# Continuum Damage Mechanics Model and Calving Law

## Elmer/Ice Users Meeting

Tuesday, April 9th, 2013

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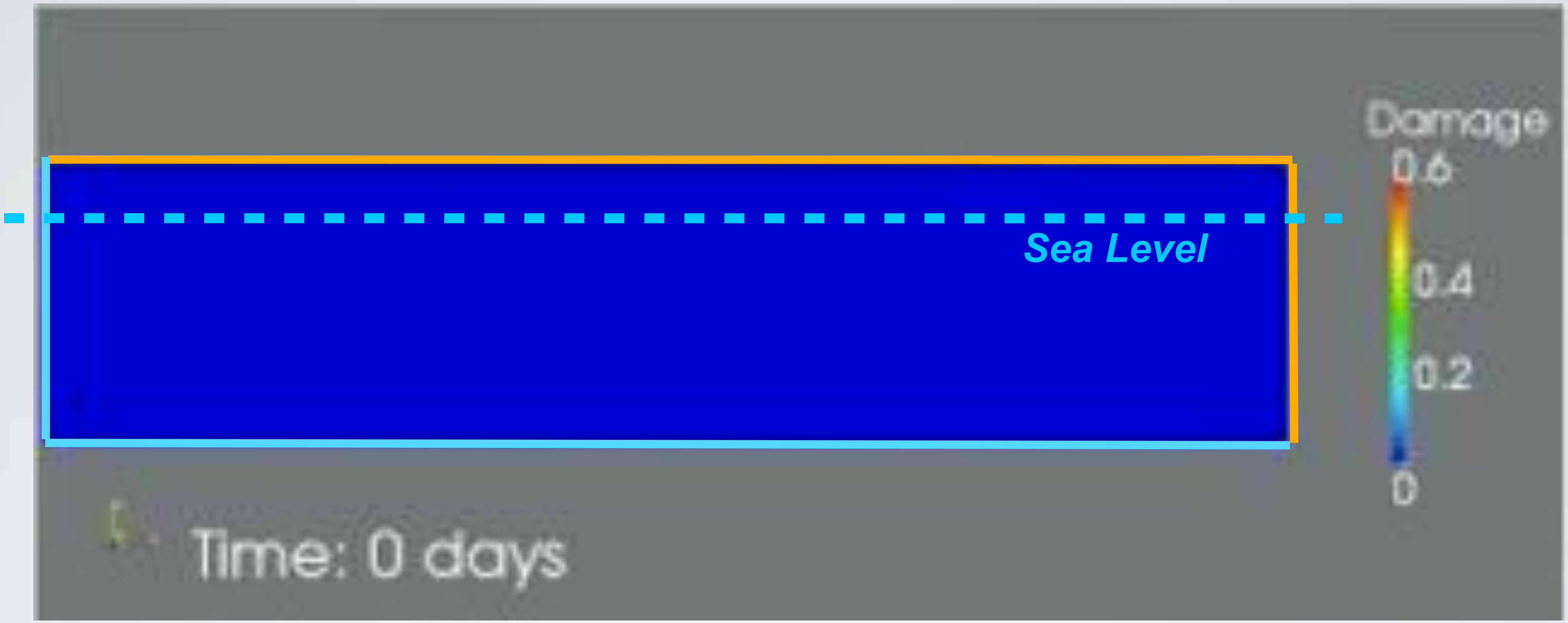
# Imagine a marine-terminated grounded glacier...



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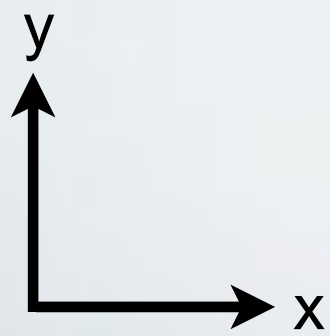

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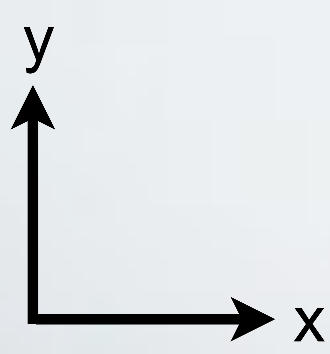


# Outline

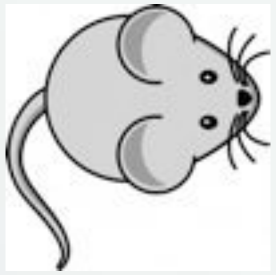
1/ Damage Model

2/ Calving Criterion

3/ Conclusion

4/ Special Bonus

Software Elmer/Ice, finite element model.
Full-Stokes model.



# Model Presentation

## Continuous Damage Mechanics Model

### Damage Mechanics

1/ Subcritical crack propagation (Weiss, 2004)

## Calving Solver

### Linear Elastic Fracture Mechanics

2/ Fracture initiation (Van der Veen, 1998ab)

3/ Fracture arrest

# Model Presentation - Continuum Damage Mechanics

Continuum Damage Mechanics (CDM) model : Local evolution of crevassing at the model grid scale, depending on the stress field and its transportation through a damage variable, which affects in turn the rheology of the ice.

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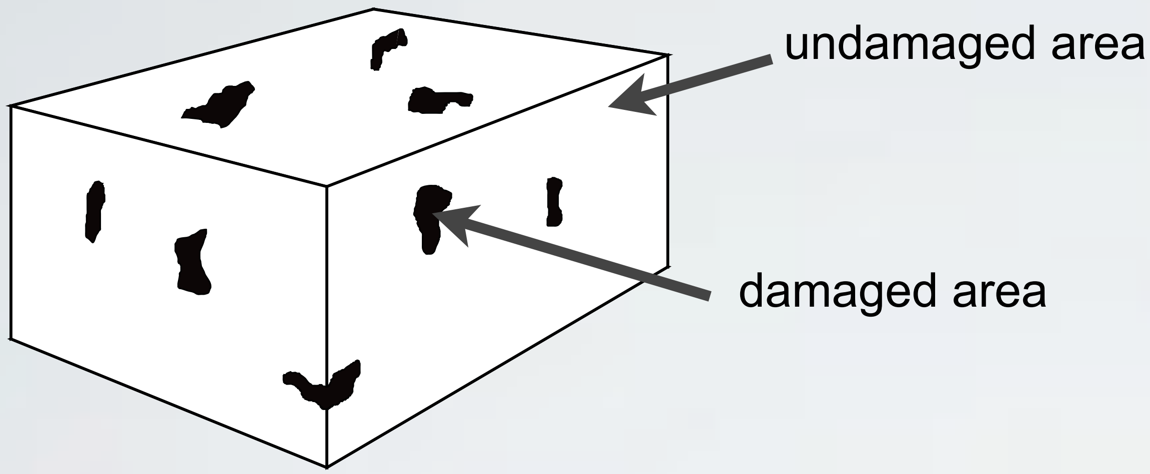


*damaged material*

Isotropic damage variable **D** :

$$\begin{cases} D = 0 & \text{Undamaged ice} \\ D = 1 & \text{Fully damaged ice} \end{cases}$$

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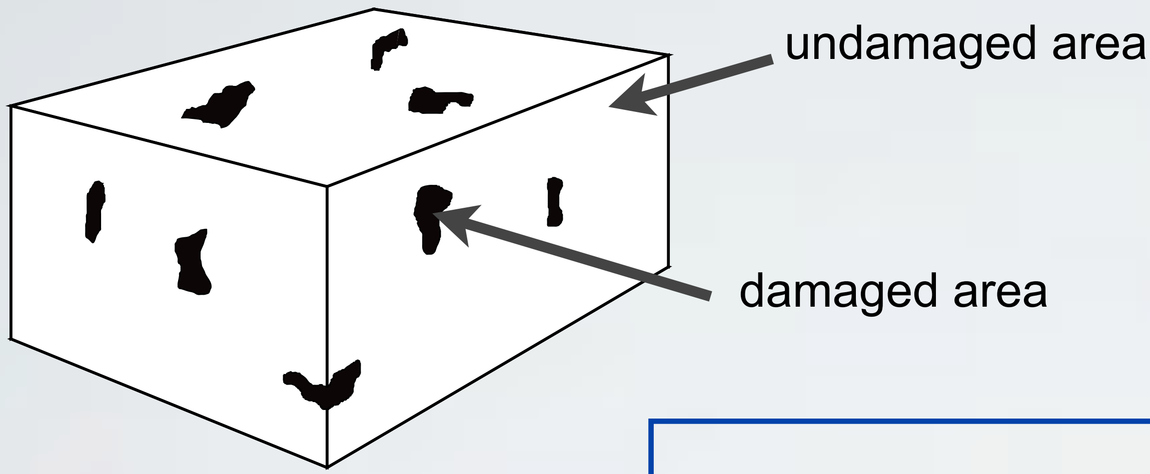


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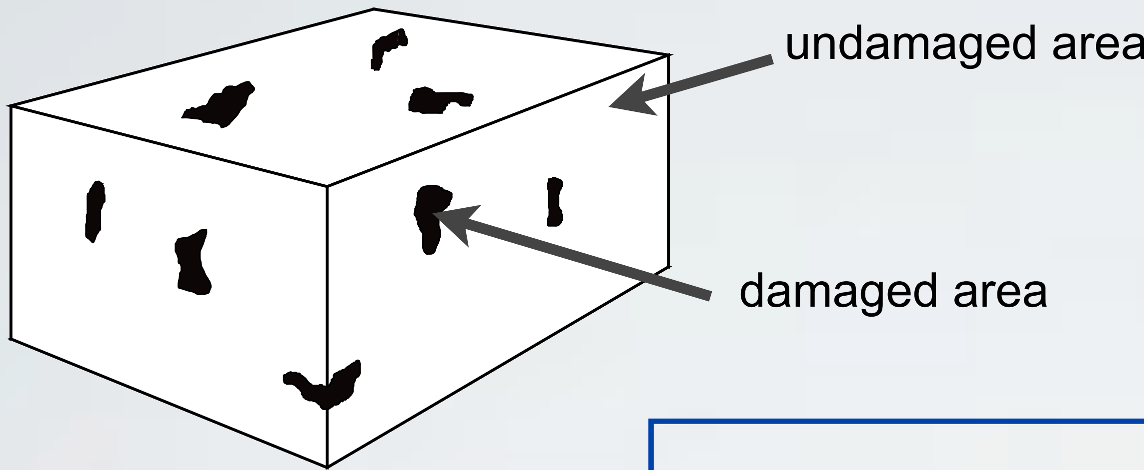


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$$\frac{\partial D}{\partial t} + \vec{u}\nabla D = f(\chi)$$

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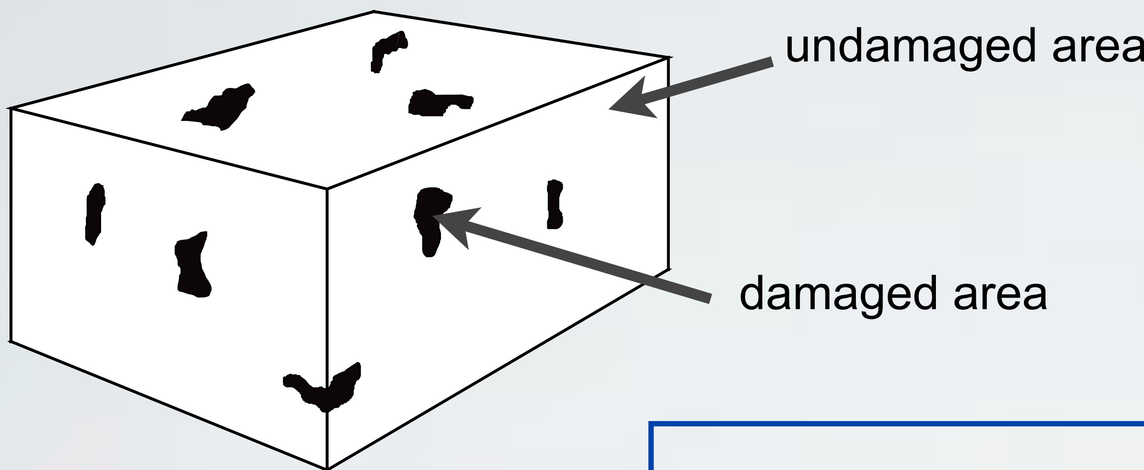


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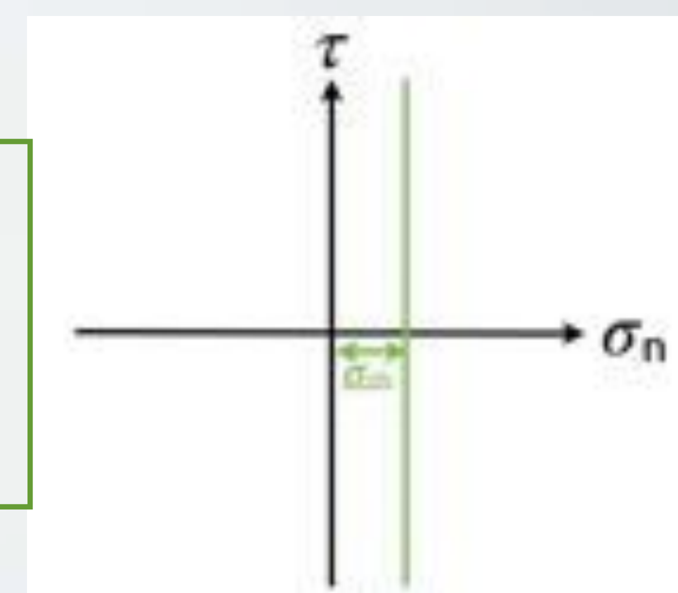
Stress criterion for damage increase :

$$\chi(\tilde{\sigma}) = \frac{\sigma_I}{(1-D)} - \sigma_{th}$$




*Limit of the damage criterion. Stresses are positive in traction*

# Model Presentation - Continuum Damage Mechanics

**Damage Mechanics**

**1/ Subcritical crack propagation (Weiss, 2004)**
2/ Fracture initiation (Van der Veen, 1998ab)
3/ Fracture arrest

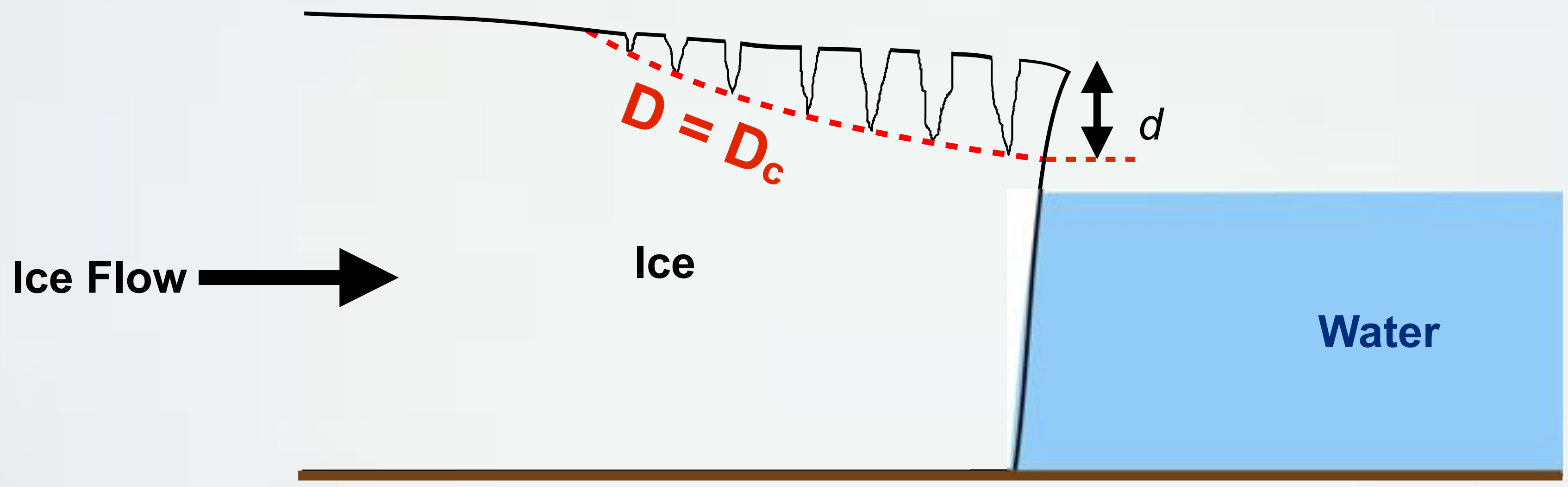
Damage is free to evolve, slowly.

Ice keeps a viscous behaviour.

The viscosity and the ice flow are affected

Looking for a critical damage contour D= Dc.

↳ Representing a pre-existing crevasse depth *d*




*Glacier shape*

# Model Presentation - Continuum Damage Mechanics

**USF_Damage.f90**

## InitDamage

Put some damage in the ice following a gaussian distribution

```
Initial Condition 1
  Pressure = Real 0.0
  Velocity 1 = Real 0.0
  Velocity 2 = Real 0.0
  DID = Variable Damage
    Real Procedure "./USF_Damage" "InitDamage"
End
```

+

Boundary Conditions

## EnhancementFactor

Change the value of Glen Enhancement Factor

$$E = \frac{1}{(1-D)^n}$$

```
Material 1
  Density = Real $rhoi

  Viscosity Model = String "Glen"
  Glen Enhancement Factor = Variable Damage
    Real Procedure "./USF_Damage" "EnhancementFactor"
```

# Model Presentation - Continuum Damage Mechanics

InitDamage ← **USF_Damage.f90** → EnhancementFactor

↓

**SourceDamage**

Compute the source term of the advection equation

**Compute Principal stresses**

↑

**ComputeDevStress**

# Model Presentation - Continuum Damage Mechanics

InitDamage ← **USF_Damage.f90** → EnhancementFactor

↓

## SourceDamage

Compute the source term of the advection equation

**Compute Principal stresses** → **Compute the damage criterion**

$$\chi(\tilde{\sigma}) = \frac{\sigma_I}{(1-D)} - \sigma_{th}$$

**ComputeDevStress** ↑ (to Compute Principal stresses)

# Model Presentation - Continuum Damage Mechanics

**InitDamage** ← **USF_Damage.f90** → **EnhancementFactor**

↓

**SourceDamage**

Compute the source term of the advection equation

**Compute Principal stresses** → **Compute the damage criterion**

$$\chi(\tilde{\sigma}) = \frac{\sigma_I}{(1-D)} - \sigma_{th}$$

→ **Compute the source term**

$$f(\chi) = B.\chi$$

**ComputeDevStress** → **Compute Principal stresses**

```
Body Force 1
  Flow BodyForce 1 = Real 0.0
  Flow BodyForce 2 = Real $gravity
  DGD Source = Variable Damage
    Real Procedure "./USF_Damage" "SourceDamage"
End
```

$$\frac{\partial D}{\partial t} + \vec{u}\nabla D = f(\chi)$$

# Model Presentation - Continuum Damage Mechanics

**USF_Damage.f90** → **InitDamage** | **SourceDamage** | **EnhancementFactor**

## SourceDamage

Compute the source term of the advection equation

**ComputeDevStress** → **Compute Principal stresses** → **Compute the damage criterion**

$$\chi(\tilde{\sigma}) = \frac{\sigma_I}{(1-D)} - \sigma_{th}$$

→ **Compute the source term**

$$f(\chi) = B.\chi$$

```
! Damage Source Parameters
Damage Criterion = string "Krug"
Damage Enhancement Factor = Real 20.0
Damage Parameter sigmath = Real 0.04
Damage Parameter lambdah = Real 0.4
```

**Criterion name**

**B**

$\sigma_{th}$

**Healing factor**

```
Body Force 1
  Flow BodyForce 1 = Real 0.0
  Flow BodyForce 2 = Real $gravity
  DGD Source = Variable Damage
    Real Procedure "./USF_Damage" "SourceDamage"
End
```

$$\frac{\partial D}{\partial t} + \vec{u}\nabla D = f(\chi)$$

# Model Presentation - Linear Elastic Fracture Mechanics

**LEFM**

1/ Subcritical crack propagation (Weiss, 2004)
**2/ Fracture initiation (Van der Veen, 1998ab)**
3/ Fracture arrest

The stress intensity factor $K_I$ is used to describe stress concentration near the tip of the crack.

$$K_I = < S_{xx} > \sqrt{\pi d}$$

opening tensile stress minus closing compressive cryostatic pressure

crevasse depth

The fracture propagates if $K_I > K_{Ic}$ (ice toughness)

# Model Presentation - Linear Elastic Fracture Mechanics

**LEFM**

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$$K_I = < S_{xx} > \sqrt{\pi d}$$

opening tensile stress minus closing compressive cryostatic pressure ($< S_{xx} >$)

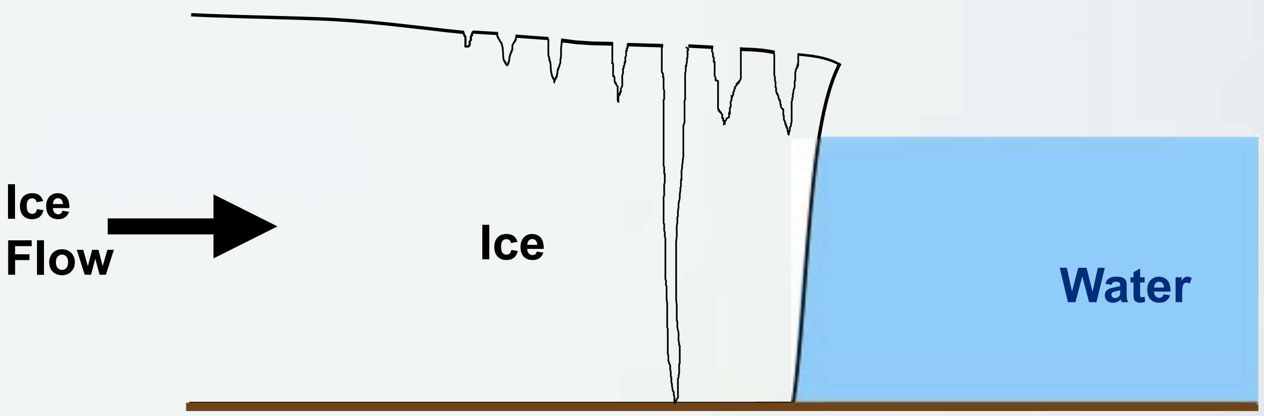
crevasse depth ($d$)

The fracture propagates if $K_I > K_{Ic}$ (ice toughness)

$K_{Ia}$ = arrest criterion

$$K_{I_a} = \alpha . K_{I_c}$$

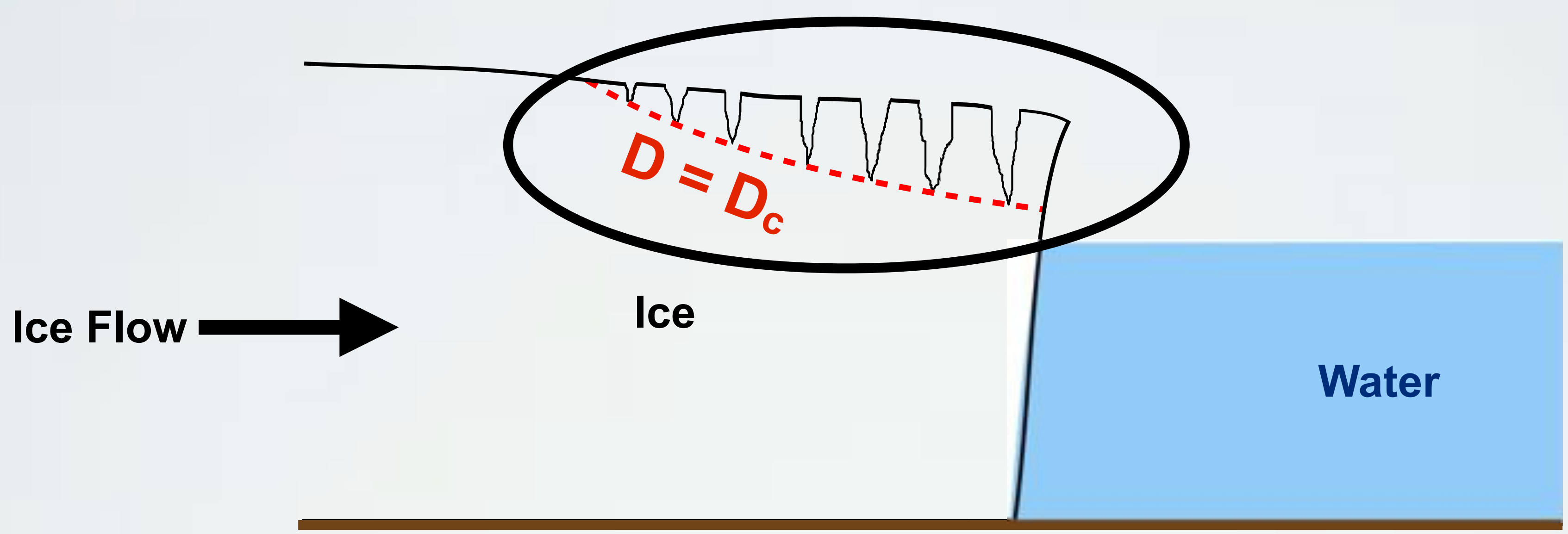
If $K_I > K_{Ia}$ at the bed, calving event occurs along a vertical line.



# Model Presentation - Linear Elastic Fracture Mechanics

**CalvingSolver.f90**

**Get Dc Contour**




*Glacier shape*

# Model Presentation - Linear Elastic Fracture Mechanics

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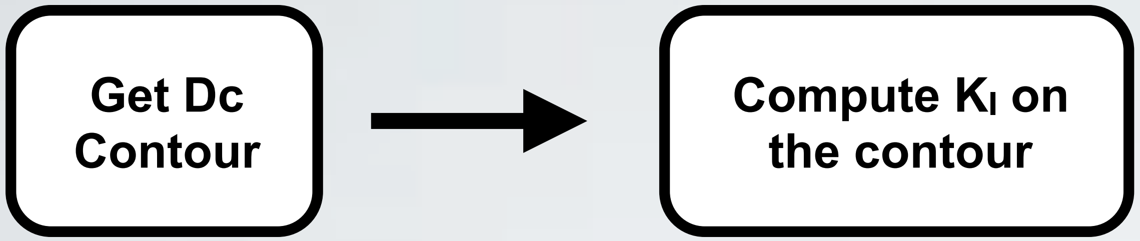
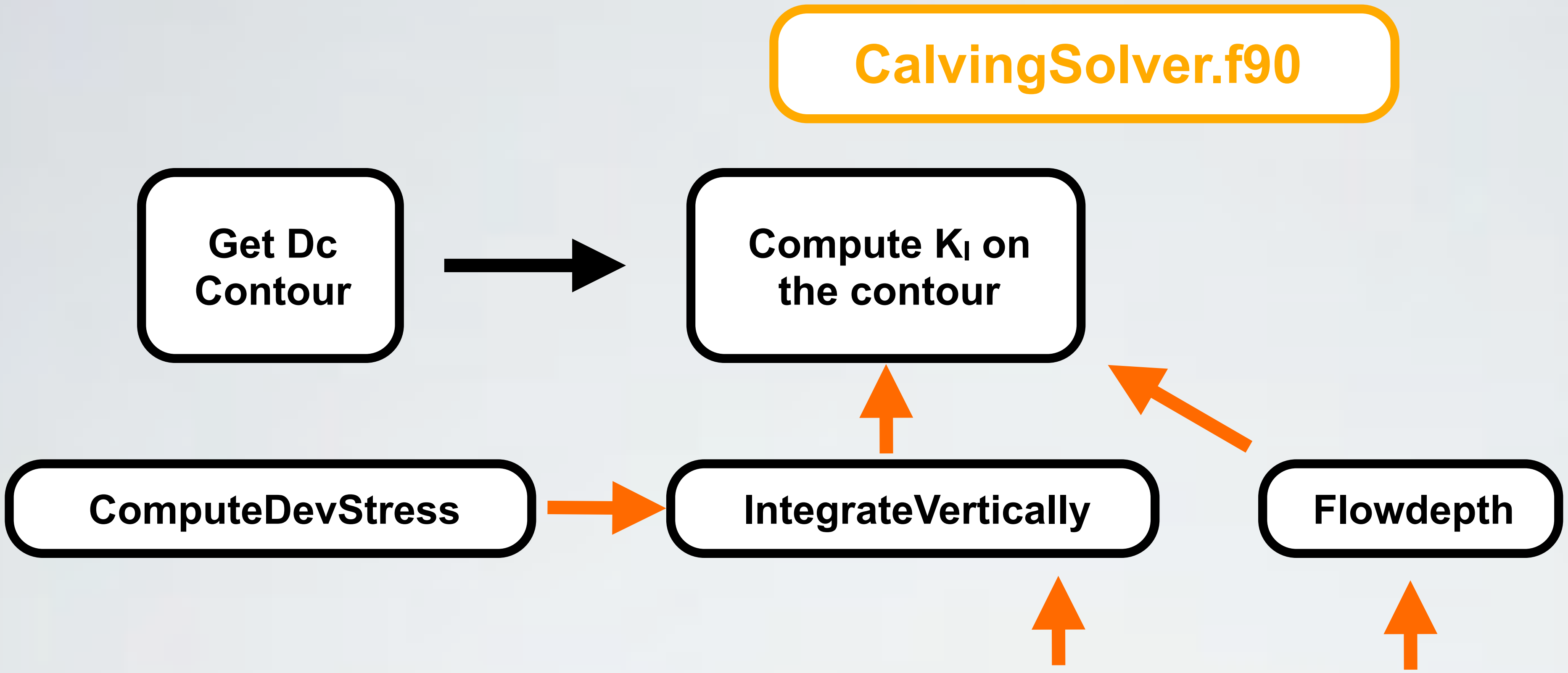



$$K_I = < S_{xx} > \sqrt{\pi d}$$

# Model Presentation - Linear Elastic Fracture Mechanics

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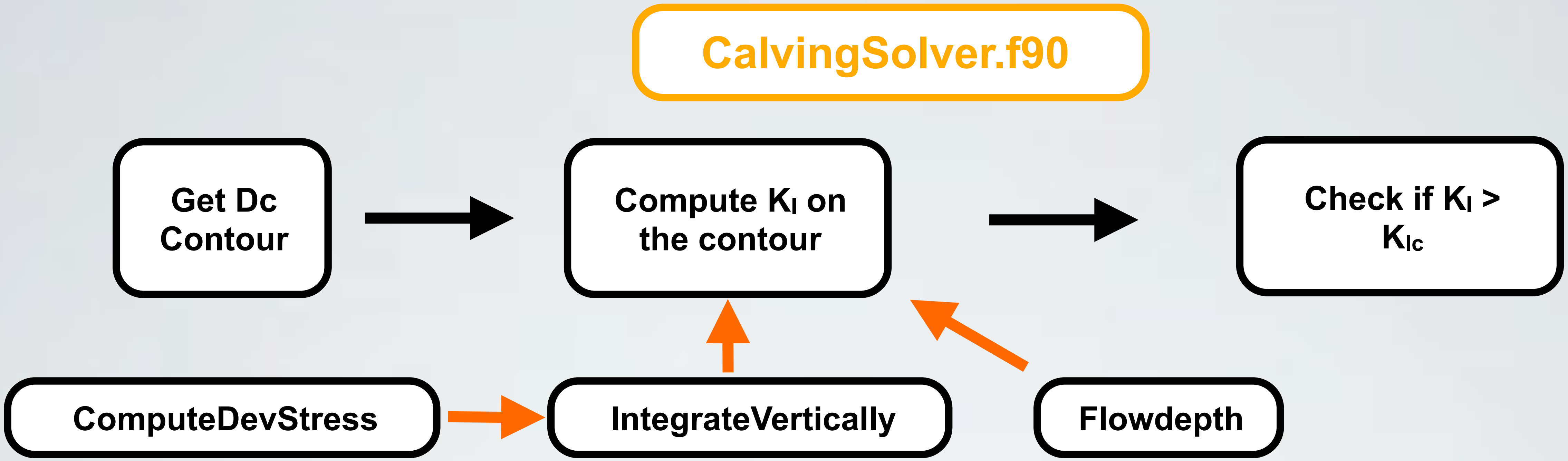

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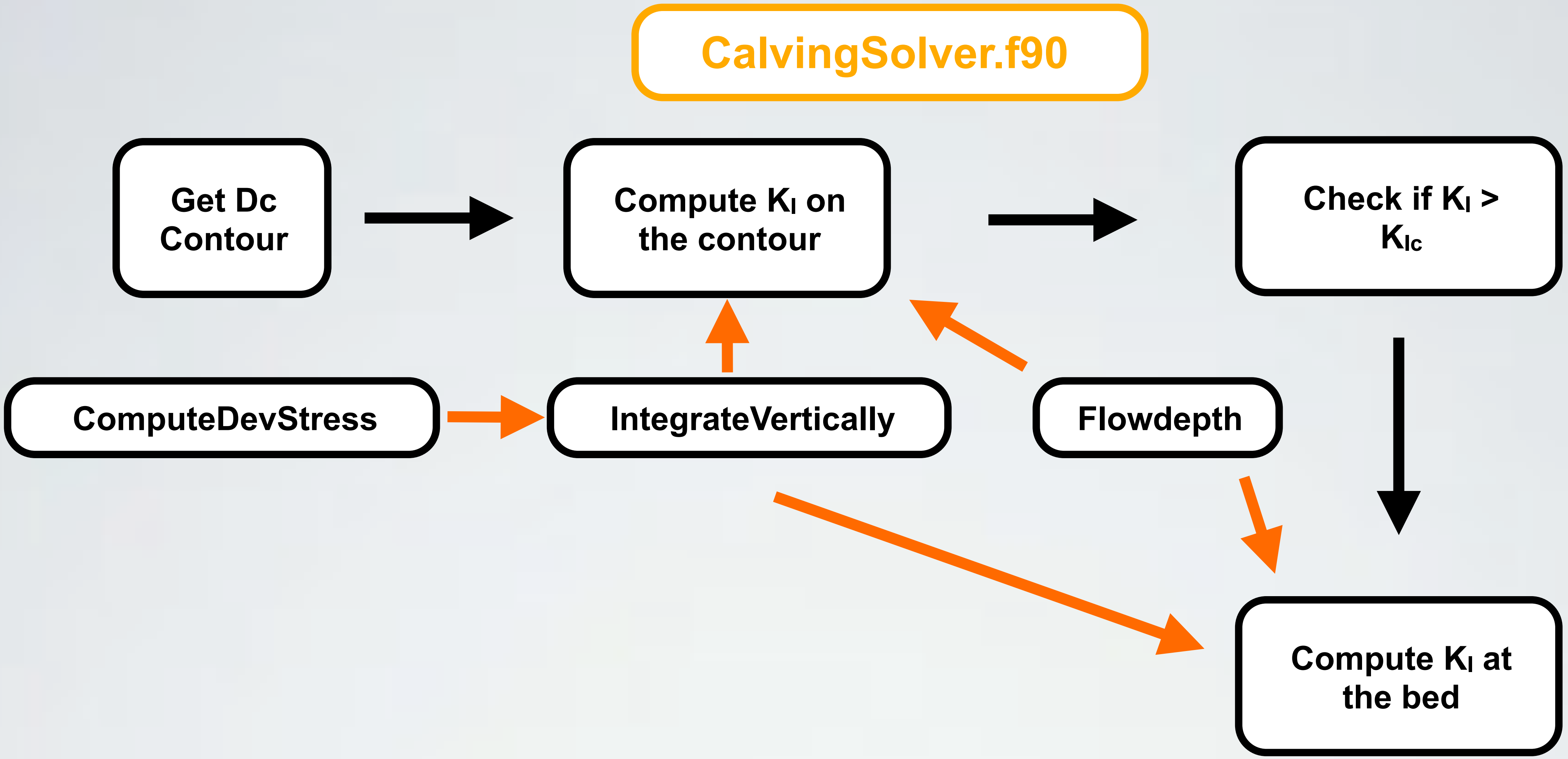

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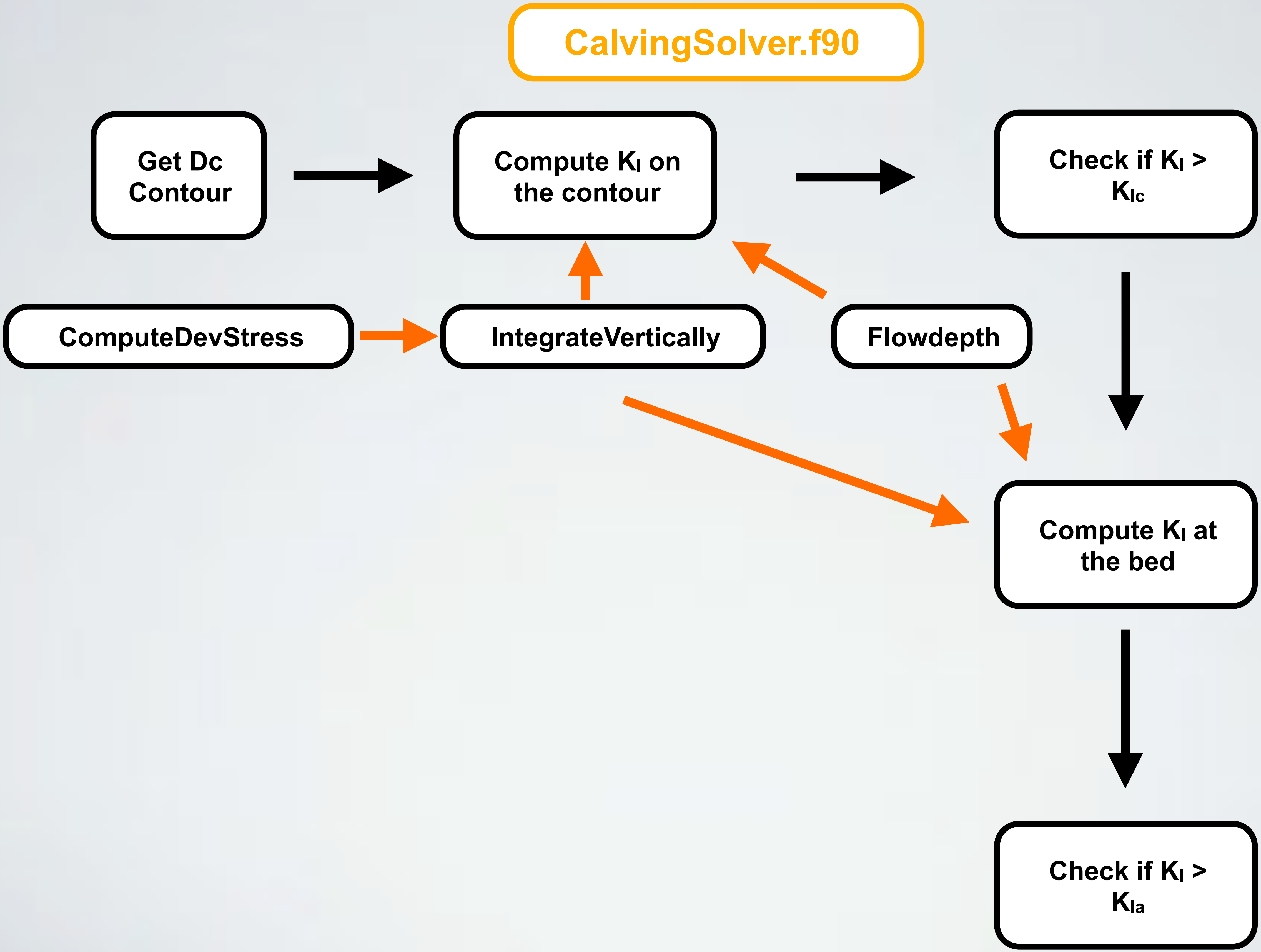

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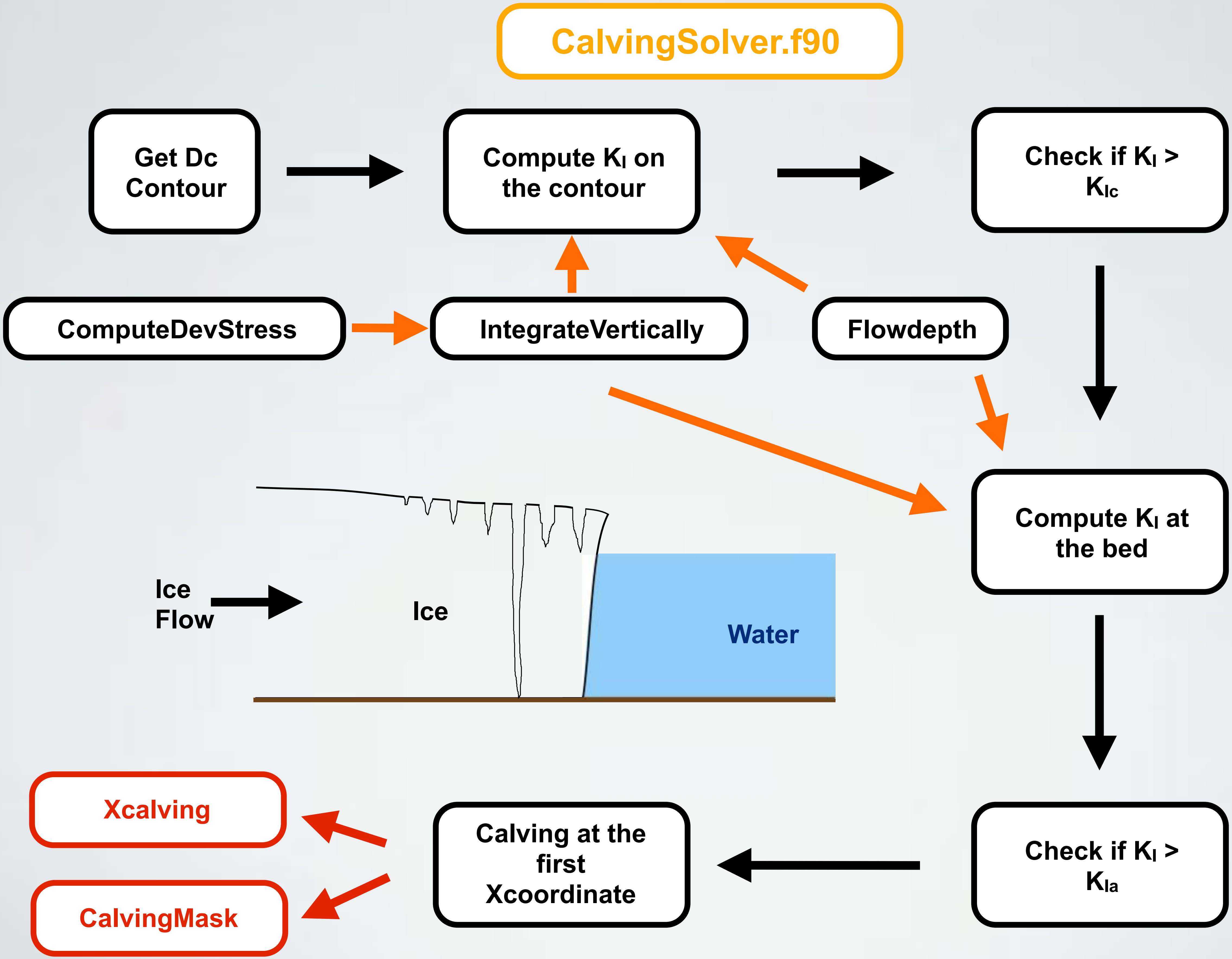


# Model Presentation - Linear Elastic Fracture Mechanics

**CalvingSolver.f90**

```
Solver 8
  !Exec Solver = Never
  Equation = "CalvingSolver"
  Variable = CalvingMask !CalvingMask
  Variable DOFs = 1
  Procedure = "./CalvingSolver" "CalvingSolver"

  Active Coordinate = Integer 2

  Critical Value = Real $CalvingCriterion !(Real 0.1)
  Critical Toughness = Real 0.150 !MPa
  Crack Arrest Parameter = Real 0.5 ! KIa = a*KIc

  Exported Variable 1 = Xcalving
  Exported Variable 1 DOFS = 1
End
```

**CalvingMask**

**Dc**

**$K_{Ic}$**

$\alpha$

with

$$K_{I_a} = \alpha . K_{I_c}$$

**Xcalving**

# Model Presentation - Linear Elastic Fracture Mechanics

**Initialization**

```
Initial Condition 2
  ! sets the initial position of the free surface variable
  Xf = Equals Coordinate 1
  ! reference to this initial condition
  ReferenceXf = Equals Coordinate 1
  Xcalving = Equals Coordinate 1
  CalvingMask = Real -1.0
End
```

**How to prescribe terminus retreat?**

```
! Calving front (x=L) This is the free surface here
Boundary Condition 4
  Target Boundaries = 2
  Body Id = 2

!  décommenter si besoin d'un front mobile
  Xf = Equals Xcalving
  Xf Condition = Equals CalvingMask

  Mesh Update 1 = Variable Xf, ReferenceXf
    Real MATC "tx(0) - tx(1)"

  Flux integrate = Logical True
```

# Conclusion

Two distincts aspects :

- Damage Model -> USF_Damage.f90
- Calving Criterion -> CalvingSolver.f90

**Work in progress !**

Prospectives / Limitations :

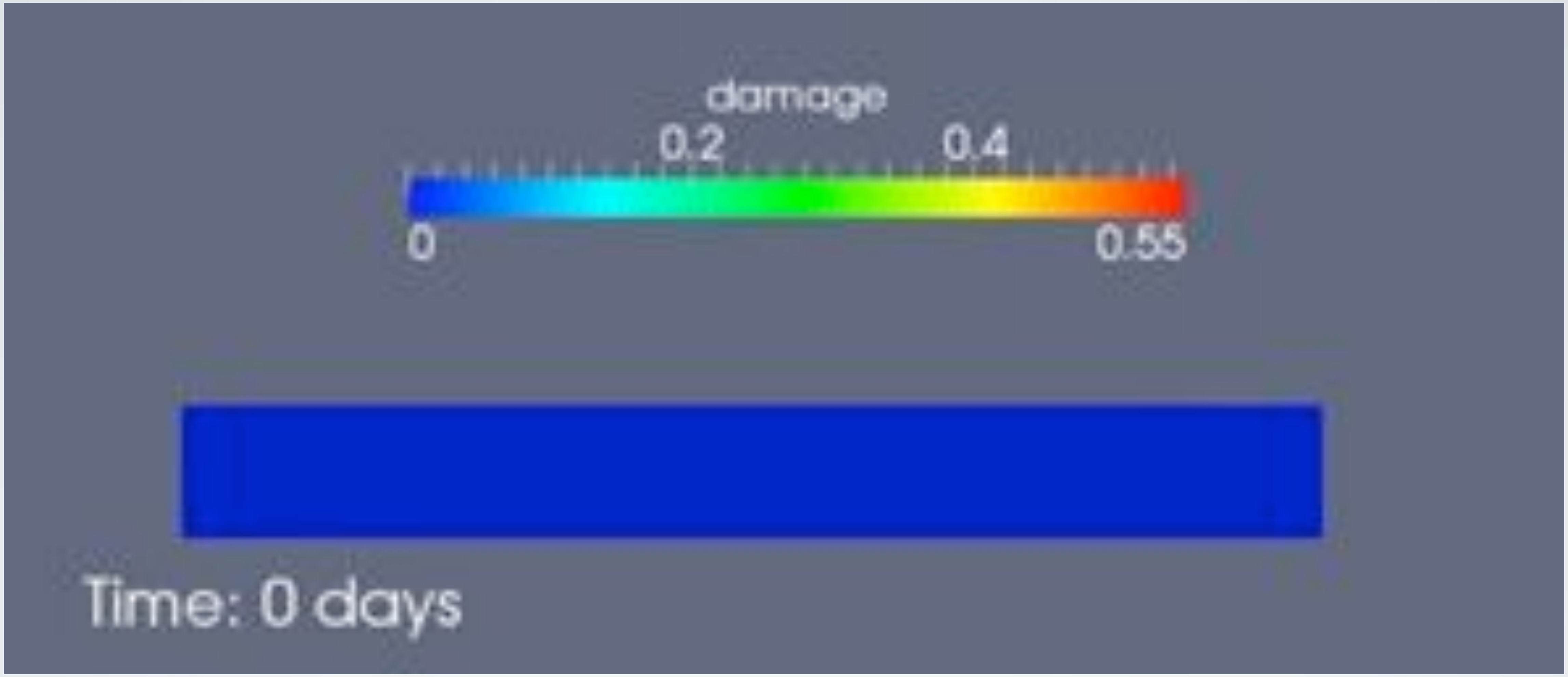
Damage Model :
- Extent the damage model for 3D/ Partitionned cases
- Improve the complexity of damage criterion
- Application on real cases

Calving Solver (not the priority):
- 3D developpement

# Some results

With a sinusoidal frontal melt rate...

# Some results




With a sinusoidal frontal melt rate...

# Some results


With a sinusoidal frontal melt rate...

# Thank you !

## References

Van der Veen, C.J. « Fracture mechanics approach to penetration of bottom crevasses on glaciers ». Cold Regions Science and Technology 27, nº 3 (june 1998): 213–223. doi:10.1016/S0165-232X(98)00006-8.

Van der Veen, C.J. « Fracture mechanics approach to penetration of surface crevasses on glaciers ». Cold Regions Science and Technology 27, nº 1 (february 1998): 31–47. doi:10.1016/S0165-232X(97)00022-0.

Weiss, Jérôme. « Subcritical crack propagation as a mechanism of crevasse formation and iceberg calving ». Journal of Glaciology 50, nº 168 (2004): 109–115. doi:10.3189/172756504781830240.