USF_Damage.f90

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This routine work in 3D. It contains 2 functions :

- EnhancementFactor : Returns a new value for the enhancement factor modified according to the value of damage. This enhancement factor modifies the viscosity of the ice (see description in section 3)
- SourceDamage : Returns the source term of the damage advection equation. This function is fully described below.

The full description of this model is given in Krug et al. (2014).

1 Advection

The ice is supposed to be isotropic. The damage is described as a scalar value which is advected through the media according to the following advection-reaction equation (the reaction term is set to zero):

$$\frac{\partial D}{\partial t} + \vec{u}\nabla D = f(\chi)$$

where $\vec{u}$ is the velocity vector, and $f(\chi)$ the source term for damage.

2 Damage production

The increase of damage in the media depends on the stress field. It occurs when the stress pattern exceed a damage threshold σ_{th}. This value is basically set around 0.01MPa. It is modified by considering the heterogeneity in the ice. Thus, to account for sub-grid scale heterogeneity, we introduce some noise on σ_{th} : $\sigma_{th} = \overline{\sigma_{th}} \pm \delta\sigma_{th}$, where $\frac{\delta\sigma_{th}}{\overline{\sigma_{th}}}$ follows a standard normal distribution with a standard deviation of 0.05 (set by the user). The value of this threshold must be discussed accordingly to some physical reference, such as the toughness of ice or its tensile strength.

First, the source term f will be described as follow:

$$f(\chi) = B \cdot \chi(\tilde{\sigma}, \sigma_{th}, D)$$

where B is a damage enhancement factor which need to be calibrated (around 1-2 MPa^{-1}). χ is called the damage criterion, and writes :

$$\chi(\tilde{\sigma}, \sigma_{th}, D) = \frac{\sigma_I}{(1-D)} - \sigma_{th}$$

In the equation above, σ_I is the maximal principal stress (maximum eigenvalue of the diagonalized cauchy stress tensor). χ can be drown is the space of Mohr circle, as shown on figure 1 :

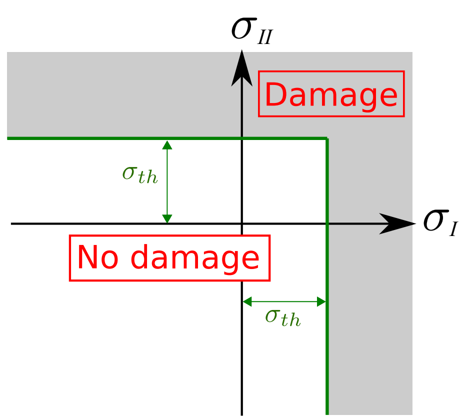

Figure 1: Critère χ dans l'espace des contraintes principales.

Here, we only consider the opening in Mode I, meaning, under pure traction. The ice is damaged when the tensile stress exceed a stress threshold.

3 Viscosity change

Damage affects the viscosity of the ice as it is advocated in the media, by changing the value of the enhancement factor used in the Glen's law. This enhancement factor is modified in Elmer/Ice accordingly to this equation:

$$E = \frac{1}{(1-D)^n}$$

Here, $D = 0$ for non-damaged ice and $0 < D < 1$ when ice is damaged. A value of $D = 1$ correspond to a fully damaged ice. In order to avoid infinite velocity resulting from the indeterminate form, the incrementation of D is strictly limited below 1.

3.1 Mathematical description of the impact of damage on ice rheology

The Glen's law writes:

$$\boldsymbol{S} = (EA)^{-1/n} \boldsymbol{I}_{\dot{\epsilon}_2}^{(1-n)/n} \dot{\boldsymbol{\epsilon}} \tag{1}$$

Where the second invariant $\boldsymbol{I}_{\dot{\epsilon}_2}^2$ is defined as:

$$\boldsymbol{I}_{\dot{\epsilon}_2}^2 = \frac{1}{2} \dot{\epsilon_{ij}} \dot{\epsilon_{ij}} \tag{2}$$

Using the equivalence principle of Lemaitre et al. (1988), strain is affected by damage through the effect of an effective deviatoric stress only (as we consider the damage to alter the viscous properties of the flow). Thus,

$$\tilde{\sigma} = \frac{\sigma}{(1 - D)} \tag{3}$$

Here, D is the ice damage. $D = 0$ for non-damaged ice and $0 < D < 1$ when ice is damaged. When introducing the damage and the effective deviatoric stress, equation (1) reads:

$$\tilde{\boldsymbol{S}} = (A)^{-1/n} \boldsymbol{I}_{\dot{\epsilon}_2}^{(1-n)/n} \dot{\boldsymbol{\epsilon}} \tag{4}$$

When re-introducing non-effective variables in the previous equation, it comes:

$$\boldsymbol{S} = (A)^{-1/n} (1 - D) \boldsymbol{I}_{\dot{\epsilon}_2}^{(1-n)/n} \dot{\boldsymbol{\epsilon}} \tag{5}$$

And by identification with Eq. (1) , one can link the enhancement factor E with the damage D, such as

$$\boxed{E = \frac{1}{(1 - D)^n}} \tag{6}$$

When $D = 0$, for undamaged ice, $E = 1$ whereas for damaged ice $E > 1$ (the flow is enhanced).

References

Krug, J., Weiss, J., Gagliardini, O., and Durand, G. (2014). Combining damage and fracture mechanics to model calving. *The Cryosphere Discussions*, 8(2):1631–1671.

Lemaitre, J., Chaboche, J., and Germain, P. (1988). *Mécanique des matériaux solides*, volume 7. Dunod Paris.