A real world application
Tête Rousse Glacier

Olivier GAGLIARDINI

LGGE - Grenoble - France

Tête Rousse Glacier

✓ **Context**

    - The history of Tête Rousse Glacier

    - The 2010 water filled-cavity

    - Analysis of the cavity roof stability (Autumn 2010)

✓ **Step 1**

    - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ **Step 2**

    - Influence of an empty cavity below Tête Rousse Glacier (diagnostic)

✓ **Step 3**

    - Rate of closure of the cavity for a given drainage scenario (prognostic)

Location (Mont Blanc Area, France Alps)

Tête Rousse Glacier

Bionnassay Glacier

74170 Saint-Gervais-les-Bains

Location (Mont Blanc Area, France Alps)

Tête Rousse glacier
3100 to 3300 m
0.08 km² (2007)
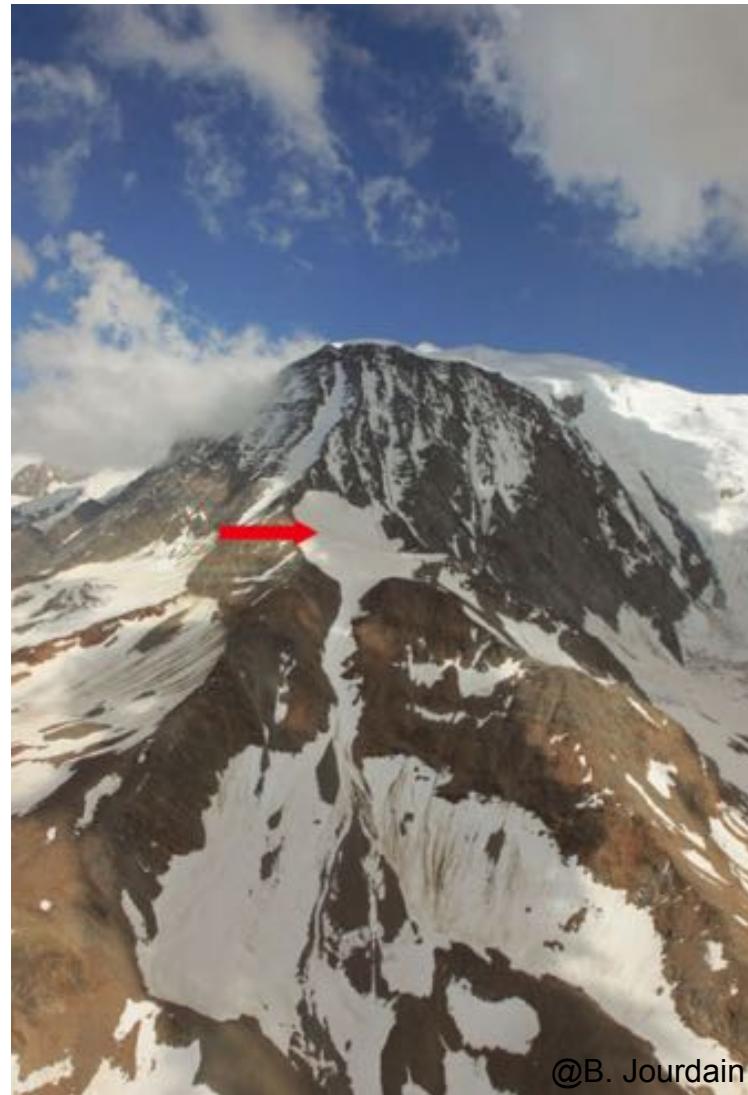
@B. Jourdain

Chronology

The Past History – The 1892 catastrophe

Contemporary history:

2007-10 - Studies to answer the question about the necessity to maintain the tunnel

07/2010 - A water filled cavity under pressure is discovered

  - Crisis – Artificial drainage

2011   - Small research program to understand the formation of the cavity

  - New crisis – Artificial drainage

2012   - New Artificial drainage needed

2016   - Still under survey – build of an artificial spillway

The 1892 catastrophe

11 July 1892

175 fatalities

100 000 m³ of water

Flood produced

800 000 m³ of sediment

LE PROGRÈS ILLUSTRÉ

LA CATASTROPHE DE SAINT-GERVAIS
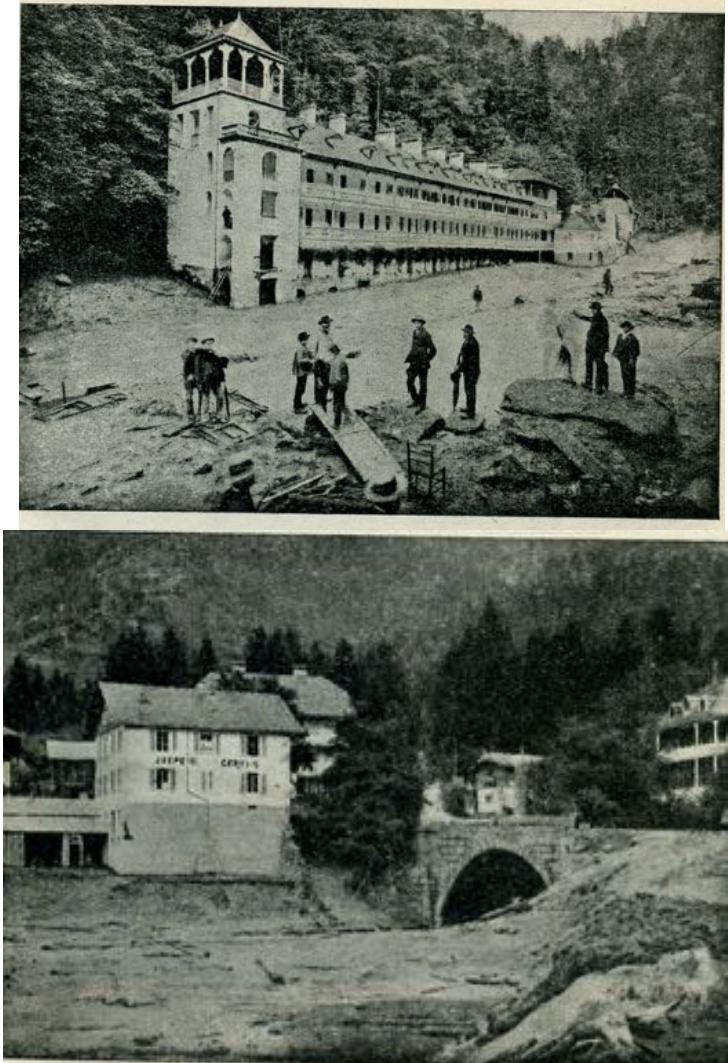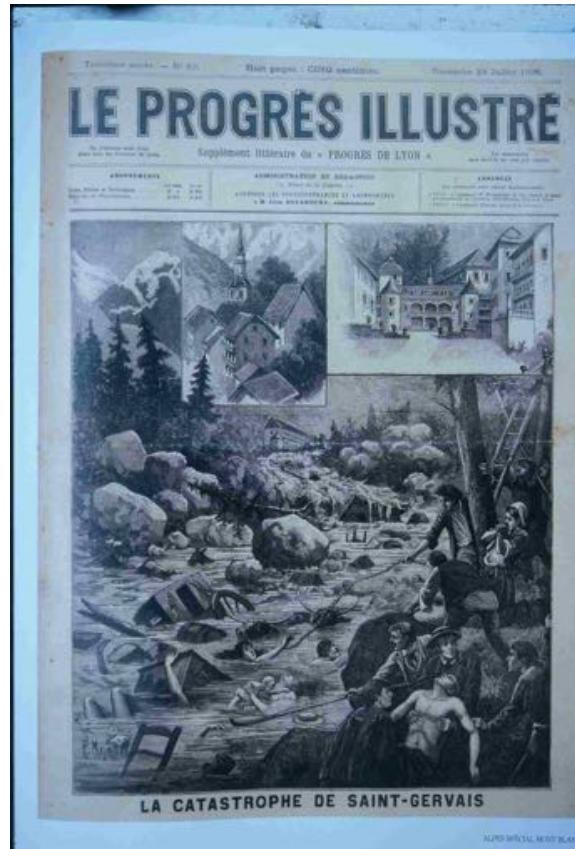
Fig. 22. — Le pont de la route départementale n° 4 tourné et submergé par la lave.
13 juillet 1892 — Cliché Kuss.

@Vincent, LGGE

The 1892 catastrophe
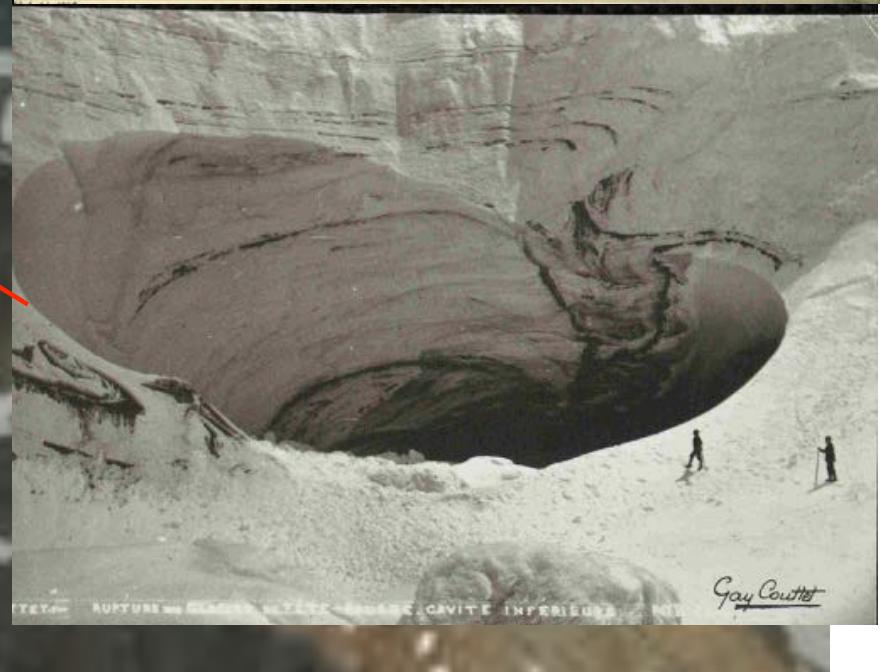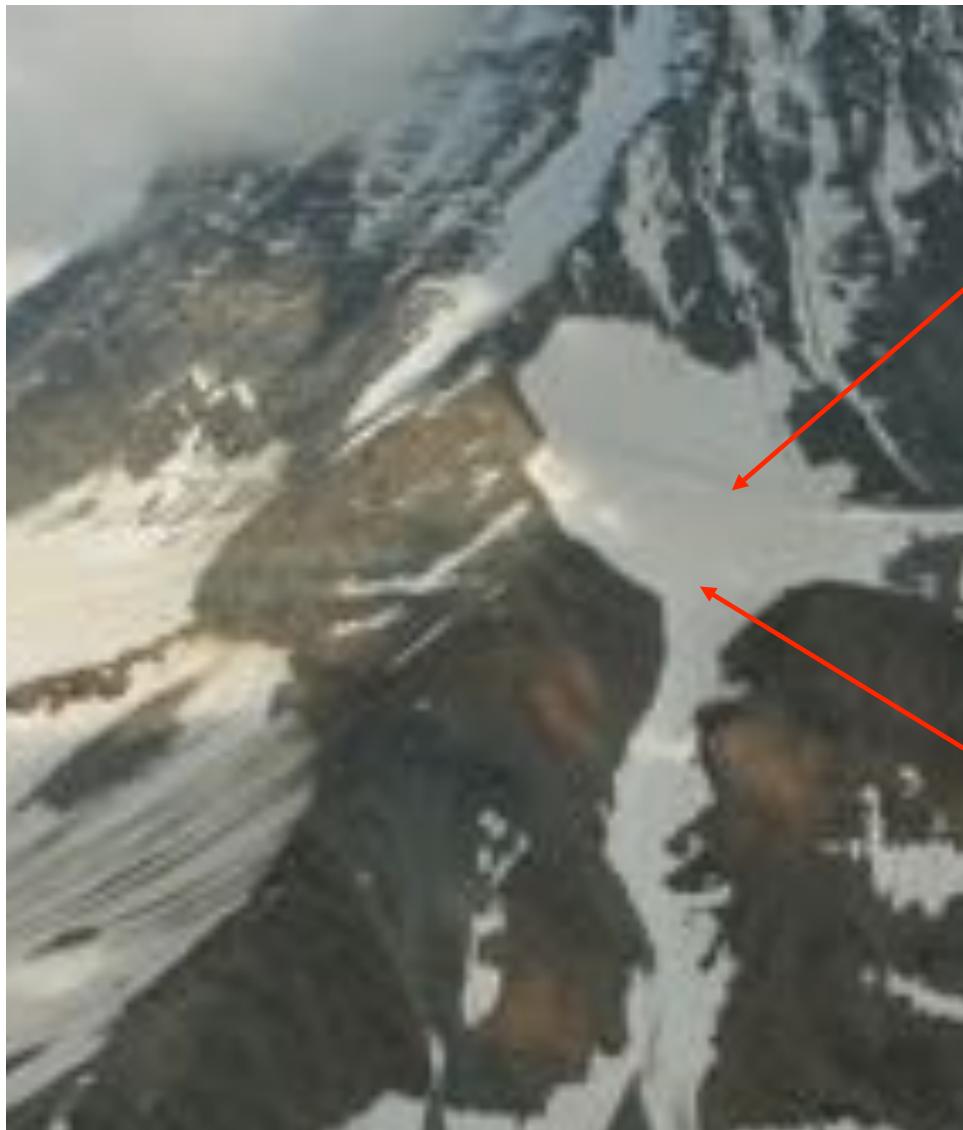
@Vincent, LGGE

Is there still a risk at Tête Rousse ?

Question asked by authorities in 2007
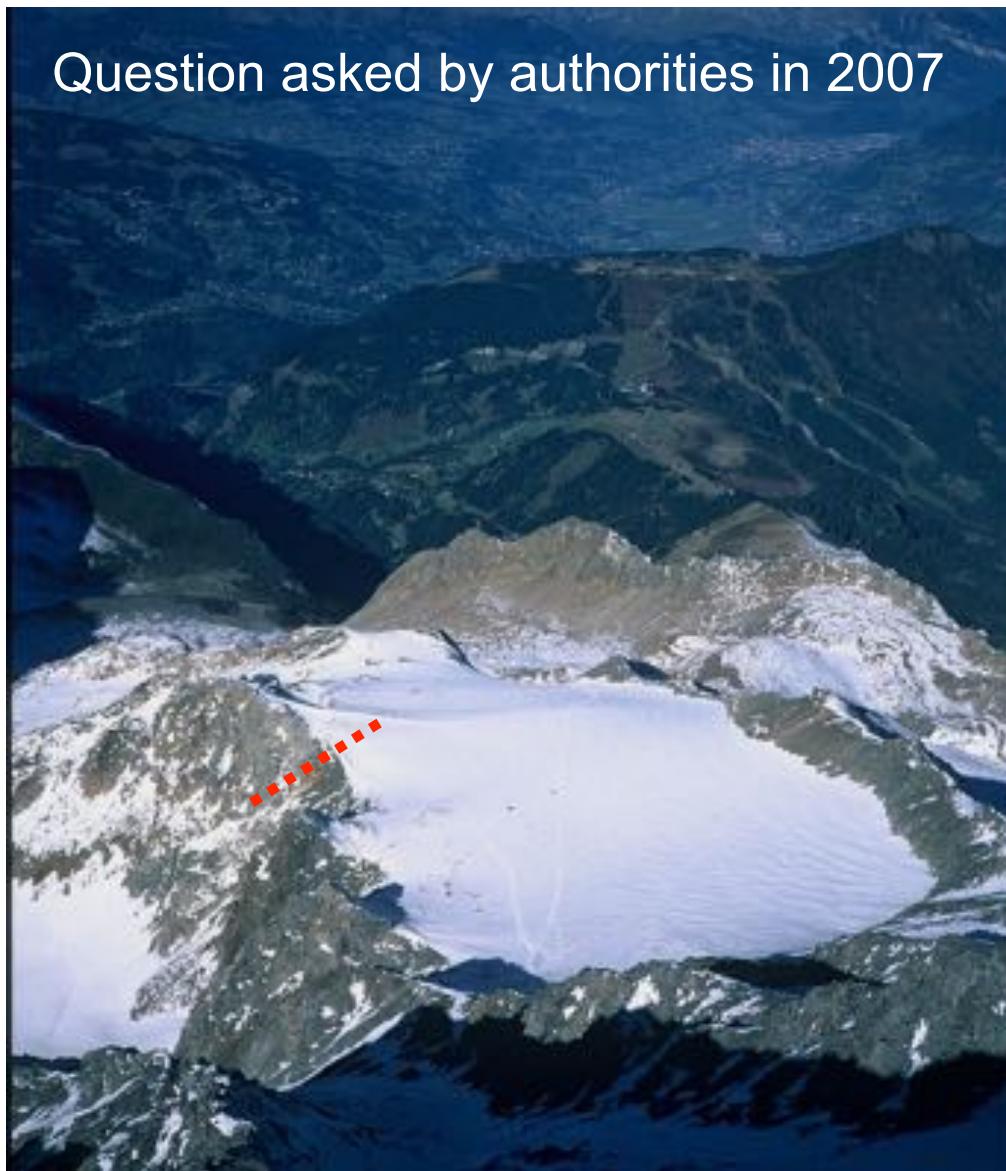
@Vincent, LGGE

Glaciological studies

. Topographic measurements

. Radar measurements

. Temperature measurements

. Mass balance measurements
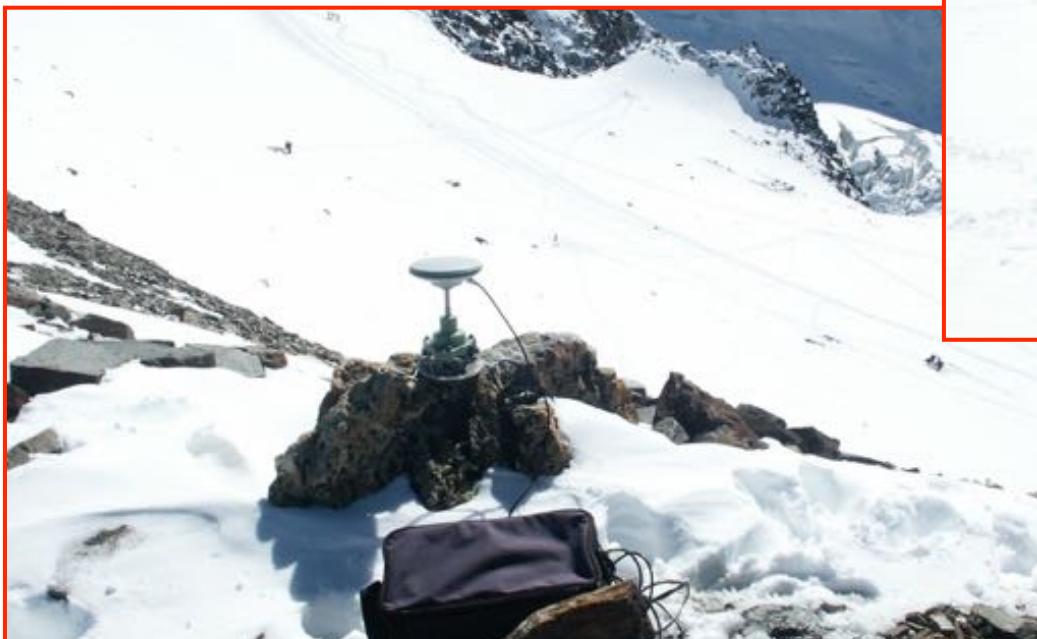
@Vincent, LGGE

Glaciological studies
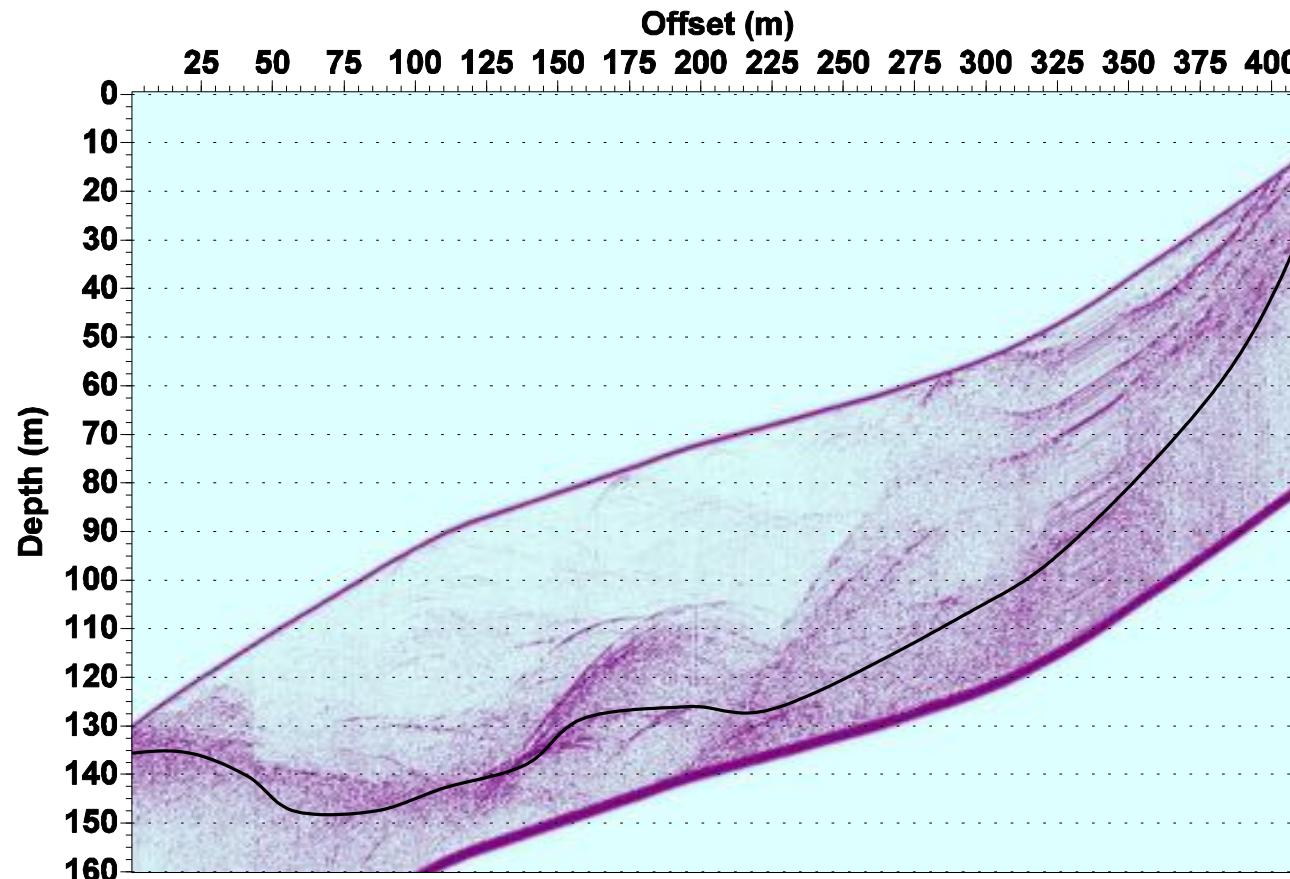
@Vincent, LGGE

The radar measurements showed a zone (volume) with an anomaly.

Glaciological studies

In Sept 2009, geophysical survey using the

Magnetic Resonance Imaging (LTHE, Grenoble)

@Vincent, LGGE

# Glaciological studies

Water content (%)

Water volume of 65 000 m^3
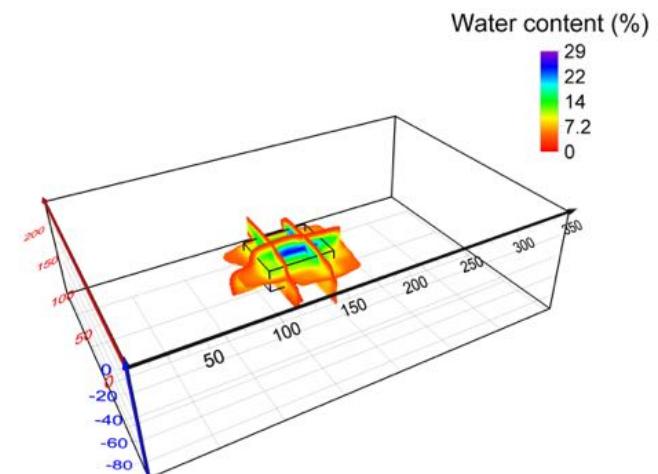
Report presented to public authorities in March 2010

@Vincent, LGGE

Pressure measurements

20 hot-water drillings performed from 29 June to 8 July 2010

Confirm the presence of a cavity

and that

the cavity is under pressure!

Decisions

The hydrostatic pressure exceeded the ice pressure due to the weight of the ice column

We could expect that the water contained in the glacier would be released suddenly

The public authorities have been warned immediately (13 July, 2010)

It has been decided to drain the subglacial lake as soon as possible, because 3000 people were threatened in the valley.

# A difficult field work

Drainage of the cavity

The artificial drainage started the 26 of August

A new risk ?
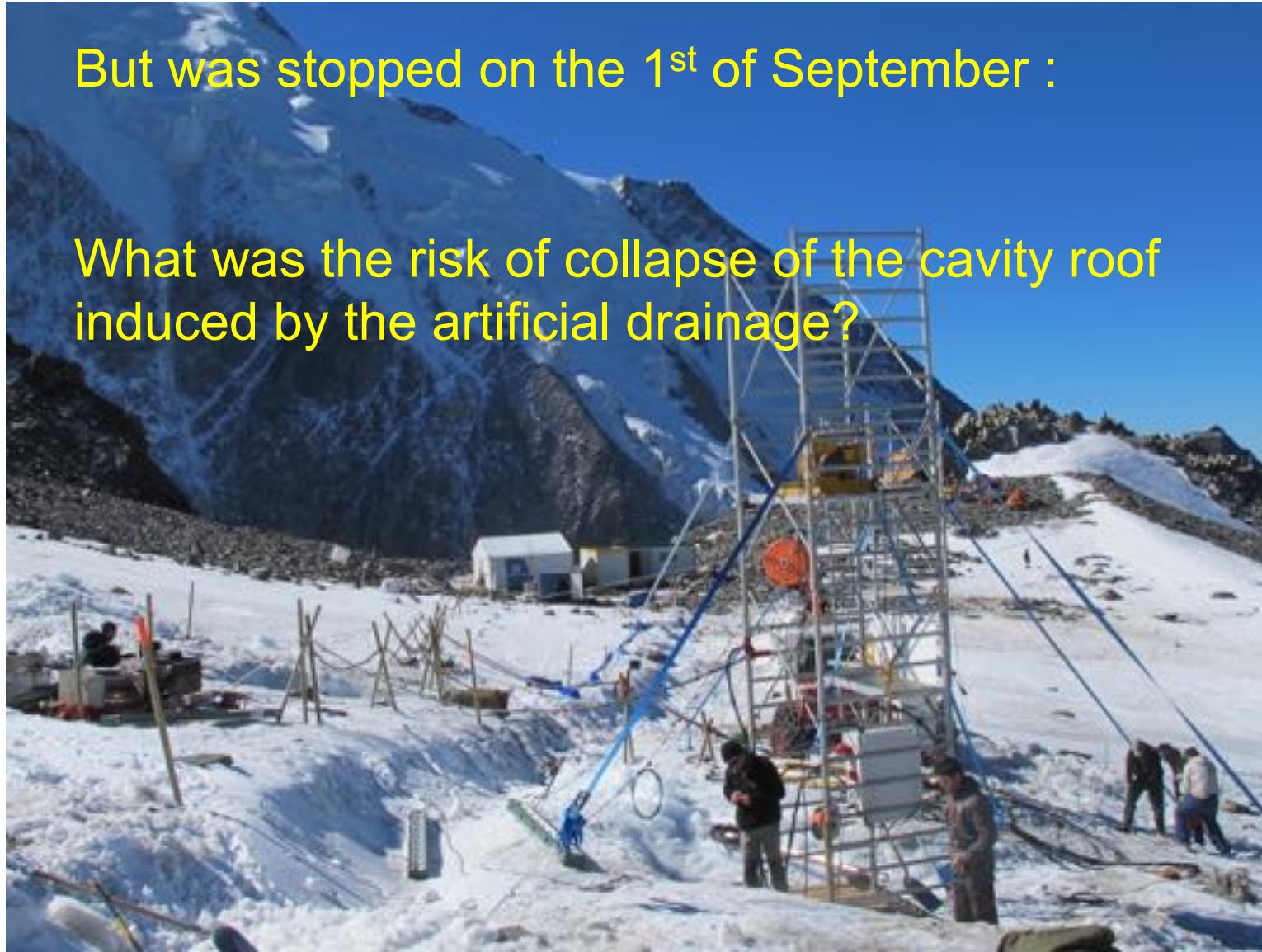
But was stopped on the 1st of September :

What was the risk of collapse of the cavity roof induced by the artificial drainage?

The 2010 cavity

Pumping of 47 700 m^3 from 25 August to 8 October 2010

Question (addressed end of August 2010):
What is the risk of break-up during the pumping phase?

right

upstream

$h \sim 50$ m
$l_y \sim 80$ m
$l_x \sim 30$ m

left
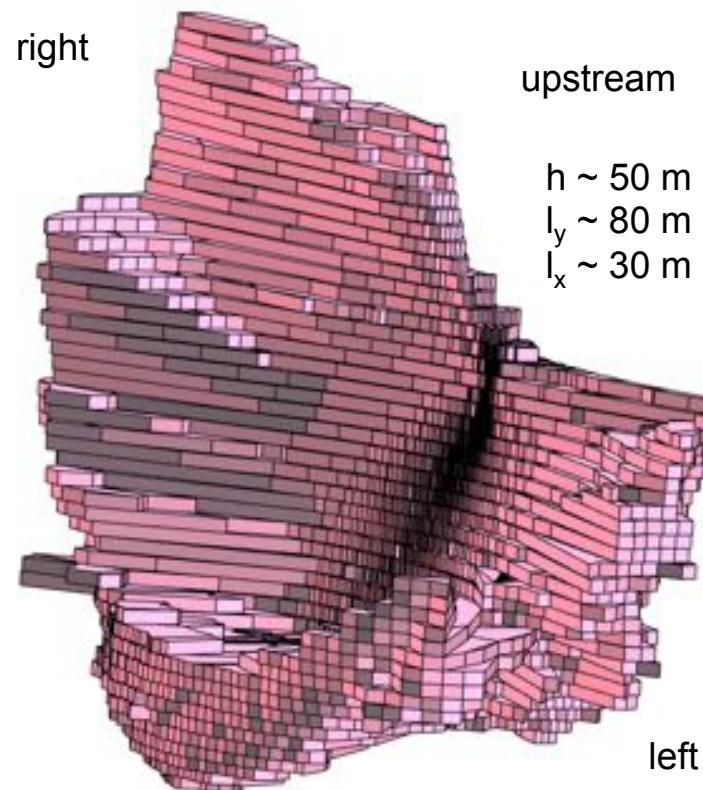
Time-line for investigations

Sonar data

Meeting with the mayor of St Gervais

Proposed application

Construct a model of the flow of Tête Rousse Glacier

- Step 1: Without the cavity (normal state)

- Step 2: Add an empty cavity (stress analysis)

- Step 3: Rate of closure of the cavity

(surface deformation analysis)

Data for ice flow modelling

- Bedrock DEM

- 2007 Surface DEM

- Cavity topography from sonar measurements

- Few surface velocities, without the empty cavity (0.6 m/a at the centre of the glacier)

- 27 Stakes to measure surface displacement during drainage
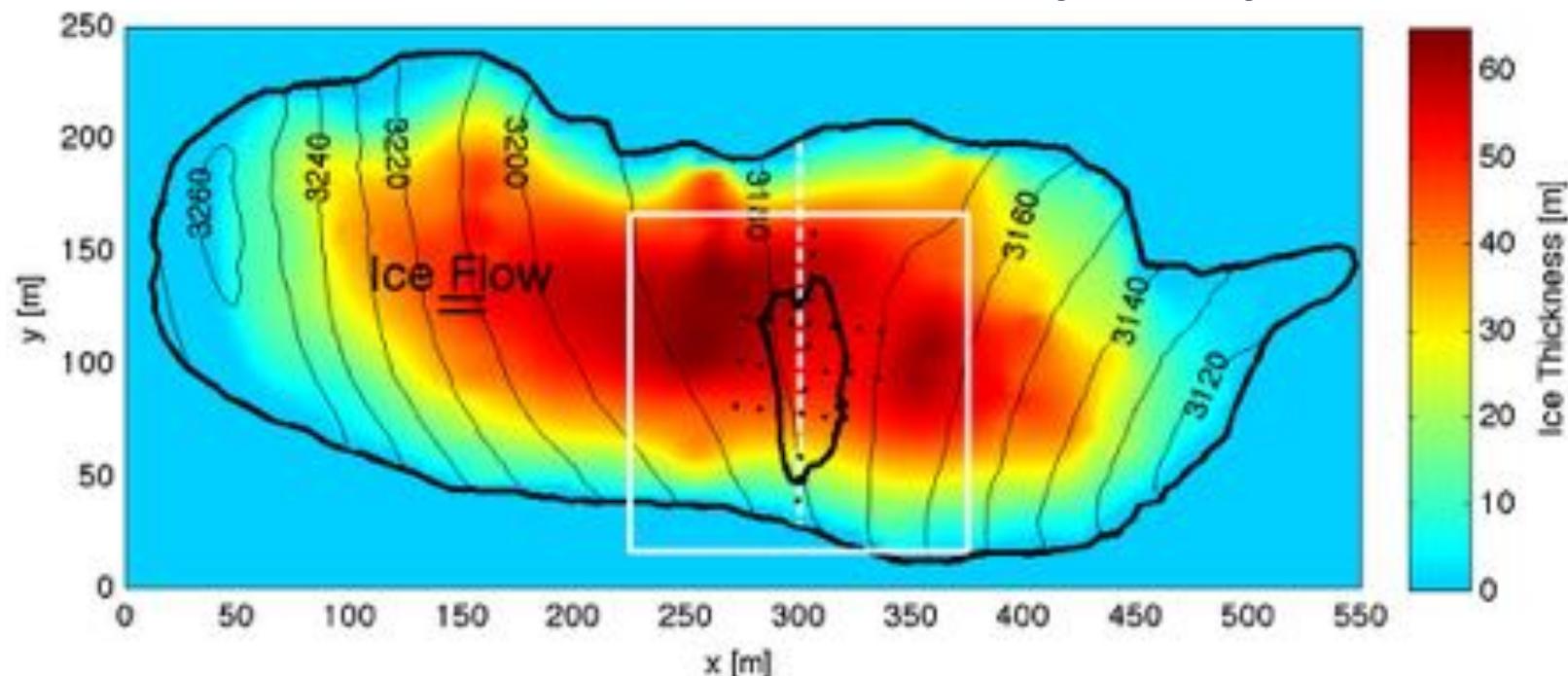
Material:

Data: Contour_TR_cavity.dat, Contour_TR_glacier.dat, DEM_TR_bed.dat, DEM_TR_cavity.dat, DEM_TR_surf.dat

PROG: USF_TR.f90

Step1: Makegeo.m, teterousse.geo, teterousse1.sif

Step2a: Makegeo_2.m, teterousse.geo, teterousse2a.sif

Step2b: teterousse2b.sif

Step3a: teterousse3a.sif

Step3b: teterousse3b.sif

Step3c: teterousse3c.sif

Modelling Tête Rousse Glacier

✓ **Step 1** - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ Step 2

      - 2a Influence of an empty cavity below Tête Rousse Glacier (diagnostic)
      - 2b Apply a water pressure in the cavity

✓ Step 3

      - 3a Rate of closure of the cavity for a given drainage scenario (prognostic)
      - 3b Add a drainage scenario

Step 1: Work to do

- create the mesh

- impose the boundary conditions in the SIF file

- test other BCs on the lateral boundary

- test sliding at the base of the glacier

# Step 1: steps to make the mesh

1/ build the `teterousse.geo` file (input file of gmsh, footprint of the glacier)

2/ `gmsh` to get `teterousse.msh` (still footprint of the glacier)

3/ `ElmerGrid` to transform into Elmer format (still footprint of the glacier)

4/ we will use the internal extrusion feature in Elmer to create a volume from this footprint (see the sif file)
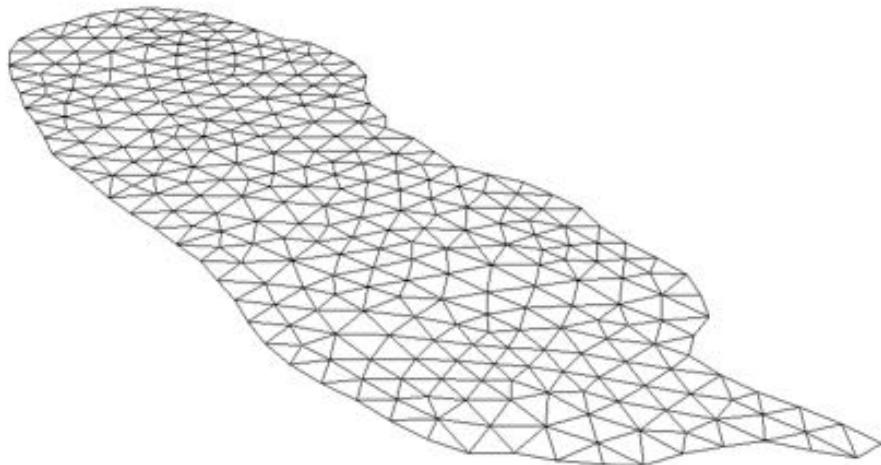
Step 1: `Makegeo.m` (create a .geo file)

```matlab
clear;
lc_out=18.0;               (size of the element in the plane)

A=dlmread('Contour_TR_glacier.dat');      (Read contour points)
fid1=fopen('teterousse.geo','w');
fprintf(fid1,'Mesh.Algorithm=5; \n');   (delaunay algorithm)

As=size(A,1);

np=0;
for ii=1:As
    np=np+1;
    fprintf(fid1,'Point(%g)={%14.7e,%14.7e,0.0,%g}; \n',np,A(ii,1),A(ii,2),lc_out);
end

fprintf(fid1,'Spline(1)={');
for ii=1:As
  fprintf(fid1,'%g,',ii);
end
fprintf(fid1,'%g}; \n',1);

fprintf(fid1,'Line Loop(2)={1}; \n');
fprintf(fid1,'Plane Surface(3) = {2}; \n');
fprintf(fid1,'Physical Line(4) = {1}; \n');
fprintf(fid1,'Physical Surface(5) = {3}; \n');

fclose(fid1)
```

Step 1a: `Makegeo.m`

```
% create teterousse.msh using gmsh
system "gmsh teterousse.geo -2"              (**)

% convert teterousse.gmsh in an Elmer type mesh
System "ElmerGrid 14 2 teterousse.msh -autoclean"              (**)
```
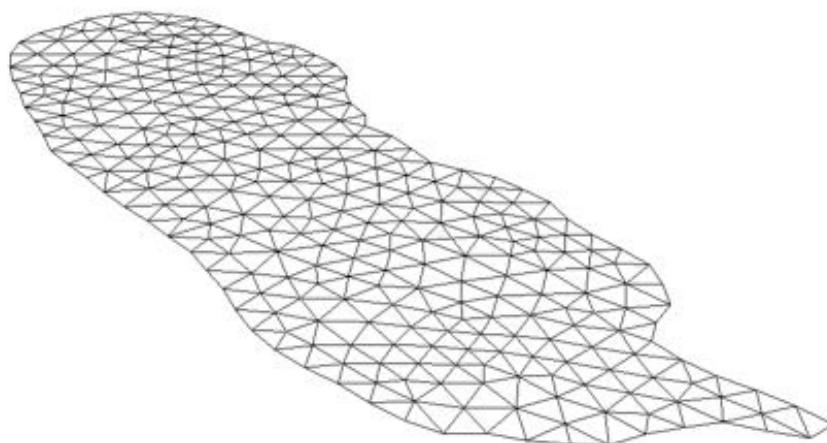

(**) if you copy and paste these lines, you might get an error as the minus sign is not always correctly understood (different coding).

Step 1: `gmsh` (create a .msh file)

```
gmsh teterousse.geo -2
```

help: http://www.geuz.org/gmsh/

line commands:
"`-2`" performs 1D and 2D mesh generation and then exit

Step 1: In the sif file

Define the number of vertical layers (Simulation section):

```
Simulation
  Coordinate System  =  Cartesian 3D
  Simulation Type = Steady

  Extruded Mesh Levels = Integer 16

  …
End
```

The second solver to be executed is the StructuredMeshMapper

```
Solver 2
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"
  Active Coordinate = Integer 3        (3d problem – mesh moves in z direction)
  Mesh Velocity Variable = String "dSdt"
  Mesh Update Variable = String "dS"
  Mesh Velocity First Zero = Logical True
  Displacement Mode = Logical False
  Correct Surface = Logical True         zs = min(zs, bed+1.0)
  Minimum Height = Real 1.0
End
```

Step 1: In the sif file

Read, interpolate and store in 2 variables the bed and surface DEM

```
Solver 1
  Exec Solver = "Before Simulation"
  Equation = "Read DEMs"
  Procedure = "ElmerIceSolvers" "Grid2DInterpolator"
  ! Bedrock DEM
  Variable 1 = String "bedDEM"
  Variable 1 data file = File "../Data/DEM_TR_bed.dat"
  Variable 1 x0 = Real 947700.0d0
  Variable 1 y0 = Real 2104850.0d0
  Variable 1 lx = Real 600.0
  Variable 1 ly = Real 350.0
  Variable 1 Nx = Integer 301
  Variable 1 Ny = Integer 176
  Variable 1 Invert = Logical False
  Variable 1 Fill = Logical False
  Variable 1 Position Tol = Real 1.0e-1
  Variable 1 No Data = Real -9999.0
  Variable 1 No Data Tol = Real 1.0

  ! Surface DEM ...
End
```

name of the DEM file

define the DEM file structure

ly

Ny +1 lines

Nx + 1 colums

(x0,y0)

lx
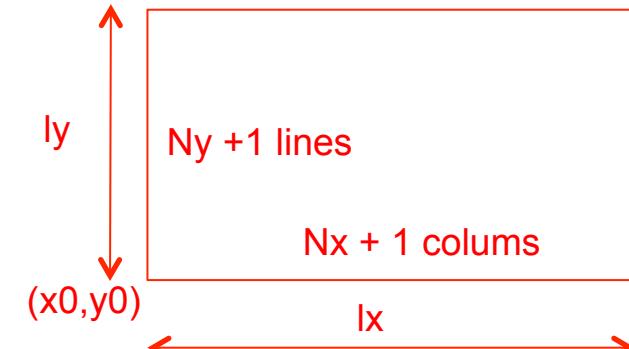
Step 1: In the sif

BedDEM and ZsDEM (variable) must be declared in one solver (Stokes for example)

```
Exported Variable 3 = -dofs 1 "BedDEM"
Exported Variable 4 = -dofs 1 "ZsDEM"
```

Keywords Bottom Surface and Top Surface (needed by the solver StructuredMeshMapper) are assigned the value of these two variables

```
!Bed rock BC
Boundary Condition 2
  Bottom Surface = Equals BedDEM

  ...

End

! Upper Surface BC
Boundary Condition 3
  Top Surface = Equals ZsDEM
End
```

Step 1: use Glen's law

$$D_{ij} = A\tau_e^{n-1}S_{ij} \quad ; \quad S_{ij} = A^{-1/n}I_{D_2}^{(1-n)/n}D_{ij}$$

$$A = A(T') = A_0\exp^{-Q/RT'}$$

$$A = A_1 = 2.89 \times 10^{-13}\ \mathrm{s}^{-1}\mathrm{Pa}^{-3}\ \text{if } T \leq -10°\mathrm{C}$$
$$A = A_2 = 2.43 \times 10^{-2}\ \mathrm{s}^{-1}\mathrm{Pa}^{-3}\ \text{if } T \geq -10°\mathrm{C}$$

$$Q = Q_1 = 60\ \mathrm{kJ\ mol}^{-1}\ \text{if } T \leq -10°\mathrm{C}$$
$$Q = Q_2 = 115\ \mathrm{kJ\ mol}^{-1}\ \text{if } T \geq -10°\mathrm{C}$$

Cuffey and Paterson (2010)

assume a constant temperature of -1°C

Paterson 2010

A* =	3.50000E-25	s^-1 Pa^-3
A1 =	2.89165E-13	s^-1 Pa^-3
A2 =	2.42736E-02	s^-1 Pa^-3
Q1 =	60000	J/mol
Q2 =	115000	J/mol

T [°C]	A [s^-1 Pa^-3]	A [a^-1 MPa^-3]
0	2.4029E-24	75.830
-1	1.9945E-24	62.942
-2	1.6533E-24	52.173
-3	1.3685E-24	43.186
-4	1.1312E-24	35.698
-5	9.3370E-25	29.465
-6	7.6958E-25	24.286
-7	6.3339E-25	19.988
-8	5.2054E-25	16.427
-9	4.2716E-25	13.480
-10	3.5000E-25	11.045
-10	3.5000E-25	11.045
-11	3.1520E-25	9.947
-12	2.8363E-25	8.951
-13	2.5501E-25	8.048
-14	2.2910E-25	7.230
-15	2.0564E-25	6.490
-16	1.8444E-25	5.820
-17	1.6528E-25	5.216
-18	1.4798E-25	4.670
-19	1.3238E-25	4.177
-20	1.1831E-25	3.734
-21	1.0565E-25	3.334
-22	9.4260E-26	2.975
-23	8.4019E-26	2.651
-24	7.4822E-26	2.361
-25	6.6570E-26	2.101
-30	3.6580E-26	1.154
-35	1.9601E-26	0.619
-40	1.0225E-26	0.323
-45	5.1843E-27	0.164
-50	2.5496E-27	0.080

Step 1: use Glen's law

```
$yearinsec = 365.25*24*60*60
$rhoi = 900.0/(1.0e6*yearinsec^2)
$rhow = 1000.0/(1.0e6*yearinsec^2)
$gravity = -9.81*yearinsec^2
! Prefactor from Cuffey&Paterson (2010) in MPa^{-3} a^{-1}
$A1 = 2.89165e-13*yearinsec*1.0e18
$A2 = 2.42736e-2*yearinsec*1.0e18
$Q1 = 60.0e3
$Q2 = 115.0e3


Material 1
  Density = Real $rhoi
  Viscosity Model = String "glen"
  Viscosity = 1.0 ! Dummy but avoid warning output
  Glen Exponent = Real 3.0
  Limit Temperature = Real -10.0
  Rate Factor 1 = Real $A1
  Rate Factor 2 = Real $A2
  Activation Energy 1 = Real $Q1
  Activation Energy 2 = Real $Q2
  Glen Enhancement Factor = Real 1.0
  Critical Shear Rate = Real 1.0e-10


  Constant Temperature = Real -1.0
End
```

$s^{-1}\,Pa^{-3}$ to $a^{-1}\,MPa^{-3}$

Step 1: Hypothesis of the modelling

Solve only the Stokes equation in a diagnostic way

3 boundary conditions

Upper Surface BC:

stress free surface
(natural BC)

Bedrock BC:

No sliding condition

Lateral BC:

zero horizontal velocities
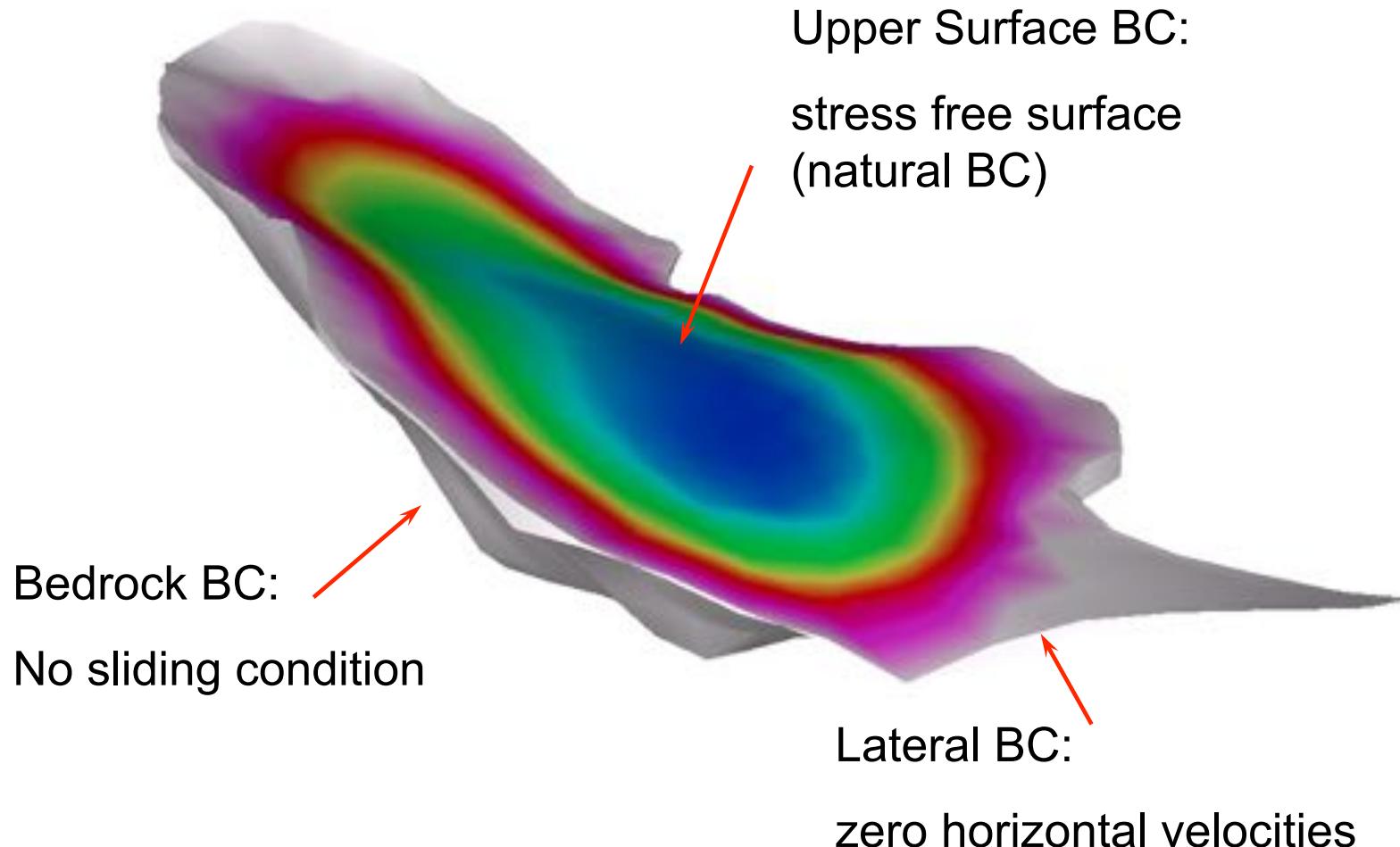
Step 1: Boundary Conditions

```
! lateral side of the glacier
Boundary Condition 1
  Target Boundaries = 1
  Velocity 1 = real 0.0             Null horizontal velocities
  Velocity 2 = real 0.0
End


! Bedrock
Boundary Condition 2
  Bottom Surface = Equals BedDEM
  Velocity 1 = Real 0.0
  Velocity 2 = Real 0.0             No sliding
  Velocity 3 = Real 0.0
End


! Upper Surface
Boundary Condition 3               Natural BC,
  Top Surface = Equals ZsDEM       nothing to do!
End
```

Step 1: Stress Solver

Objective: compute the stress field as

$$\int_V S_{ij}\Phi\,\mathrm{d}V = 2\int_V \eta D_{ij}\Phi\,\mathrm{d}V$$

where D_{ij} and η are calculated from the nodal velocities using the derivative of the basis functions

```
Solver 4
  Equation = Sij
  Procedure = "ElmerIceSolvers" "ComputeDevStress"
  Variable = -nooutput "Sij"
  Variable DOFs = 1
  Exported Variable 1 = Stress
  Exported Variable 1 DOFs = 6

  Flow Solver Name = String "Flow Solution"

  Linear System Solver = Direct
  Linear System Direct Method = umfpack
End
```

Step 1: Stress Solver

- Tell you want the Cauchy stress to be computed (Material Section)
      (else you will get the deviatoric stress)

```
Material 1
    Cauchy Stress = Logical True
End
```

- Output :   negative stress = Compressive stress
                  positive stress = Tensile stress

Stress.1	→	S_{xx}	Stress.4	→	S_{xy}
Stress.2	→	S_{yy}	Stress.5	→	S_{yz}
Stress.3	→	S_{zz}	Stress.6	→	S_{xz}

Step 1: Eigenvalues Solver

Objective: compute the eigenvalues of the Cauchy stress tensor

```
Solver 5
   Equation = "EigenStresses"
   Procedure = "ElmerIceSolvers" "ComputeEigenValues"
   Variable = -nooutput dumy
   Variable DOFs = 1

   ! The 3 eigenvalues
   Exported Variable 1 = EigenStress
   Exported Variable 1 DOFS = 3

   ! The 3 eigenvectors (Option)
   Exported Variable 2 = EigenVector1
   Exported Variable 2 DOFS = 3
   Exported Variable 3 = EigenVector2
   Exported Variable 3 DOFS =  3
   Exported Variable 4 = EigenVector3
   Exported Variable 4 DOFS = 3
End
```

Step 1: Eigenvalues Solver

Output :   negative stress = Compressive stress
                         positive stress = Tensile stress
                         ordered → `Eigenstress.3` gives the maximal tensile stress

$$\texttt{Eigenstress.1} \quad \rightarrow \quad S_1$$
$$\texttt{Eigenstress.2} \quad \rightarrow \quad S_2$$
$$\texttt{Eigenstress.3} \quad \rightarrow \quad S_3$$

Step 1: Add sliding on the bedrock

Friction law in Elmer:

$$C_i u_i = \sigma_{ij} n_j \ (i = 1, 2)$$

➡ $C_t u_t = \sigma_{nt} \ ; \ C_n u_n = \sigma_{nn}$

where n is the surface normal vector

$$n = e_{n/t_1}$$

$$t = e_{n/t_2}$$
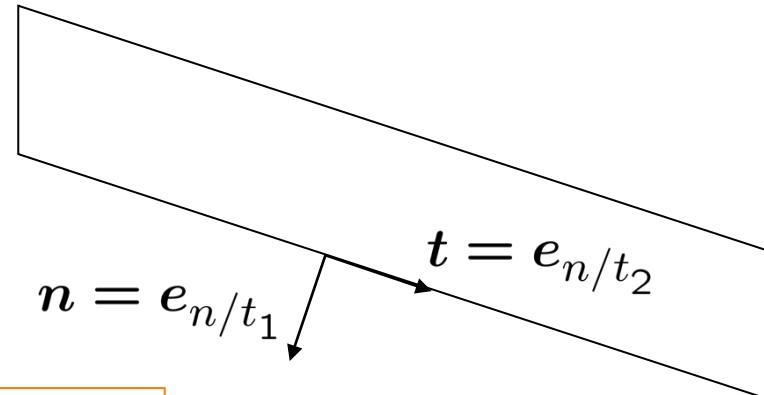
```
! Bedrock BC
Boundary Condition 2
  . . .

  Flow Force BC = Logical True
  Normal-Tangential Velocity = Logical True

  Velocity 1 = Real 0.0e0
  Slip Coefficient 2 = Real 0.1
  Slip Coefficient 3 = Real 0.1
End
```

How to evaluate the Slip Coefficient ?

Step 1: Other BCs for the lateral boundary

```
! lateral side of the glacier
Boundary Condition 1
  Target Boundaries = 1
End
```

Natural BC

```
! lateral side of the glacier
Boundary Condition 1
  Target Boundaries = 1
  Velocity 1 = real 0.0
  Velocity 2 = real 0.0
  Velocity 3 = real 0.0
End
```

zero velocity

Conclusion ?

Modelling Tête Rousse Glacier

✓ **Step 1** - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ **Step 2**
- 2a Influence of an empty cavity below Tête Rousse Glacier (diagnostic)
- 2b Apply a water pressure in the cavity

✓ **Step 3**
- 3a Rate of closure of the cavity for a given drainage scenario (prognostic)
- 3b Add a drainage scenario

Step 2a: Add the cavity (empty of water)

The initial problem

Ice

The bottom surface for the mesh

Only one BC for the base: no sliding

The new problem

Ice

The bottom surface for the mesh
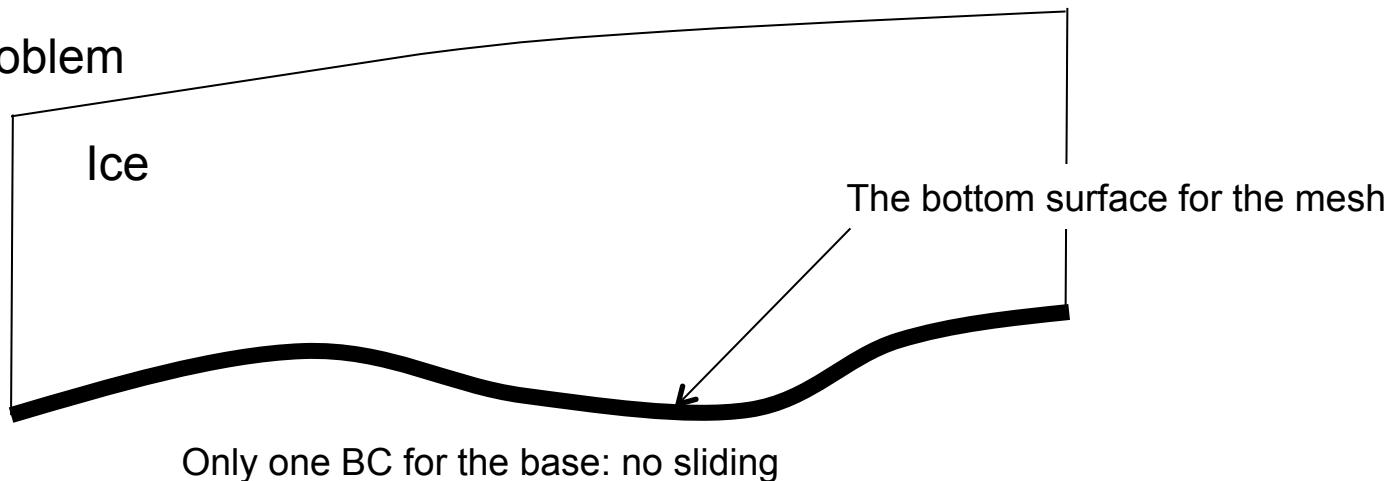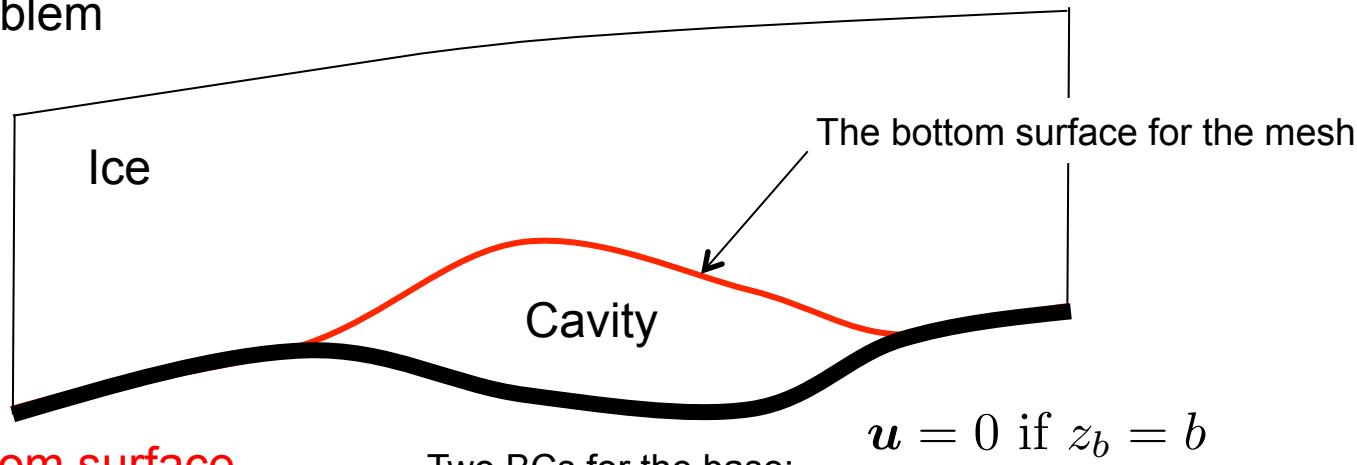
Cavity

!!! The ice bottom surface is not anymore given by the bedrock DEM !!!

Two BCs for the base:

$$\boldsymbol{u} = 0 \text{ if } z_b = b$$
$$\sigma_{nn} = p_w \text{ if } z_b > b$$

$p_w = 0$ if the cavity is empty of water

Step 2a: new Bottom Surface definition

The bottom surface is now given by the DEM_TR_cavity.dat DEM file.

Change the StructuredMeshMapper to read this DEM and store it in the ZbDEM variable

Declare this new variable (in Stokes solver section):
```
Exported Variable 5 = -dofs 1 "ZbDEM"
```

Change the boundary condition 2:
```
Boundary Condition 2
  Bottom Surface = Equals ZbDEM
End
```

Step 2a: Make a new mesh

We will use the cavity contour to have smaller size elements in the vicinity of the cavity
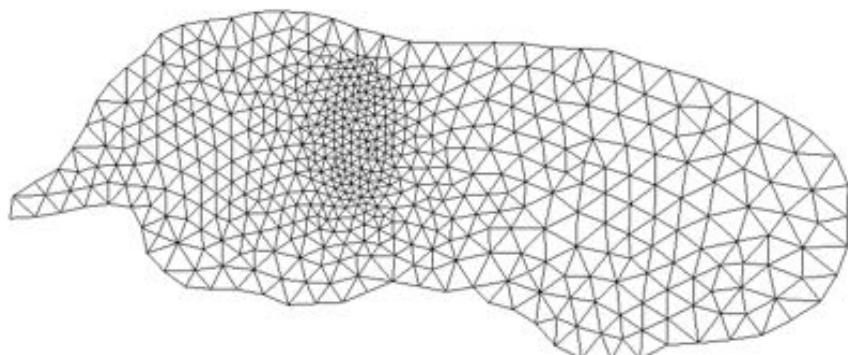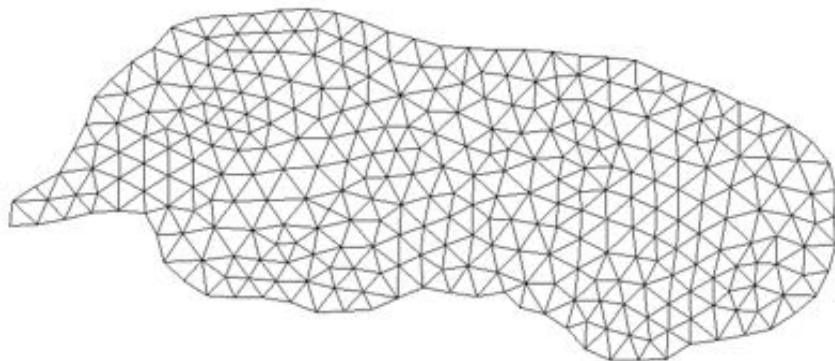
Work to do : modify the `Makegeo.m` file to create this new mesh.

Step 2a: Make a new mesh (Makegeo_2.m) 1/2

```
clear;
lc_out=18.0;
lc_in=6.0;
A=dlmread('Contour_TR_glacier.dat');
B=dlmread('Contour_TR_cavity.dat');
fid1=fopen('teterousse.geo','w');
As=size(A,1);
Bs=size(B,1);
np=0;
for ii=1:As
    np=np+1;
    fprintf(fid1,'Point(%g)={%14.7e,%14.7e,0.0,%g}; \n',np,A(ii,1),A(ii,2),lc_out);
end
for ii=1:Bs
    np=np+1;
    fprintf(fid1,'Point(%g)={%14.7e,%14.7e,0.0,%g}; \n',np,B(ii,1),B(ii,2),lc_in);
end

fprintf(fid1,'Spline(1)={');
for ii=1:As
  fprintf(fid1,'%g,',ii);
end
fprintf(fid1,'%g}; \n',1);

fprintf(fid1,'Spline(2)={');
for ii=1:Bs
  fprintf(fid1,'%g,',As+ii);
end
fprintf(fid1,'%g}; \n',As+1);
```

Step 2a: Make a new mesh `(Makegeo_2.m) 2/2`

```
fprintf(fid1,'Line Loop(3)={1}; \n');
fprintf(fid1,'Line Loop(4) = {2}; \n');
fprintf(fid1,'Plane Surface(5) = {3, 4}; \n');
fprintf(fid1,'Plane Surface(6) = {4}; \n');
fprintf(fid1,'Physical Line(7) = {1}; \n');
fprintf(fid1,'Physical Surface(8) = {5,6}; \n');
fclose(fid1)


% create teterousse.msh using gmsh
system "gmsh teterousse.geo -1 -2"

% convert teterousse.gmsh in an Elmer type mesh
System "ElmerGrid 14 2 teterousse.msh –autoclean"
```

Step 2a: Change in the basal BC

The basal BC will be of the form:

```
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable ZbDEM, BedDEM
   Real MATC "-(tx(0) > tx(1))"
```

Velocity 1 Condition = -1 if zbDEM > BedDEM   => Don't apply "Velocity 1 = 0"
                     = +1 if zbDEM <= BedDEM   => Apply "Velocity 1 = 0"

And the same for `Velocity 2` **and** `Velocity 3`.

Visualize the results in ElmerPost.
What does it change in term of velocity and stress?

Step 2a: Change in the basal BC

2nd Solution – use a f90 user function instead of MATC language

The basal BC will be of the form:
```
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable ZbDEM, BedDEM
  Real Procedure "../PROG/USF_TR" "MaskCavity"
```

Compile the user function USF_TR.f90 in PROG/

```
elmerf90 USF_TR.f90 -o USF_TR
```

Best solution for large problem:
          a f90 user function is much faster than MATC coding in the sif!

Step 2a: MaskCavity user function

```fortran
FUNCTION MaskCavity ( Model, nodenumber, Input) RESULT(Mask)
    USE types
    USE CoordinateSystems
    USE SolverUtils
    USE ElementDescription
    USE DefUtils
    IMPLICIT NONE
    TYPE(Model_t) :: Model
    TYPE(Solver_t), TARGET :: Solver
    INTEGER :: nodenumber
    REAL(KIND=dp) :: Input(2), Mask
    REAL(KIND=dp) :: znode, zbed

    znode = Input(1)
    zbed = Input(2)

    IF (znode > zbed+0.1) THEN
       Mask = -1.0
    ELSE
       Mask = 1.0
    END IF
END FUNCTION MaskCavity
```

Modelling Tête Rousse Glacier

✓ **Step 1** - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ **Step 2**

- 2a Influence of an empty cavity below Tête Rousse Glacier (diagnostic)
- **2b Apply a water pressure in the cavity**

✓ **Step 3**

- 3a Rate of closure of the cavity for a given drainage scenario (prognostic)
- 3b Add a drainage scenario

Step 2b: Add a water pressure

Modify the SIF to add a water pressure

water load h_w

$$p_w = \rho_w g(h_w - z)$$
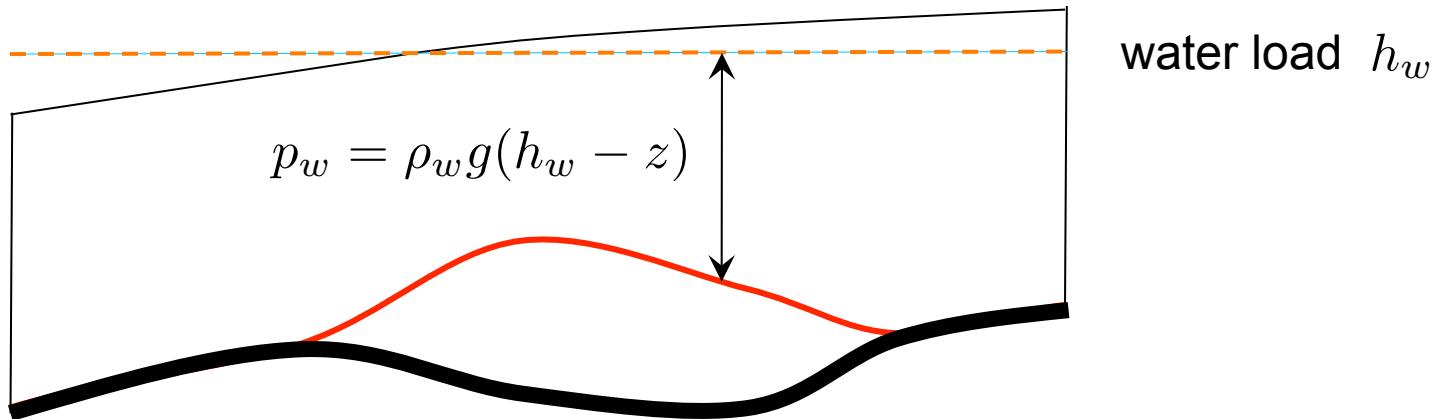
`$hw = 3176.0`   the water load

In the bedrock BC
```
Flow Force BC = Logical True
External Pressure = Variable Coordinate 3
   Real MATC "rhow*gravity*(hw-tx)
```

will only apply where a Dirichlet BC is not applied i.e. in the cavity

Modelling Tête Rousse Glacier

✓ **Step 1** - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ **Step 2**

- 2a Influence of an empty cavity below Tête Rousse Glacier (diagnostic)
- 2b Apply a water pressure in the cavity

✓ **Step 3**

- 3a Rate of closure of the cavity for a given drainage scenario (prognostic)
- 3b Add a drainage scenario

Step 3a: Move to prognostic

Will do it in two steps
- Move to prognostic assuming the cavity is empty of water at t=0
  (big step, need 2 new solvers!)

- Prescribe the observed drainage scenario for the water pressure

To move from a diagnostic to a prognostic simulations:
- Add the FreeSurface solver (here 2 times, since we have 2 FS)
- Add one body per FS (new Initial Condition and Equation Sections)
- Modifications in the Simulation and Boundary Condition Sections

Only shown for the upper free surface here

Step 3a – Steady to transient

The simulation Section has to be modified:

```
Simulation Type = Transient

Timestepping Method = "bdf"          ───────→  Backward Differences Formulae
BDF Order = 1
Output Intervals = 1
Timestep Intervals = 50
Timestep Sizes = $10.0/365.25
```

To control the "implicity" of the solution over one time step (here 1 means explicit)

```
Steady State Min Iterations = 1      ───→
Steady State Max Iterations = 1
```

```
Restart File = "../../Step2a/teterousse/teterousse_Step2a_.result"
Restart Position = 0
Restart Time = Real 0.0
Restart Before Initial Conditions = Logical True
```
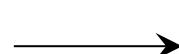
───────→ We need a restart to have the ZsDEM and ZbDEM variables for the initial condition of Zs and Zb

Step 3a – Sketch of a transient simulation

Geometry + Mesh $\longrightarrow$ Degrees of freedom

Linear System: $A.x = b$ - Direct
  - Iterative ϵ_L

ϵ_{NL}

Non Linear iterations: $A = A(x)$

Solver k

ϵ_C

Coupled iteration over dt (implicit scheme)

$t = t + \mathrm{d}t$

$\epsilon_L < \epsilon_{NL} < \epsilon_C$
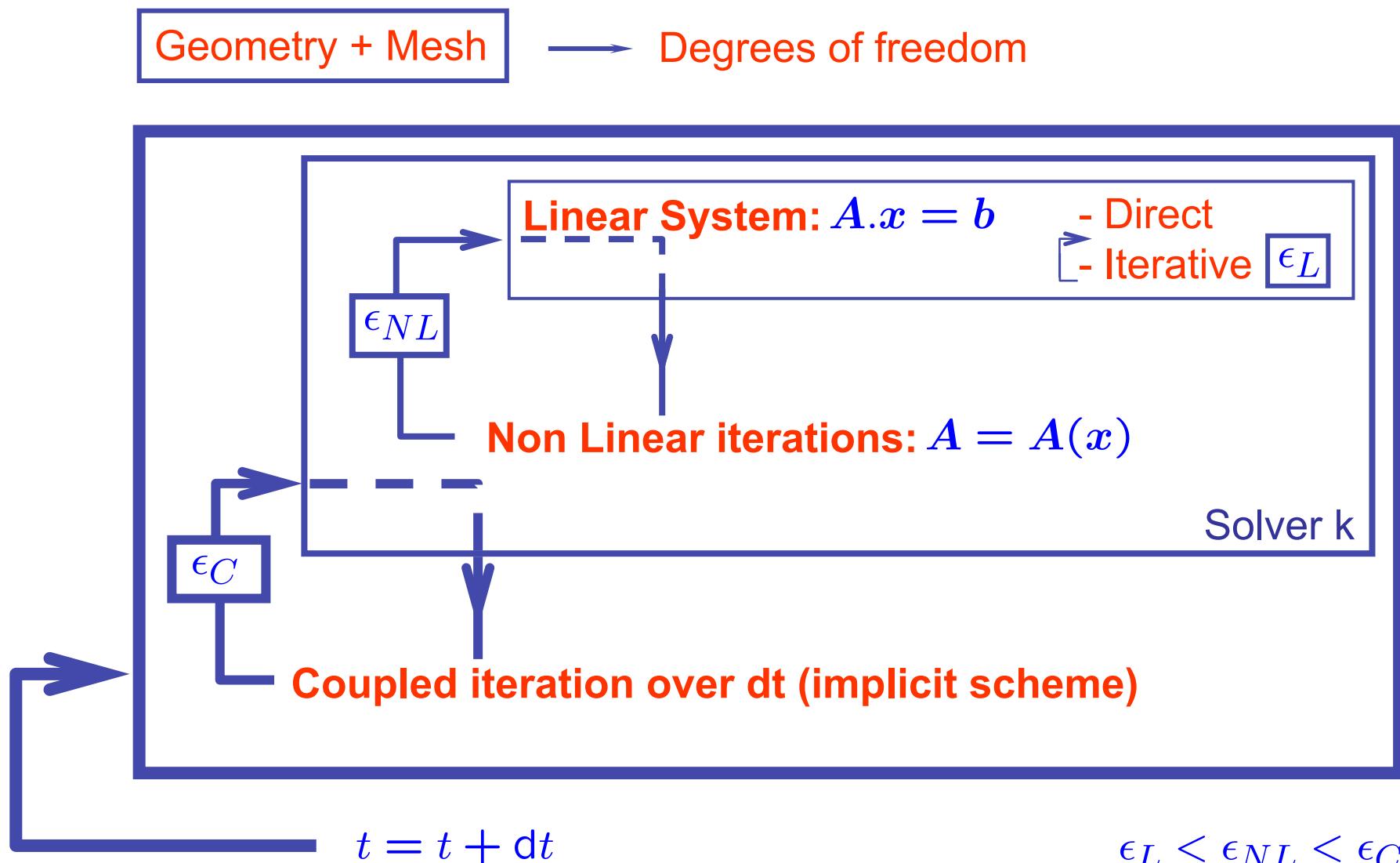
Step 3a – Free surface Solver

The free surface solver only apply to the boundary 3 (upper surface)
➡ Define a 2nd body which is the boundary 3.

```
Body 2
  Equation = 2
  Body Force = 2
  Material = 1
  Initial Condition = 2
End
```

where `Equation 2, Body Force 2 and Initial Condition 2` **are** defined for the free surface equation of the upper surface.

Tell in BC2 that this is the body 2:
```
Boundary Condition 3
  Body Id = 2
        …
End
```

Step 3a – Add the Free surface Solver

```
Solver 4
  Equation = "Free Surface Top"
  Variable = String "Zs"
  Variable DOFs =  1
  Exported Variable 1 = String "Zs Residual"
  Exported Variable 1 DOFs = 1

  Procedure = "FreeSurfaceSolver" "FreeSurfaceSolver"
  Before Linsolve = "EliminateDirichlet" "EliminateDirichlet"

  Linear System Solver = Iterative
  Linear System Max Iterations = 1500
  Linear System Iterative Method = BiCGStab
  Linear System Preconditioning = ILU0
  Linear System Convergence Tolerance = Real 1.0e-9
  Linear System Abort Not Converged = False
  Linear System Residual Output = 1

  Nonlinear System Max Iterations = 100
  Nonlinear System Convergence Tolerance  = 1.0e-6
  Nonlinear System Relaxation Factor = 1.00

  Steady State Convergence Tolerance = 1.0e-03

  Stabilization Method = Bubbles
  Apply Dirichlet = Logical False

! How much the free surface is relaxed
  Relaxation Factor = Real 1.00
End
```

Step 3a – Upper Surface

Body Force 2:

```
Body Force 2
  Zs Accumulation Flux 1 = Real 0.0e0
  Zs Accumulation Flux 2 = Real 0.0e0
  Zs Accumulation Flux 3 = Real 0.0e0
End
```

Equation 2:

```
Equation 2
  Active Solvers(1) = 2
  Flow Solution Name = String "Flow Solution"
  Convection = String Computed
End
```

Initial Condition 2: (tell that $z_s(x, 0)$ is given by the surface DEM)

```
Initial Condition 2
  Zs = Equals ZsDEM
End
```

Step 3a - StructuredMeshMapper

We say in StructuredMeshMapper that the top (and bottom) surface is defined by the variable zs:

```
Solver 1
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"

  Active Coordinate = Integer 3
  Mesh Velocity Variable = String "dSdt"
  Mesh Update Variable = String "dS"
  Mesh Velocity First Zero = Logical True


  Top Surface Variable Name = String "Zs"
  Bottom Surface Variable Name = String "Zb"


End
```

And delete from the BC the initial definition of the top (and bottom) surface:

```
Boundary Condition 3
!!! this BC is equal to body no. 2 !!!
  Body Id = 2

  Top Surface = Equals ZsDEM
End
```

Step 3a – Same for the bedrock

Name of the variable: `Zb`
Add solver : `Solver 5`
Add equation: `Equation 3`

Modify the the Bottom surface BC (3):

```
Boundary Condition 2
    Body Id = 3
    Bottom Surface = Equals ZbDEM
End
```

Add a limiter to ensure that $z_b \geq b$
In the material section

```
Min Zb = Equals BedDEM
Max Zb = Real +1.0e10
```

+ in the Free Surface solver : `Apply Dirichlet = Logical `True

Step 3a – Newton linearization

If you want to use Newton linearization for the non-linear iterations, don't forget to reset the conditions used to move from Picard to Newton at each time step, by adding:

```
Solver 3
  Equation = "Navier-Stokes"
  Nonlinear System Reset Newton = Logical True

End
```

Step 3a – Two subtleties...

1/ Need of the restart...

        Initial conditions are set before the first solver is executed

        Impossible then to initialize with an other variable

        This is then done by using a restart and specifying:

```
Restart Before Initial Conditions = Logical True
```

2/ Problem when a solver is called two time in the same sif...

        Need to make a copy of the object file to avoid mixing of the saved variables in the solver from two different calls:

```
cp $ELMER_HOME/share/elmersolver/lib/FreeSurfaceSolver.so
MyFreeSurfaceSolver
```

        Use a different call in the sif file for Zb:

```
Procedure = "./MyFreeSurfaceSolver" "FreeSurfaceSolver"
```

Modelling Tête Rousse Glacier

✓ **Step 1** - Tête Rousse Glacier flow without a water filled-cavity (diagnostic)

✓ **Step 2**
- 2a Influence of an empty cavity below Tête Rousse Glacier (diagnostic)
- 2b Apply a water pressure in the cavity

✓ **Step 3**
- 3a Rate of closure of the cavity for a given drainage scenario (prognostic)
- **3b Add a drainage scenario**

Step 3b – Add a drainage scenario

Add an evolution of the water load of the linear form:

$$h_w = 3170.0 - t * \Delta h_w / \Delta t$$

Work to do:
- write a MATC function hw to prescribe the water load evolution

- write a User Function to do the same (see USF_TR.f90)

Step 3b – Add a drainage scenario

MATC function `hw` to prescribe the water load evolution:

```
! Water load function of time (in year)
! Decrease by DH in DT and h > 3100.0
$ function hw(t) {\
   DH = 70.0;\
   DT = 20.0;\
   h = 3170.0 - t*365.25*DH/DT ;\
   if (h > 3100.0) {\
     _hw = h ;\
   } else {\
     _hw = 3100.0 ;\
   }\
}
```

Call in the bedrock BC

```
External Pressure = Variable time, Coordinate 3
   Real MATC "rhow*gravity*(hw(tx(0))-tx(1))
```

More Steps ???

Some ideas:

- go parallel

- add the Savedata solver to get upper and/or lower surfaces ASCII output

- add the StructuredProjectToPlane solver

```
Solver 2
  Equation = "HeightDepth"
  Procedure = "StructuredProjectToPlane" "StructuredProjectToPlane"
  Active Coordinate = Integer 3

  Operator 1 = depth
  Operator 2 = height
End
```

- write a f90 user function to give the water pressure as a function of time (Step3b)
                        For the 2 last points, see Step3c

0 days

Vz (m/a)
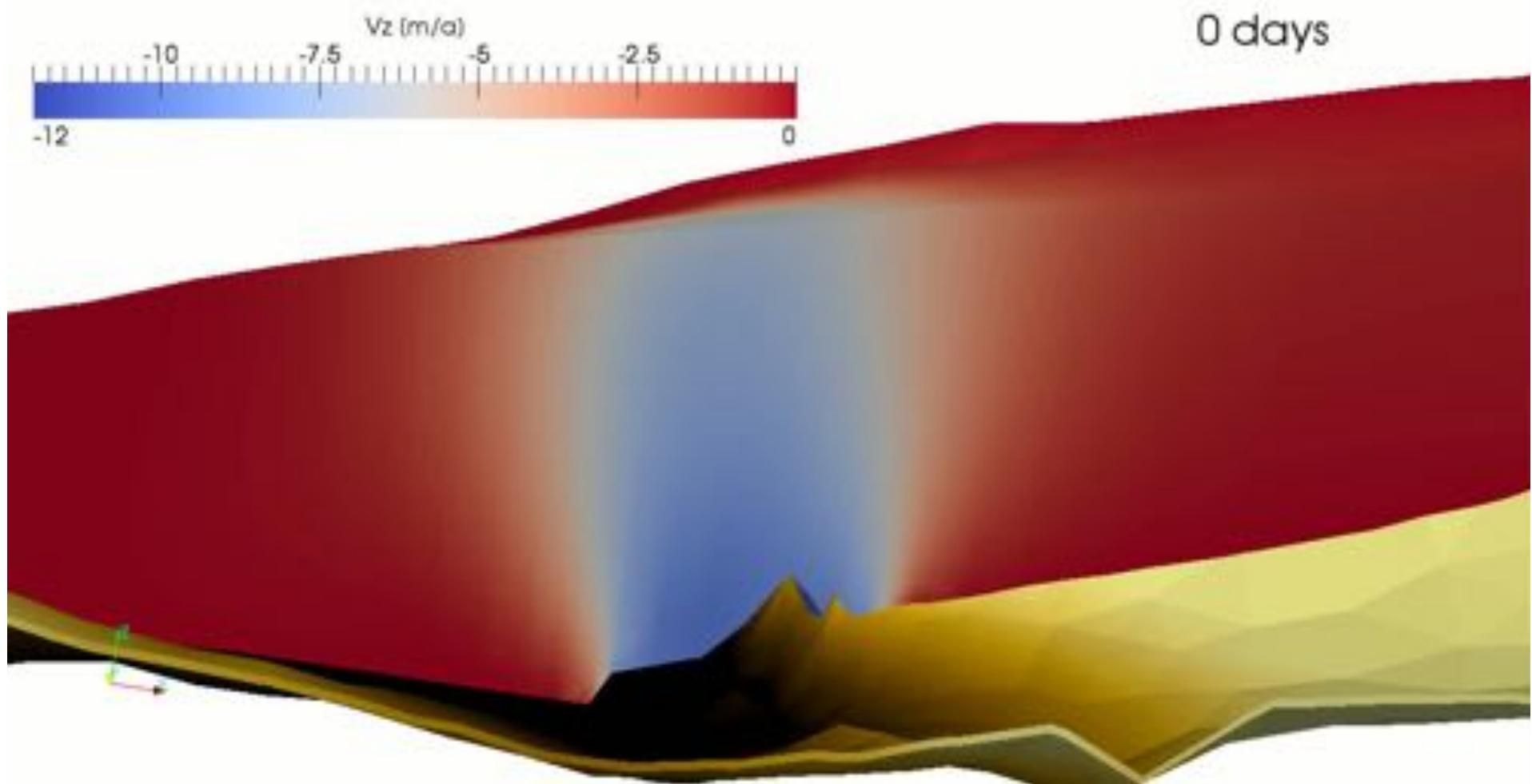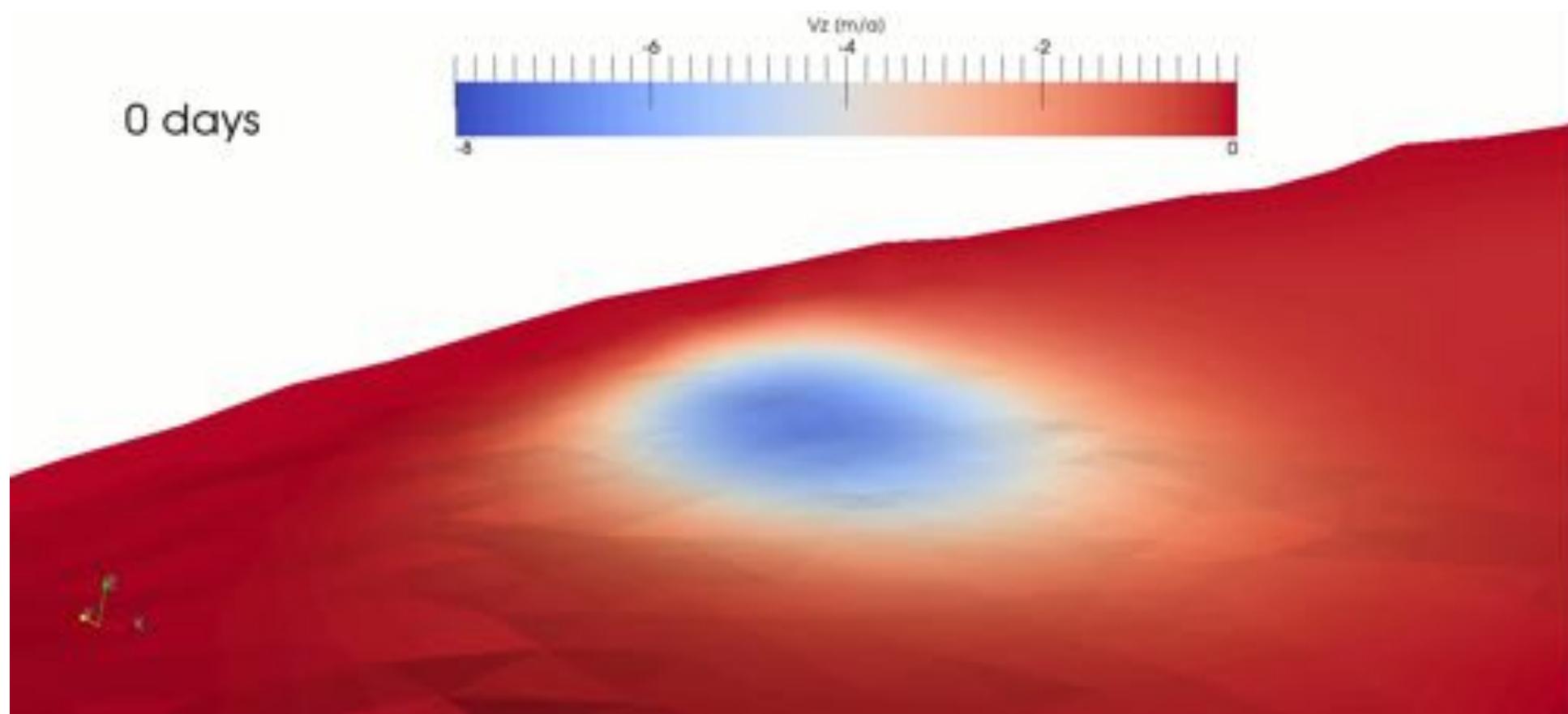
References

Gagliardini O., F. Gillet-Chaulet, G. Durand, C. Vincent and P. Duval, 2011. Estimating the risk of glacier cavity collapse during artificial drainage: the case of Tête Rousse Glacier. Geophys. Res. Lett., 38, L10505, doi:10.1029/2011GL047536.