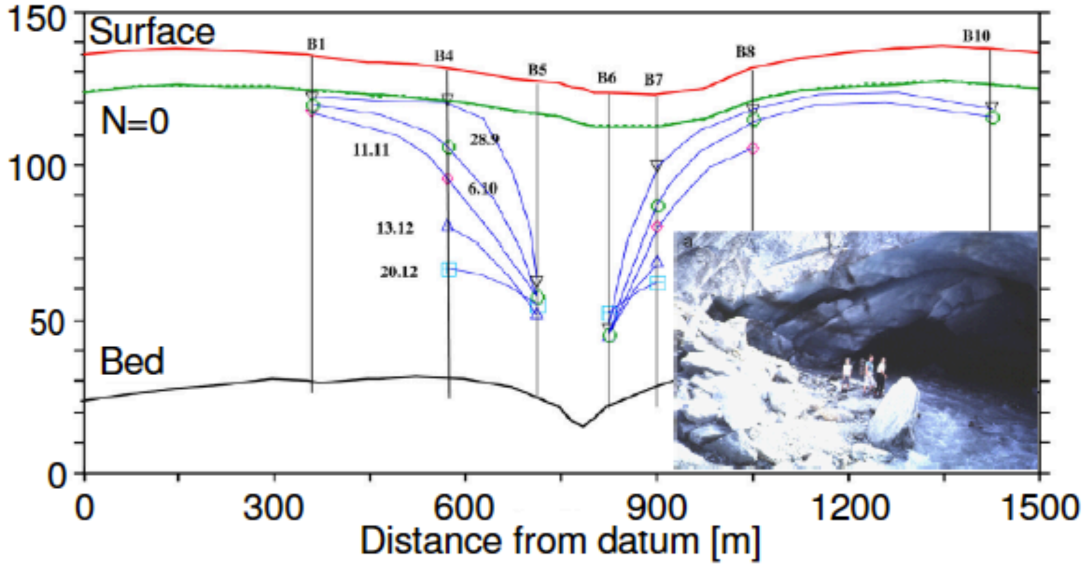
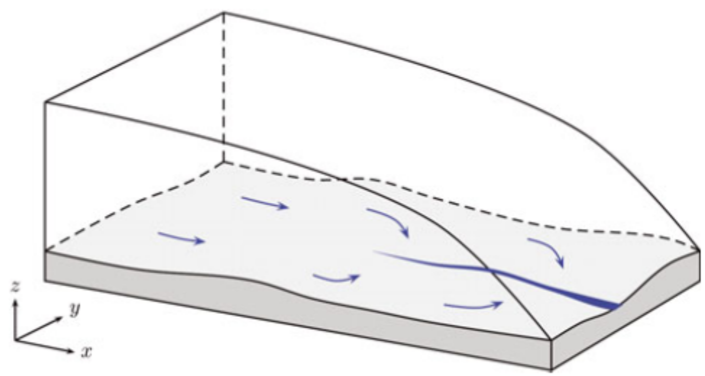
Two tightly-related systems

The link between inefficient and efficient systems is observable in the field

- As a spatial variation

Water load observed across Breidamerkrujökull during Automn Winter transition





From [Hewitt, 2011]

Figure adapted from [Boulton et al., 2007]

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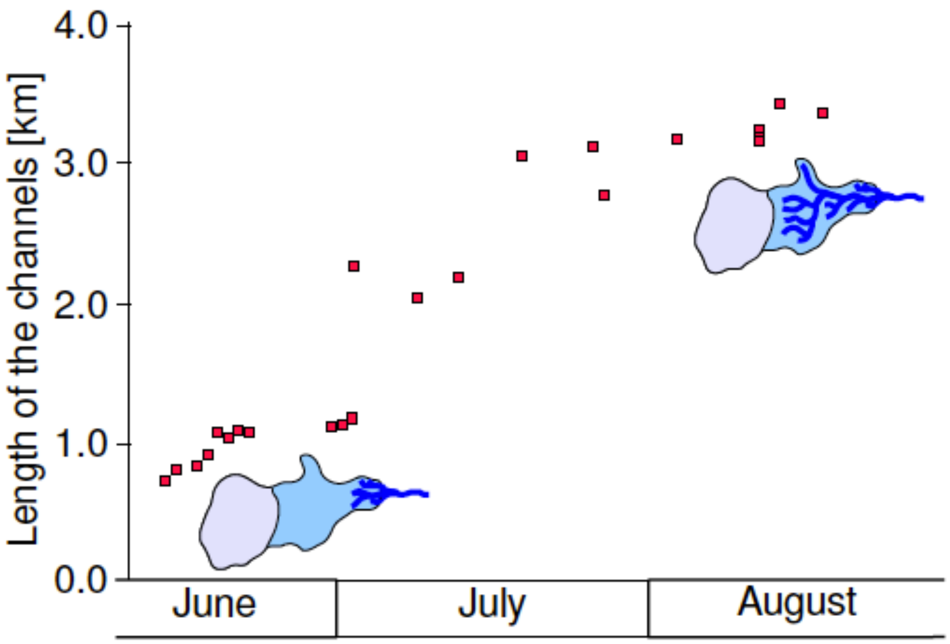
Two tightly-related systems

The link between inefficient and efficient systems is observable in the field

- As a spatial variation
- As a temporal evolution



Channel length evolution during summer on Arolla glacier (Switzerland)

Figure adapted from [Nienow et al., 1998]

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  - Linear friction law
  - Weertman type friction law
  - Water-pressure dependant friction laws
  - Double continuum hydrology model
  - GlaDS model
- ✓ **Implementation in Elmer/Ice**
  - Various friction laws
- ✓ **Examples**

Friction laws

A friction law is a relation that gives the basal shear stress as a function of the sliding velocity and other variables (effective pressure, ...):

$$\begin{cases} \tau_b = \boldsymbol{t} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \\ u_b = \boldsymbol{u} \cdot \boldsymbol{t} \\ \sigma_{nn} = \boldsymbol{n} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{n} \\ N = -\sigma_{nn} - p_w \end{cases}$$

$$\tau_b + f(u_b, N, ..) = 0$$

Linear friction laws:

$$\tau_b + \beta u_b = 0$$ β Drag factor or friction parameter

$$u_b + C\tau_b = 0$$ C Sliding parameter

Weertman type friction law (non-linear):

$$\tau_b + (u_b/A_s)^{1/n} = 0$$

A_s Sliding parameter

n Glen's flow law exponent

Friction laws – water pressure dependant

The friction should depend on the water pressure $N = -\sigma_{nn} - p_w$

Raymond and Harrison, 1987, Bindschadler (1983), Budd et al. (1984) :

$$u_b + k\tau_b^p N^{-q} = 0 \qquad \text{e.g. } p = n = 3,\ q = 1$$

Iken's bound, 1981:

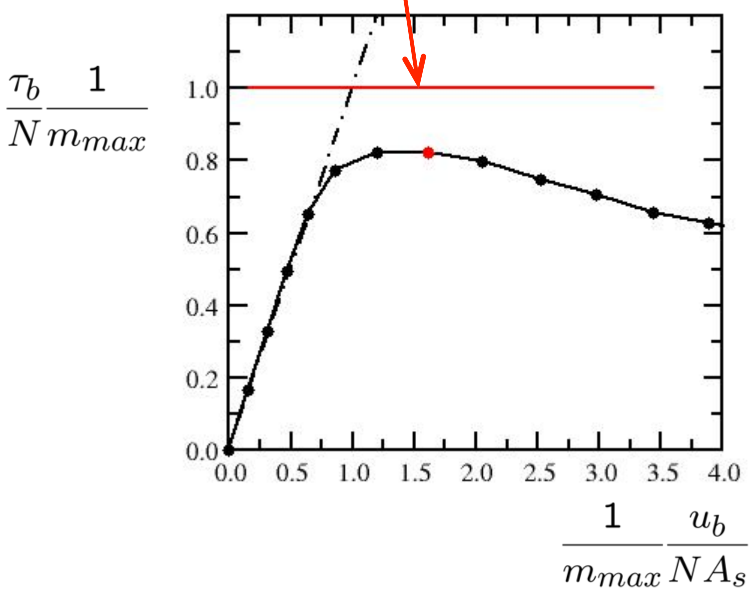
$$\tau_b / N < m_{\max} \qquad m_{\max} \text{ the maximum up-slope of the bed}$$

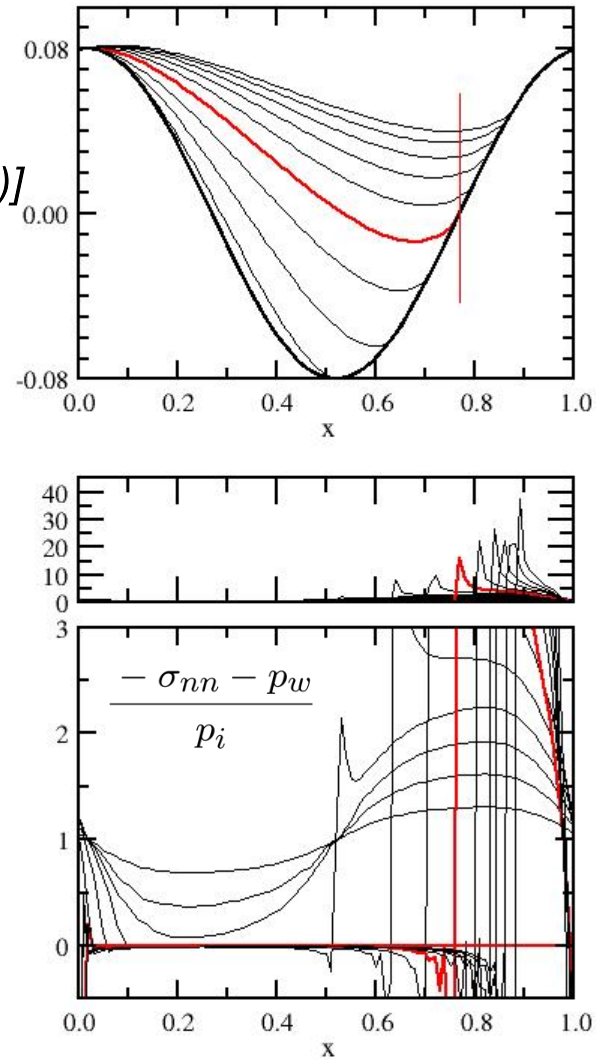
not fulfilled by the previous law

Illustration of Iken's bound

Iken’s bound : $\frac{\tau_b}{N} \leq m_{max}$

[Iken (1981), Fowler (1986), Schoof (2005)]





[Gagliardini et al., 2007]

Coulomb-type friction law

Schoof (2005), Gagliardini et al., 2007:

$$\frac{\tau_b}{N} + C\left(\frac{\chi}{1+\alpha\chi^m}\right)^{1/n} = 0$$

where

$$\begin{cases} \chi = \dfrac{u_b}{C^n N^n A_s} \\ \alpha = \dfrac{(m-1)^{m-1}}{m^m} \end{cases}$$

Fulfills the Iken's bound: $0 < \frac{\tau_b}{N} \leq C \leq m_{max}$

3 parameters:

$$\begin{cases} A_s \quad [m MPa^{-n} a^{-1}] \\ C \leq m_{max} \\ m \geq 1 \end{cases}$$

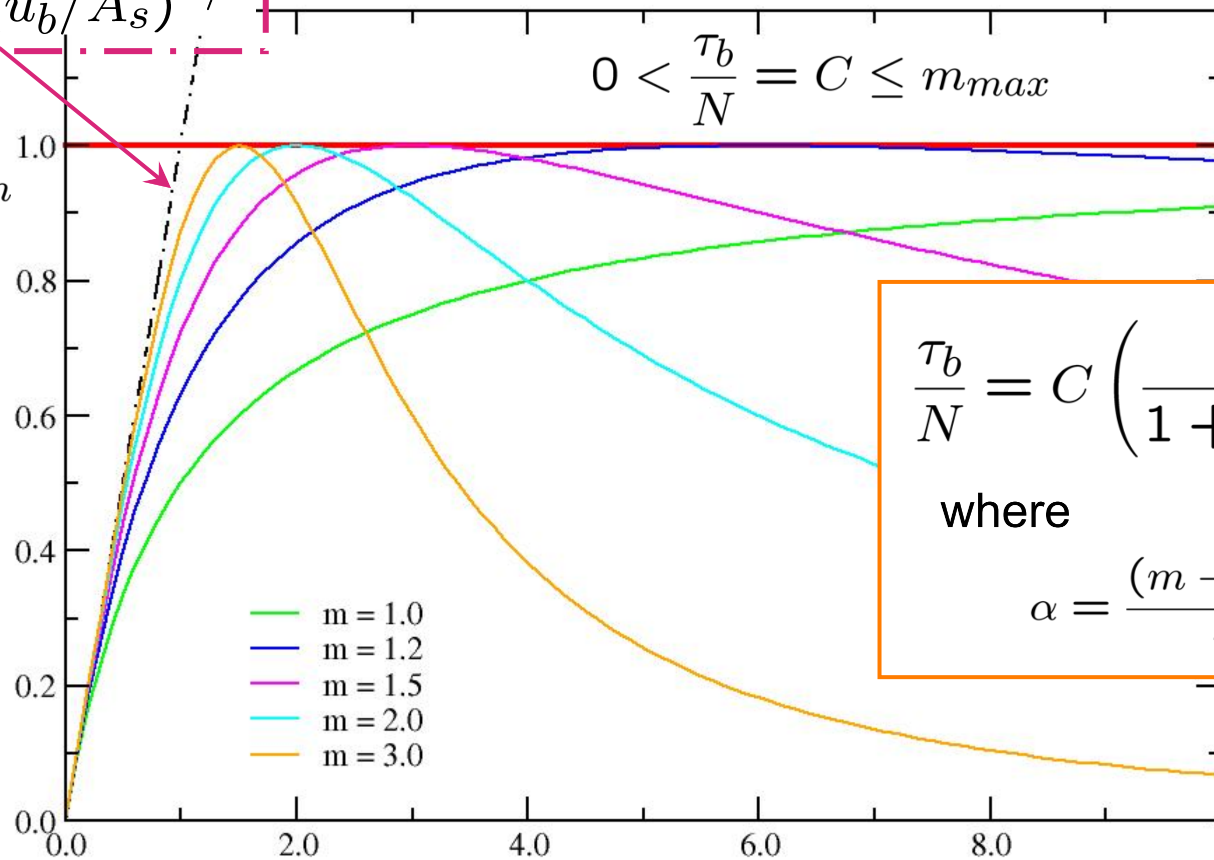
- A_s: Sliding parameter in absence of cavitation
- C: Maximum value of τ_b/N
- m: Post-peak exponent

Coulomb-type friction law

$$\tau_b = (u_b/A_s)^{1/n}$$

$$0 < \frac{\tau_b}{N} = C \leq m_{max}$$

$$\left(\frac{\tau_b}{CN}\right)^n$$



$$\frac{\tau_b}{N} = C\left(\frac{\chi}{1+\alpha\chi^m}\right)^{1/n}$$

where

$$\alpha = \frac{(m-1)^{m-1}}{m^m}$$

$$\chi = \frac{u_b}{C^n N^n A_s}$$

[Schoof (2005), Gagliardini et al., 2007]

Tsai Coulomb law

$$\tau_b + \min((u_b/A_s)^{1/n}\,;\,fN) = 0$$

[Tsai et al., 2015]

Fulfills the Iken's bound: $0 < \dfrac{\tau_b}{N} \leq f$

2 parameters:

$$\begin{cases} A_s & [mMPa^{-n}a^{-1}] \\ f & \end{cases}$$

- A_s $[mMPa^{-n}a^{-1}]$ — Sliding parameter in absence of cavitation
- f — friction coefficient

Comparison

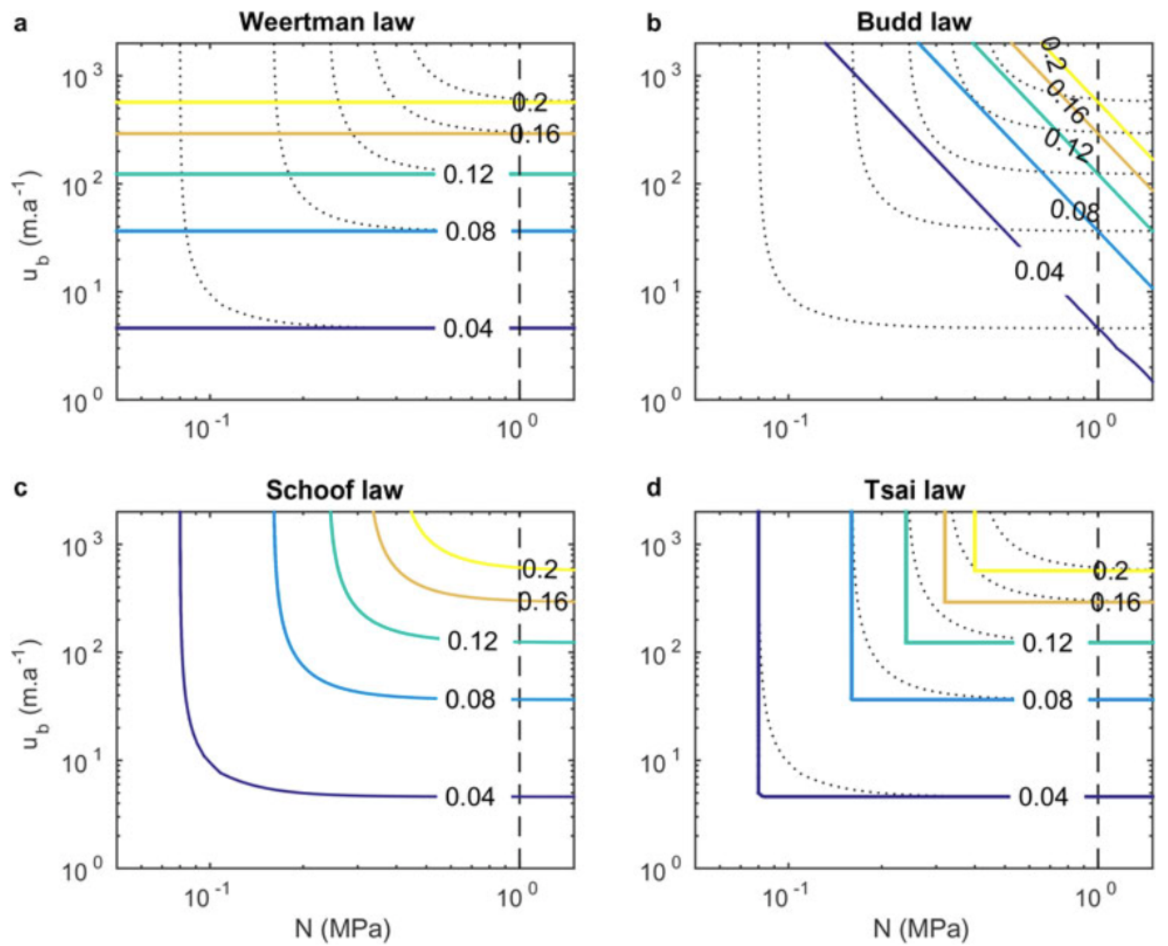
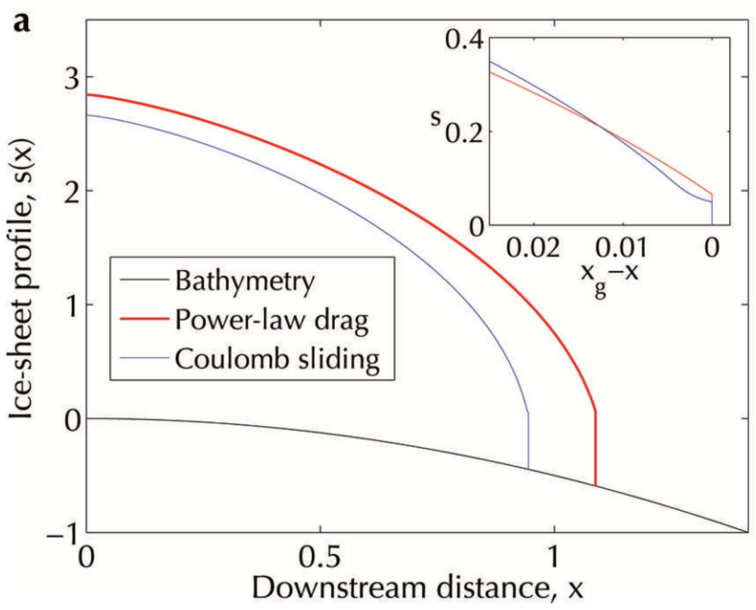


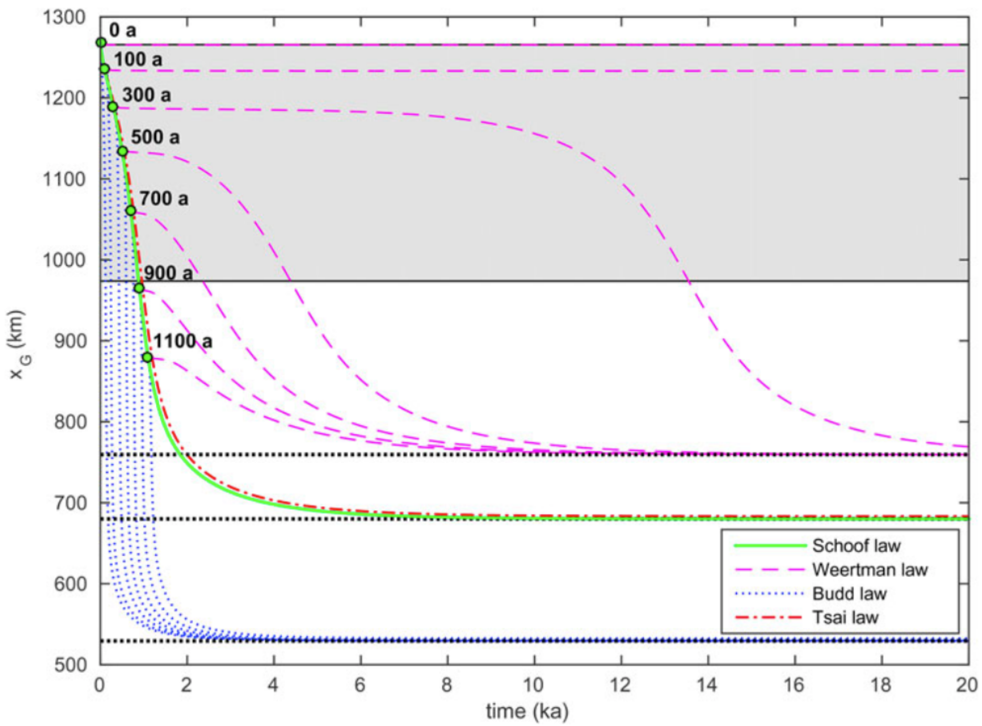
Fig. 1. Iso-values of τ_b ranging from 0.04 to 0.2 MPa given in the basal velocity-effective pressure log–log plane with: (a) Weertman, (b) Budd, (c) Schoof and (d) Tsai laws (Eqns (1)–(4)), for $m = 1/3$, $q = 1$, $C_W = C_B = C_S = 7.624 \times 10^6$ S.I. and $f = C_{max} = 0.5$. Dotted black lines reported on each plot are the iso-values of τ_b given with the Schoof law. The vertical black dotted line corresponds to $N = 1$ MPa.

[Brondex et al., 2017]

Influence of Friction law on GL



[Tsai et al., 2015]



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Two approaches in Elmer/Ice

Double continuum approach

- Implemented by Basile de Fleurian
- in the distribution
- http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:hydrologydc

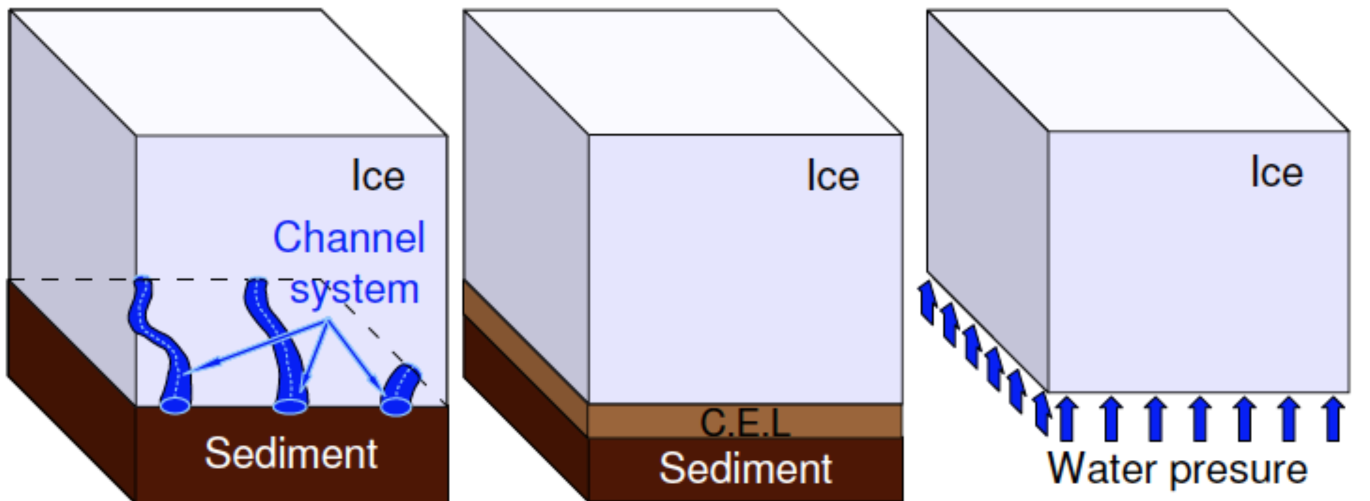
Cavity sheet and discrete channels

- Model developed by Mauro Werder (Werder et al., 2013)
- Implemented in Elmer by O. Gagliardini
- In the distribution
- http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:glads

The double continuum approach

Karstified hydrology methods developed while facing difficulties to model conduit drainage [Teutsch and Sauter, 1991]

2 systems	Similar media	Verticaly
≠ characteristics	≠ conductivities	integrated



C.E.L : Channel Equivalent Layer

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Computation of the water load

Vertically-integrated computation of the water load h_w

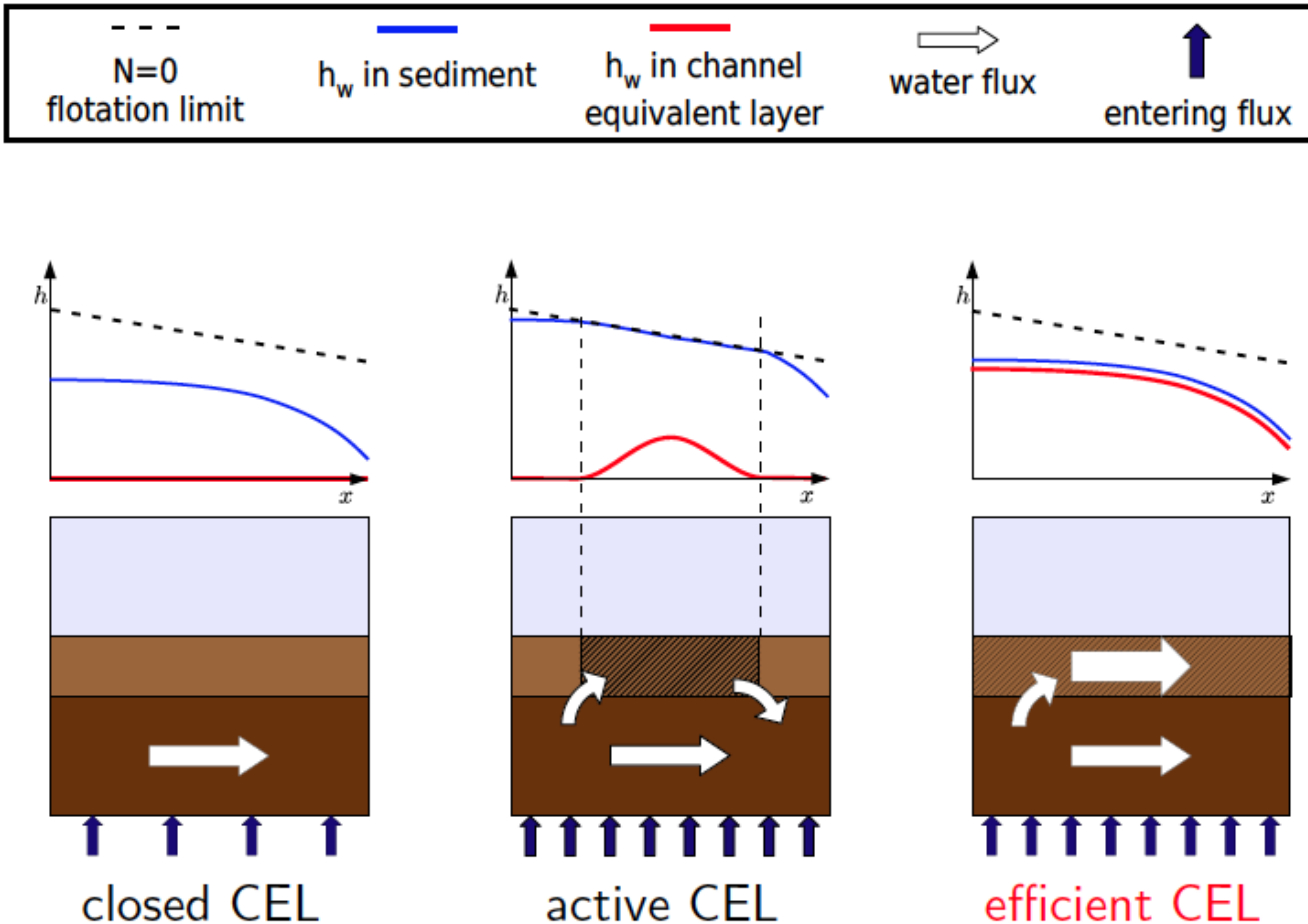
$$\mathrm{div}\left[\mathbf{T}\,\mathrm{grad}\,h_w\right] = S\frac{\partial h_w}{\partial t} + qe$$

Relies on the transmitivity $\mathbf{T}$ and storage coeficient S of the aquifer

$$\mathbf{T} = \mathbf{K}e\,;\; S = \rho_w g e \omega \left[\beta_w + \frac{\alpha}{\omega}\right]$$

ρ_w	Water density	q	Sink/Source term
e	Layer thickness	ω	porosity
$\mathbf{K}$	Sediment conductivity	α	Porous media compressibility
β_w	Water compressibility		

3 states of the Channel Equivalent Layer

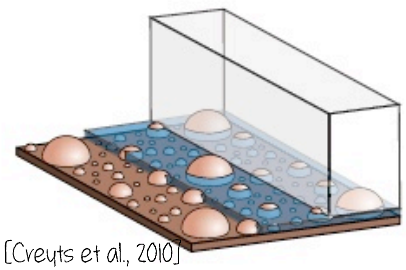


GlaDS model

(Werder et al., 2013)

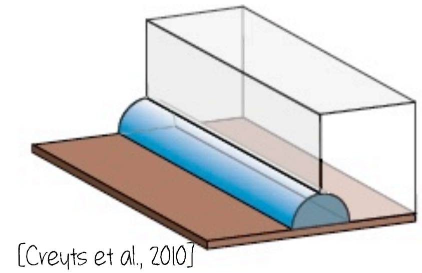
Two components system, 3 variables: Φ, h, S

Network of cavities

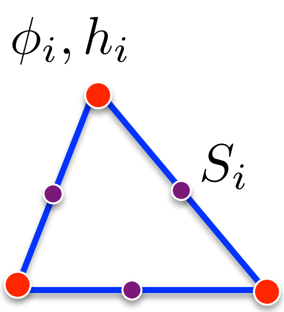


Cavity thickness h
(nodal variable)

Channels



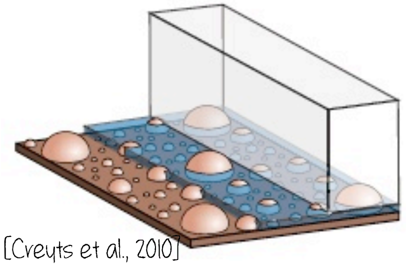
Channel cross-sectional area : S
(edge variable)



$$\sum_i \int_{\Omega_i} \left[\theta \frac{e_v}{\rho_w g}\frac{\partial \phi}{\partial t} - \nabla\theta\cdot\boldsymbol{q} + \theta\,(w - m_b - v)\right] \mathrm{d}\Omega$$
$$+ \sum_j \int_{\Gamma_j}\left[-\frac{\partial\theta}{\partial s}Q + \theta\left(\frac{\Xi-\Pi}{L}\left(\frac{1}{\rho_i}-\frac{1}{\rho_w}\right) - v_c\right)\right] \mathrm{d}\Gamma$$
$$+ \int_{\partial\Omega_N}\theta q_N \,\mathrm{d}\Gamma - \sum_k \theta\left(-\frac{A_m^k}{\rho_w g}\frac{\partial\Phi}{\partial t} + Q_s^k\right) = 0\,,$$

GlaDS model, cavities

(Werder et al., 2013)


[Creyts et al., 2010]

Discharge (Darcy-Weisbach law) :

$$\boldsymbol{q} = -kh^{\alpha}|\operatorname{grad}\phi|^{\beta-2}\operatorname{grad}\phi$$

Cavity thickness evolution :

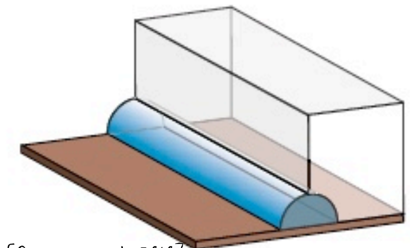
$$\frac{\partial h}{\partial t} = w(h) - v(h,\phi)$$

with

$$\begin{cases} v(h,\phi) = \tilde{A}h|N|^{n-1}N & \text{creep, closing (opening)} \\ w(h) = \max(0; \frac{u_b}{l_r}(h_r - h)) & \text{opening term} \end{cases}$$

GlaDS model, Channels

(Werder et al., 2013)



Discharge (Darcy-Weisbach law) :

$$Q = -k_c S^{\alpha_c} \left| \frac{\partial \phi}{\partial s} \right|^{\beta_c - 2} \frac{\partial \phi}{\partial s}$$

Channel cross-sectional area evolution :

$$\frac{\partial S}{\partial t} = \frac{\Xi(S,\phi) - \Pi(S,\phi)}{\rho_i L} - v_c(S,\phi)$$

with

$$v_c(S,\phi) = \tilde{A}_c S |N|^{n-1} N$$ Creep, closing (opening)

$$\Xi(\phi) = \left| Q \frac{\partial \phi}{\partial s} \right| + \left| l_c q_c \frac{\partial \phi}{\partial s} \right|$$ Energy dissipated

$$\Pi(S,\phi) = -c_t c_w \rho_w (Q + f l_c q_c) \frac{\partial \phi - \phi_m}{\partial s}$$ Sensible heat change

Comparison

	Double Continuum	GlaDS
Cavity only	=	=
Channels	continuous	discrete
Coupling	$N \rightarrow u$	$N \leftrightarrow u$
Channels closing	No	Yes

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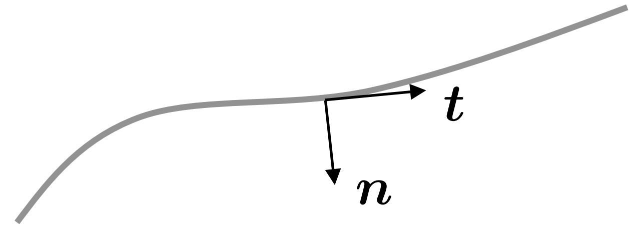
Friction laws in Elmer/Ice

Friction law in Elmer:

$$C_i u_i = \sigma_{ij} n_j \text{ with } i = 1, 2, 3$$

where $\boldsymbol{n}$ is the surface normal vector



In Normal-Tangential coordinate : $\boldsymbol{n} = (1, 0, 0)$

and
$$\begin{cases} C_n u_n = \sigma_{nn} \\ C_{t_1} u_{t_1} = \sigma_{nt_1} \\ C_{t_2} u_{t_2} = \sigma_{nt_2} \end{cases}$$

Friction law applied through the two Slip Coefficients 2 and 3

```
! Bedrock BC
Boundary Condition 1
  Target Boundaries = 1

  Flow Force BC = Logical True
  Normal-Tangential Velocity = Logical True

  Velocity 1 = Real 0.0e0
  Slip Coefficient 2 = Real 0.1
  Slip Coefficient 3 = Real 0.1
End
```

Friction laws in Elmer/Ice

Linear friction laws:

$$\tau_b = \beta u_b$$

```
$beta = 0.1

Slip Coefficient 2 = Real $ beta
Slip Coefficient 3 = Real $ beta
```

Non-Linear friction laws:

Need a User Function to evaluate the Slip Coefficient

Rewrite the friction law in the form $\tau_b = C_t(u_b) u_b$
where $C_t(u_b)$ is the `Slip Coefficient` estimated through a user function

Weertman: $$C_t(u_b) = u_b^{(1-n)/n} / A_s^{1-n}$$

Schoof, 2005
Gagliardini et al., 2007

$$C_t(u_b) = CN \left(\frac{\chi u_b^{-n}}{1 + \chi^m} \right)^{1/n}$$

with $\chi = \dfrac{u_b}{C^n N^n A_s}$

Friction laws in Elmer/ice

Problem when $u_b \to 0$

The law is linearized for small velocity:

$$\begin{cases} C_t(u_b) = C_t(u_b) \text{ for } u_b > u_{t0} \\ C_t(u_b) = C_t(u_{t0}) \text{ for } u_b \leq u_{t0} \end{cases}$$

Example of a call (File USF_Sliding.f90):

```
 Normal-Tangential Velocity = Logical True
 Flow Force BC = Logical True

!! Water pressure given through the Stokes 'External Pressure' parameter
!! (Negative = Compressive)
 External Pressure = Equals Water Pressure

 Velocity 1 = Real 0.0
 Slip Coefficient 2 = Variable Coordinate 1
     Real Procedure ”ElmerIceUSF" "Friction_Coulomb”

!! PARAMETERS NEEDED FOR THE BASAL SLIDING LAW
 Friction Law Sliding Coefficient = Real $As
 Friction Law Post-Peak Exponent  = Real $m
 Friction Law Maximum Value = Real $C
 Friction Law PowerLaw Exponent = Real $n
 Friction Law Linear Velocity = Real $ut0
```

GlaDS solvers

Three solvers:

- GlaDSCoupledSolver: main solver, all 3 variables ϕ, h, S are solved in a coupled way
- GlaDSsheetThickDummy: just here to declare h (to save previous values)
- GlaDSchannelOut: just here to export in VTU format edge type variables (not accounted for by ResultOutput solver).

Coupling with SSA: user functions HorizontalVelo and OverburdenPressure.

Including Moulins (101 boundary elements) in serial or parallel meshes : python tool makemoulin.py in Meshers\

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Examples

Friction :

Weertman

- Tests/GL_MISMIP, Tests/Contact, Tests/Friction_Weertman.
- http://elmerice.elmerfem.org/wiki/doku.php?id=userfunctions:weertman

Coulomb

- Tests : Tests/Friction_Coulomb and Tests/Friction_Coulomb_Pw
- http://elmerice.elmerfem.org/wiki/doku.php?id=userfunctions:coulomb

Budd

- http://elmerice.elmerfem.org/wiki/doku.php?id=userfunctions:budd#general_description

Hydrology :

Double continuum approach

- Tests/Hydro_SedOnly and Tests/Hydro_Coupled
- http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:hydrologydc

Cavity sheet and discrete channels

- Tests/GlaDS and Tests/GlaDS_SSA
- http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:glads

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