First Elmer/Ice course

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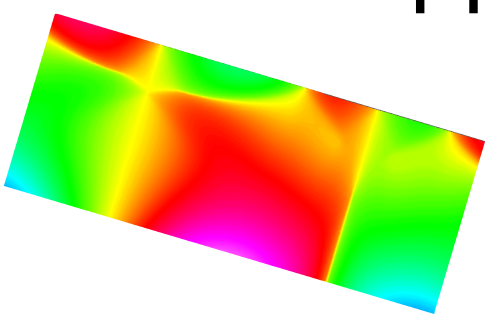
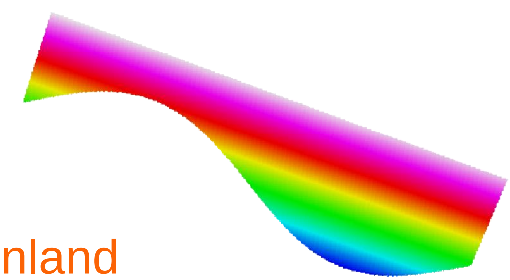
Application to ISMIP HOM[3] tests B and D



Step by step !



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Outline

Step 0 Start from a very **simple test case**. What are we solving? (Glen's law, …)

Step 1 Introducing **periodic boundaries** and **unit systems**:
Move to test ISMIP-HOM B020 (mesh, periodic BC)

Step 2 **Visualization and Data**:
-Add SaveData (SaveLine) solver to get ASCII output on the BC
-Add SaveData (SaveScalars) solver to get ASCII for CPU-time and volume
-Add ComputeDevStress solver to get the stress field

Step 3 Sliding law from **user function** or **inline MATC-function**:
-Move to test ISMIP-HOM D020
-Add ResultOutput solver to results additionally in VTU format
-Short Demo of *ParaView*

Step 4 Restart from Step 2: Move to prognostic ISMIP B020.
- Move from a steady to a transient simulation
- Free surface solver

Step 5 Move to **Prognostic** ISMIP D020.

Step 0

Create a `My_ISMIP_Appli` directory

Copy the directory *Step0 in `My_ISMIP_Appli`*

- Make the mesh : > **`ElmerGrid 1 2 square.grd`**

- Run the test : > **`ElmerSolver ismip_step0.sif`**

- Watch the results : > **`ElmerPost`** and open `square\ismip_step0.ep`

- What are we solving?

Stokes:

$$\operatorname{div} \boldsymbol{\sigma} + \rho \boldsymbol{g} = 0$$

$$u_{i,i} = 0$$

Navier-Stokes with convection and acceleration terms neglected :

```
Flow Model = String Stokes
```

in the Stokes solver section

Step 0 – Glen's law and Elmer

In glaciology, you can find (at least) two definitions for Glen's law:

$$D_{ij} = \frac{B}{2}\tau_e^{n-1} S_{ij} \quad ; \quad S_{ij} = 2B^{-1/n}\dot{\gamma}^{(1-n)/n} D_{ij}$$

$$D_{ij} = A\tau_e^{n-1} S_{ij} \quad ; \quad S_{ij} = A^{-1/n} I_{D_2}^{(1-n)/n} D_{ij}$$ ISMIP notation

where $I_{D_2}^2 = D_{ij}D_{ij}/2$ and $\dot{\gamma}^2 = 2D_{ij}D_{ij}$

The power-law implemented in Elmer writes: $S_{ij} = 2\eta_0 \dot{\gamma}^{m-1} D_{ij}$

$$\eta_0 = B^{-1/n} = (2A)^{-1/n}$$

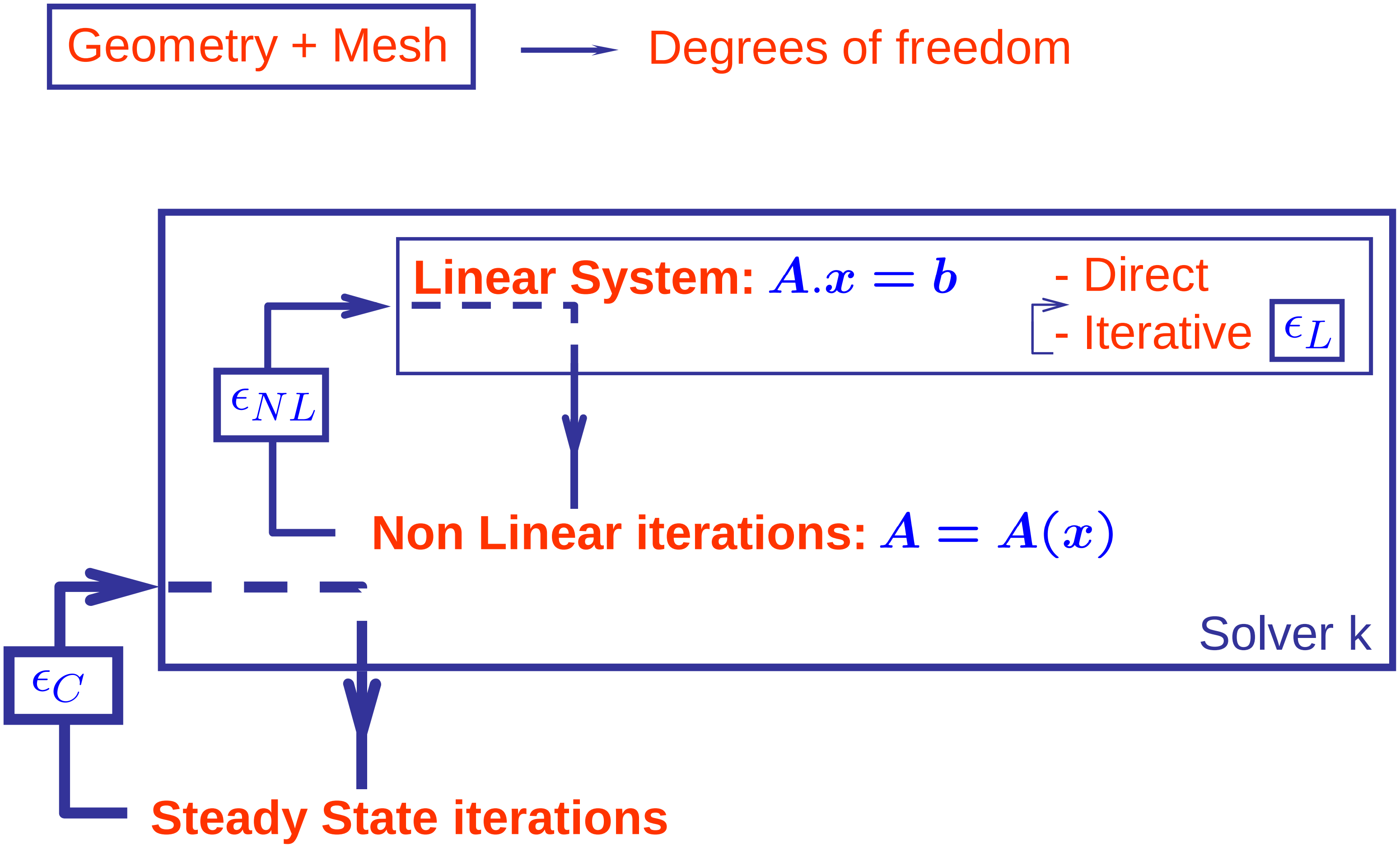
$$m = 1/n$$

$$\dot{\gamma}^2 \geq \dot{\gamma}_c^2$$

In Material Section:

```
Viscosity Model = String "power law"
Viscosity = Real η0
Viscosity Exponent = Real m
Critical Shear Rate = Real γ̇c
```

Step 0 – Sketch of a Steady simulation



$$\epsilon_L < \epsilon_{NL} < \epsilon_C$$

Step 0 – Numerical methods

In the NS Solver Section: (see Chapters 3 and 4 of Elmer Solver Manual)

- Solution for the Linear System:

```
Linear System Solver = Direct
Linear System Direct Method = umfpack
```

- Non-Linear System :

Picard

Newton

```
Nonlinear System Max Iterations = 100
Nonlinear System Convergence Tolerance  = 1.0e-5
Nonlinear System Newton After Iterations = 5
Nonlinear System Newton After Tolerance = 1.0e-02
Nonlinear System Relaxation Factor = 1.00
```

$= \epsilon_{NL}$

- Coupled problem (not needed here in fact…):

```
Steady State Convergence Tolerance = Real 1.0e-3
```

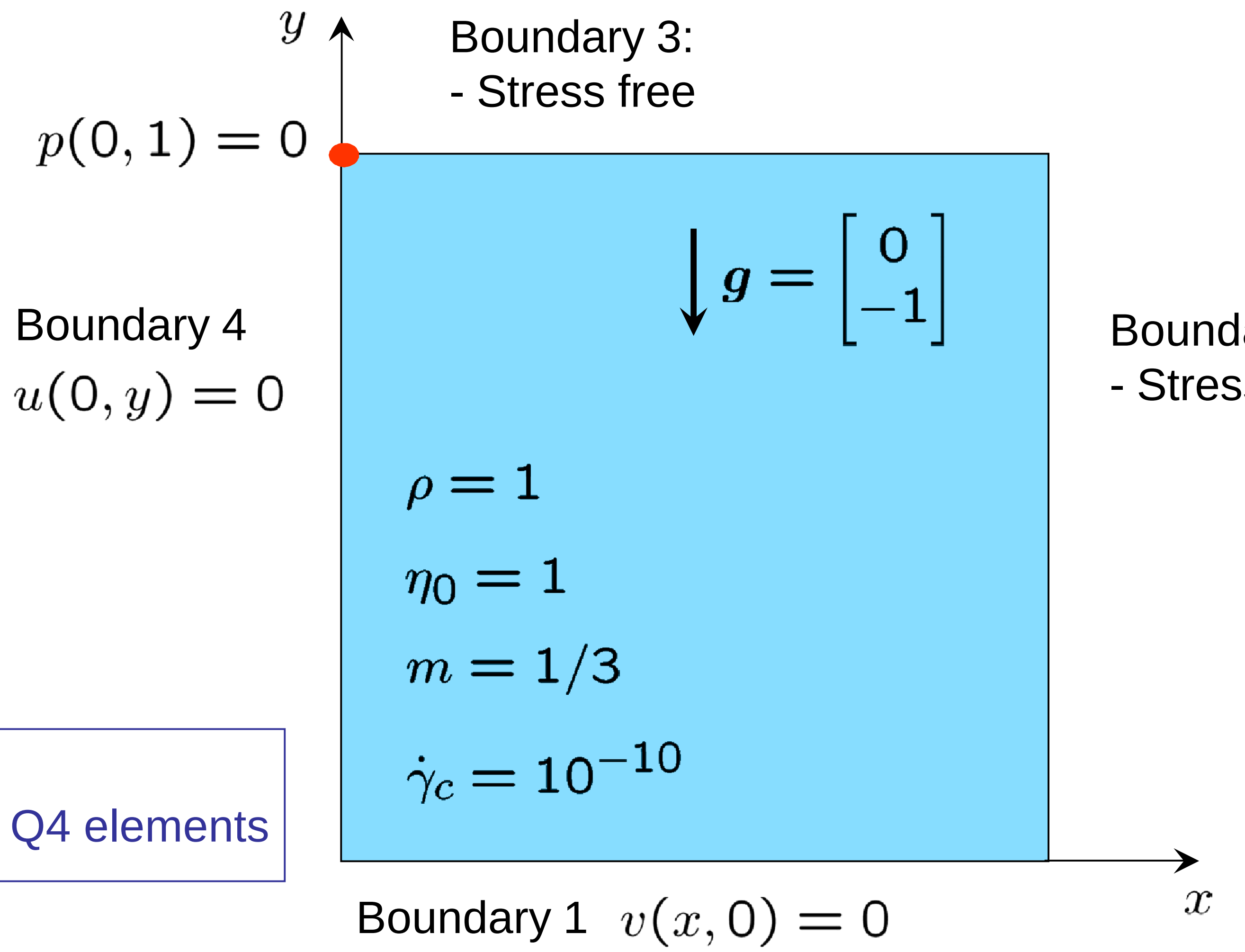
$= \epsilon_C$

- Stabilization of the Stokes equations:

```
Stabilization Method = String Bubbles
```

(other options: `Stabilized, P2P1`)

Step 0 – What are we solving?



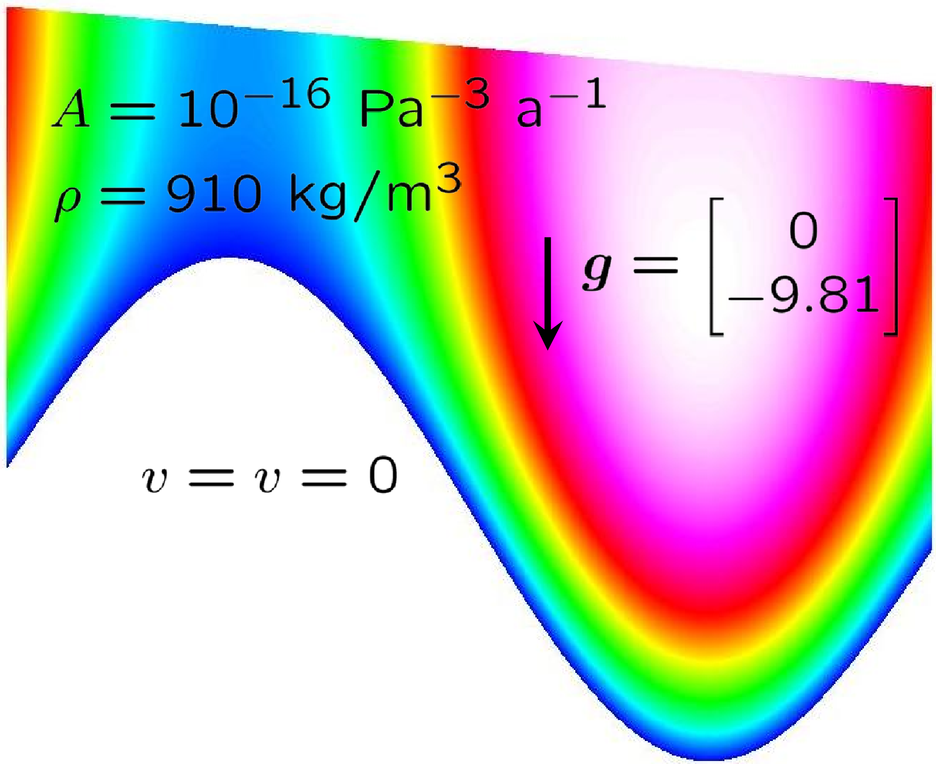
Mesh:
20 x 20 Q4 elements

Step 1 – Move to ISMIP-HOM B020

What we have to solve :

Stress free $z_s(x) = -x\,\tan(0.5^\circ)$

Periodic BC



$A = 10^{-16}\ \mathrm{Pa}^{-3}\ \mathrm{a}^{-1}$

$\rho = 910\ \mathrm{kg/m}^3$

$\boldsymbol{g} = \begin{bmatrix} 0 \\ -9.81 \end{bmatrix}$

Periodic BC

$v = v = 0$

$L = 20 \times 10^3\ \mathrm{m}$

$$z_b(x) = z_s(x) - 1000 + 500\,\sin\left(\frac{2\pi}{L}x\right)$$

Step 1 – Changes from Step0

- New directory, new names !
  e.g.: `square.grd -> rectangle.grd`

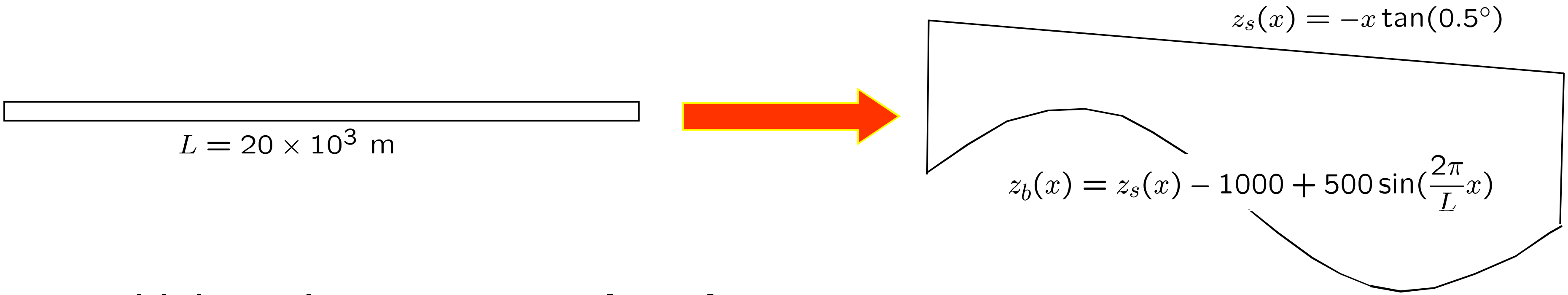
- Make the mesh : use of the `StructuredMeshMapper` solver

- Right values for the different constants (which system of Units ?)

- Add the periodic boundary conditions

Step 1 – StructuredMeshMapper

- Start from rectangular mesh L x 1m and use the solver StructuredMeshMapper to produce the ISMIP B geometry



- Add the solver **StructuredMeshMapper**

```
Solver 1
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"
  Active Coordinate = Integer 2
  Mesh Velocity Variable = String ”dSdt”
  Mesh Update Variable = String ”dS”
  Mesh Velocity First Zero = Logical True
End
```

(Not really needed in steady)

Step 1 – StructuredMeshMapper

Declare mapping for **Bottom Surface** and **Top Surface** in Boundary Condition 1 and 2, respectively:

```
$Slope = 0.5 * pi / 180.0
$L = 20000.0

! Bedrock
Boundary Condition 1
  Target Boundaries = 1
  Velocity 1 = Real 0.0e0
  Velocity 2 = Real 0.0e0
  Bottom Surface = Variable Coordinate 1
    Real MATC "-tx*tan(Slope)-1000.0+500.0*sin(2.0*pi*tx/L)"
End

! Upper Surface
Boundary Condition 3
  Target Boundaries = 3
  Top Surface = Variable Coordinate 1
    Real MATC "-tx*tan(Slope)"
End
```

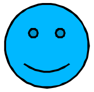
Step 1 – Elmer and Units

Elmer does not force any choice of units – they just have to be coherent.
Minimal influence on results, as the system matrix is normalized.

For the Stokes problem, one should give values for:

- the density: ρ $(= 910 \text{ kg/m}^3)$
- the gravity: g $(= 9.81 \text{ m s}^{-2})$
- the viscosity: η_0 $(\text{Pa s}^{1/n})$ $(1 \text{ Pa} = 1 \text{ kg s}^{-2} \text{ m}^{-1})$

kg – m – s [SI] : velocity in m/s and time-step in seconds 

kg – m – a : velocity in m/a and time-step in years  $1 \text{ a} = 31\,557\,600 \text{ s}$

MPa – m – a : velocity in m/a and Stress in MPa 

(What I will use in the following)

Step 1 – Value of the ISMIP constants

For ISMIP tests A-D, the value for the constants are

- the density: $\rho = 910\ \mathrm{kg/m^3}$
- the gravity: $g = 9.81\ \mathrm{m\ s^{-2}}$
- the fluidity: $A = 10^{-16}\ \mathrm{Pa^{-3}\ a^{-1}}$

	USI kg - m - s		kg - m - a		MPa - m - a	
g =	9.81	m / s^2	9.7692E+15	m / a^2	9.7692E+15	m / a^2
ρ =	910	kg / m^3	910	kg / m^3	9.1380E-19	MPa m^{-2} a^2
A =	3.1689E-24	kg^{-3} m^3 s^5	1.0126E-61	kg^{-3} m^3 a^5	100	MPa^{-3} a^{-1}
η =	5.4037E+07	kg m^{-1} s$^{-5/3}$	1.7029E+20	kg m^{-1} a$^{-5/3}$	0.1710	MPa a$^{1/3}$

$$\eta_0 = B^{-1/n} = (2A)^{-1/n}$$

$$m = 1/n$$

$$\dot\gamma^2 \geq \dot\gamma_c^2$$

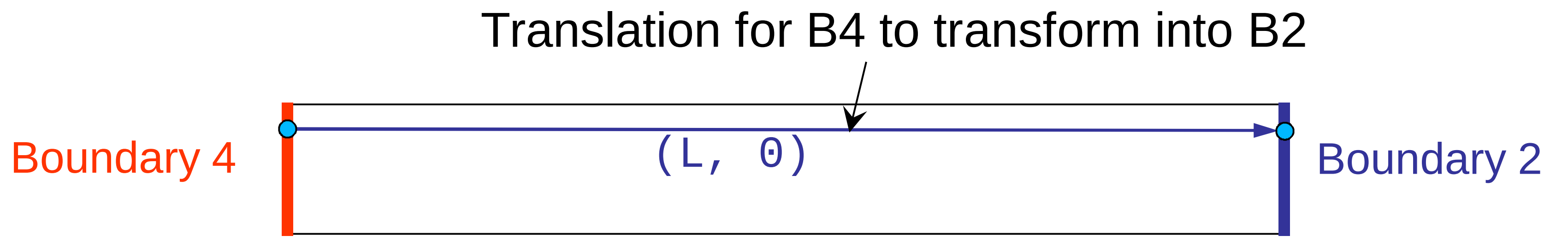
Step 1 – Value of the ISMIP constants

One can use MATC coding to get the correct value of the parameters

```
$yearinsec = 365.25*24*60*60
$rhoi = 900.0/(1.0e6*yearinsec^2)
$gravity = -9.81*yearinsec^2
$n = 3.0
$eta = (2.0*100.0)^(-1.0/n)
```

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Body Force 1
  Flow BodyForce 1 = Real 0.0
  Flow BodyForce 2 = Real $gravity
End

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
Material 1
  Density = Real $rhoi

  Viscosity Model = String "power law"
  Viscosity = Real $eta
  Viscosity Exponent = Real $1.0/n
  Critical Shear Rate = Real 1.0e-10
End
```

Step 1 – Periodic boundary conditions



Declare at the top of the sif:

```
$L = 20.0e3
        …
Boundary Condition 2
  Target Boundaries = 2
  Periodic BC = 4
  Periodic BC Translate(2) = Real $L 0.0
  Periodic BC Velocity 1  = Logical True
  Periodic BC Velocity 2  = Logical True
  Periodic BC Pressure  = Logical True
End
Boundary Condition 4
  Target Boundaries = 4
End
```

Nothing to declare for BC4 !

Step 2 – Add ComputeDevStress

Objective: compute the stress field as $\int_V S_{ij} \Phi \, \mathrm{d}V = 2 \int_V \eta D_{ij} \Phi \, \mathrm{d}V$

where D_{ij} and η are calculated from the nodal velocities using the derivative of the basis functions

- Add a Solver

```
Solver 3
  Equation = Sij
  Variable = -nooutput "Sij"
  Variable DOFs = 1
  Exported Variable 1 = Stress[Sxx:1 Syy:1 Szz:1 Sxy:1]
  Exported Variable 1 DOFs = 4
  Stress Variable Name = String “Stress”
  Procedure = ”ElmerIceSolvers" "ComputeDevStress”
  Flow Solver Name = String “Flow Solution”
  Linear System Solver = Direct
  Linear System Direct Method = umfpack
End
```

- Add in the material section:

```
Cauchy = Logical False
```

Step 2 – Add SaveData Solver (SaveLine)

Objective: save the variables on the top surface (ASCII matrix file)

- Add a new solver

```
Solver 4
  Exec Solver =  After All
  Procedure = File “SaveData” “SaveLine”
  Filename =  “ismip_surface.dat”
  File Append = Logical False
End
```

- Tell in which BC you want to save the data

```
Boundary Condition 3
  Target Boundaries = 3
  Save Line = Logical True
End
```

- Ordering of the variables: see file `ismip_surface.dat.names`

Step 2 – Add SaveData Solver (SaveLine)

SaveLine can also be used to save data at a ‘drilling site’ (a line which is not a boundary). Here, the data are saved at x = 10km.

- Change the solver section

```
Solver 4
  Exec Solver =  After All
  Procedure = File “SaveData” “SaveLine”
  Filename =  “ismip_drilling.dat”
  Polyline Coordinates(2,2) = Real $ (0.5*L) -1000. (0.5*L) 0.0
  File Append = Logical False !overwrites existing files
End
```

- And don’t forget to comment the `Save line = Logical True` in BC3

Step 2 – Add SaveScalars

SaveScalars allows to save scalars and derived quantities. Here, we will save:

1/ the volume of the domain (surface),
2/ the maximum value of the absolute horizontal velocity,
3/ the flux on the 3 boundaries 2, 3 and 4.
4/ the CPU time,
5/ the CPU memory

Step 2 – Add SaveScalars

-Add a new solver

```
Solver 5
  Exec Solver =  After
  Procedure = “SaveData” “SaveScalars”
  Filename = “ismip_scalars.dat”
  File Append = Logical True
  Variable 1 = String “flow solution”
  Operator 1 = String “Volume”
  Variable 2 = String “Velocity 1”
  Operator 2 = String “max abs"
  Variable 3 = String “flow solution”
  Operator 3 = String “Convective flux”
  Operator 4 = String “cpu time”
  Operator 5 = String “cpu memory”
End
```

- Tell at which boundaries you want to save the flux

```
Flux Integrate = Logical True
```

Step 3 – Add ResultOutput

- ResultOutput allows to export the result in vtu (Visulation Toolkit unstructured grid) format and use Paraview for post-treatment

```
Solver 6
  Exec Solver = After TimeStep
  Exec Interval = 1
  Equation = "result output"
  Procedure = "ResultOutputSolve" "ResultOutputSolver"
  Output File Name = String "ismip$Step".vtu"
  Output Format = String vtu
End
```

- For all these added solvers, modify the Equation section:

```
Equation 1
  Active Solvers(6) = 1 2 3 4 5 6
End
```

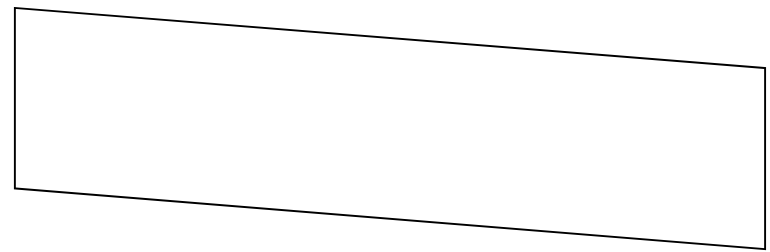
Step 3 – Remark on VTU output

– If one is not interested in the legacy format for ElmerPost, but only in VTU:
  - In `Simulation` just exchange the file suffix from `.ep` to `.vtu`
  - `Post File = “name.vtu”`
  - This runs the `ResultOutputSolver` instead of writing ElmerPost-output
  - No selection on output variables can be made (everything will be dumped)

Step 3 – Move to ISMIP-HOM D020

Changes from B020:

- geometry of domain



$$\begin{cases} z_s(x,y) = -x \tan(0.1^\circ) \\ z_b(x,y) = z_s(x,y) - 1000 \end{cases}$$

→ modify the `Top Surface` and `Bottom Surface` variables

- boundary condition at the bedrock interface

$$\begin{cases} \tau_{nt} = \beta^2 u_t \\ u_n = \boldsymbol{u} \cdot \boldsymbol{n} = 0 \end{cases}$$

with $\beta^2(x) = 1000 + 1000 \sin(\frac{2\pi}{L} x)$

in [Pa a m^{-1}] !

→ modify Boundary Condition 1

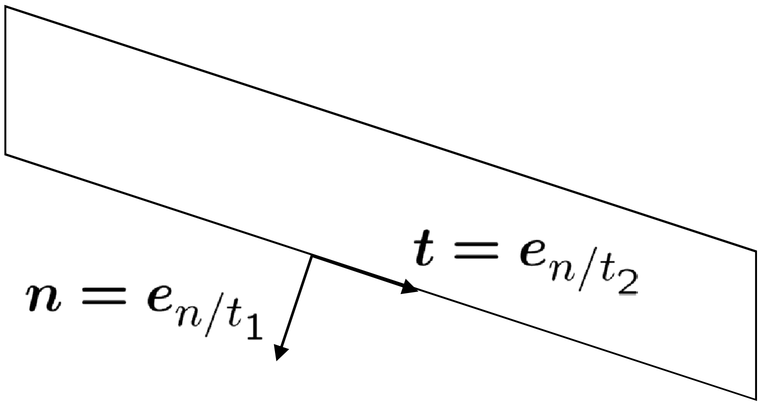
Step 3 – Move to ISMIP-HOM D020

Friction law in Elmer:

$$C_i u_i = \sigma_{ij} n_j \; (i = 1, 2)$$

➡ $C_t u_t = \sigma_{nt}$; $C_n u_n = \sigma_{nn}$

where $\boldsymbol{n}$ is the surface normal vector



Modification of the Boundary Condition 1:

- First Solution: MATC definition of Ct

```
Boundary Condition 1
  Target Boundaries = 1
  Flow Force BC = Logical True
  Normal-Tangential Velocity = Logical True

  Velocity 1 = Real 0.0e0

  Slip Coefficient 2 = Variable coordinate 1
    Real  MATC "1.0e-3*(1.0 + sin(2.0*pi* tx / L ))
End
```

Stress condition defined in a normal-tangential coordinate system

$u_n = 0$

$C_t = \ldots$

in [MPa a m^{-1}] !

Step 3 – Move to ISMIP-HOM D020

- Second Solution: User Function to define Ct

```
Boundary Condition 1
  …
  Slip Coefficient 2 = Variable coordinate 1
     Real Procedure "./ISMIP_D" "Sliding"
End
```

where Sliding is a User Function defined in the file `ISMIP_D.f90`
(see next slide)

Compilation:

```
> elmerf90 ISMIP_D.f90 -o ISMIP_D
```

Step 3 – Move to ISMIP-HOM D020

```
FUNCTION Sliding ( Model, nodenumber, x) RESULT(C)
   USE Types

   IMPLICIT NONE
   TYPE(Model_t) :: Model
   INTEGER :: nodenumber, i
   REAL(KIND=dp) :: x, C, L
   LOGICAL :: FirstTime=.True.

   SAVE FirstTime, L

   IF (FirstTime) THEN
       FirstTime=.False.
       L = MAXVAL(Model % Nodes % x) !the highest occurring x-coord
   END IF

   x = Model % Nodes % x(nodenumber)
   C  =  1000.0d-06*(1.0_dp + SIN(2.0_dp * PI * x/ L))
                        ! in MPa a /m
END FUNCTION Sliding
```

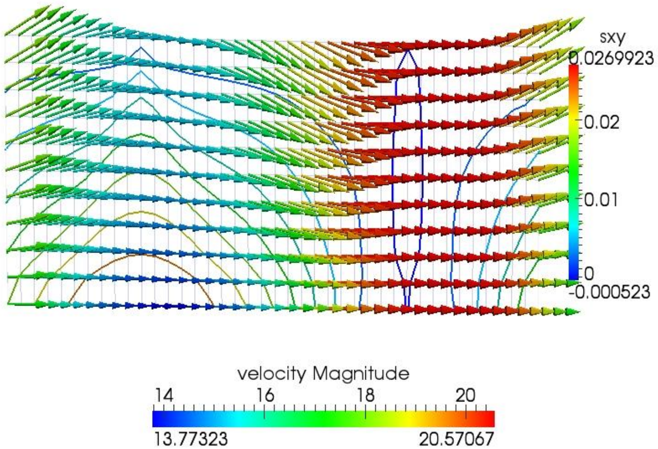
Step 3 – Remark on VTU output

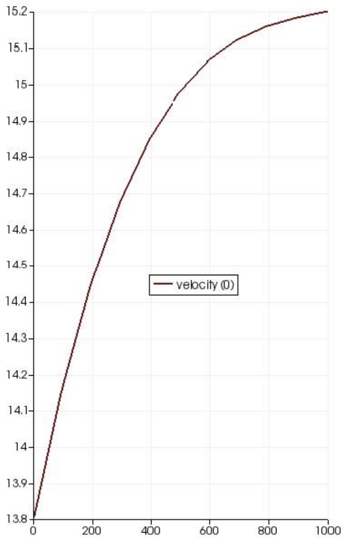
– If one is not interested in the legacy format for ElmerPost, but only in VTU:
  - In `Simulation` just exchange the file suffix from `.ep` to `.vtu`
  - `Post File = "name.vtu"`
  - This runs the `ResultOutputSolver` instead of writing ElmerPost-output
  - No selection on output variables can be made (everything will be dumped)

Step 3 – ParaView

– Launch ParaView:

- **`> paraview`**
- Load the result file `ismip3.vtu0001.vtu`
- “ice-core” in ParaView:
  - Use filter “PlotOverLine”
  - Set Point1 = (5000,-1000,0); Point2=(5000,0,0)





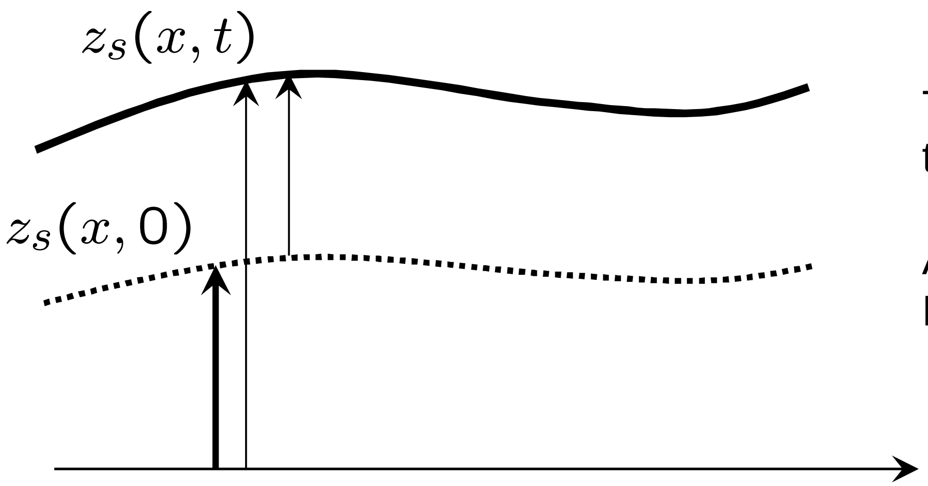
Step 4 – Move to prognostic B020

Move from a diagnostic to a prognostic simulations:

- Steady to transient

- Add the **free surface** solver

$$\frac{\partial z_s}{\partial t} + u_x \frac{\partial z_s}{\partial x} - u_z = a$$



The mesh is vertically deformed using the StructuredMeshMapper solver.

An alternative is to use the MeshUpdate solver (see older courses)

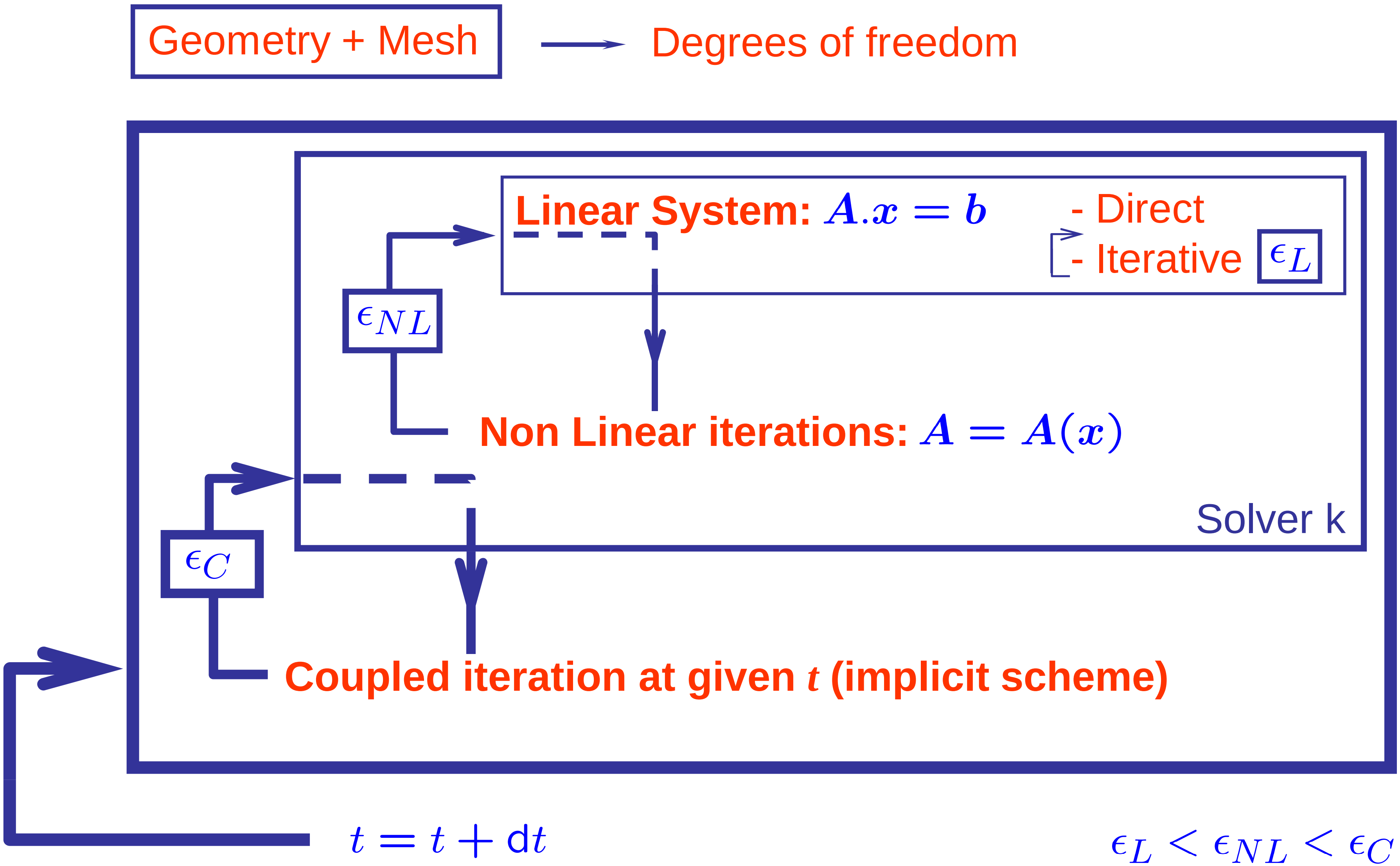
Step 4 – Steady to transient

The simulation Section has to be modified:

```
Simulation Type = Transient

Timestepping Method = “bdf”
BDF Order = 1
Output Intervals = 1
Timestep Intervals = 200
Timestep Sizes = 1.0

Steady State Min Iterations = 1
Steady State Max Iterations = 10
```

- Timestepping Method → Backward Differences Formula
- BDF Order → BDF Order 1 = Backward Euler
- Output Intervals → I/O intervals in .ep/.vtu file
- Steady State Max Iterations → To control the “implicity” of the solution over one time step (see example bellow).

Step 4 – Sketch of a transient simulation



Step 4 – Free surface Solver

The free surface solver only applies to the boundary 3 (top surface)

➡ Define a 2nd (lower dimensional) body on boundary 3.

```
Body 2
  Equation = 2
  Body Force = 2
  Material = 1
  Initial Condition = 2
End
```

where `Equation 2, Body Force 2 and Initial Condition 2` are defined for the free surface equation.

Tell in BC3 that this is the body 2:

```
Boundary Condition 3
  Target Boundaries = 3
        …
  !!! this BC is equal to body no. 2 !!!
  Body Id = 2
        …
End
```

Step 4 – Free surface Solver

Add the Free Surface Solver:

The minimum is presented here, you can add limits not to be penetrated by the free surface

```
Solver 2
   Equation = "Free Surface"
   Variable = String Zs
   Variable DOFs =  1
   Exported Variable 1 = String “Zs Residual"
   Exported Variable 1 DOFs = 1

   Procedure = "FreeSurfaceSolver" "FreeSurfaceSolver"
   Before Linsolve = “EliminateDirichlet” “EliminateDirichlet” ! Not in parallel

   Linear System Solver = Iterative
   Linear System Max Iterations = 1500
   Linear System Iterative Method = BiCGStab
   Linear System Preconditioning = ILU0
   Linear System Convergence Tolerance = Real 1.0e-5
   Linear System Abort Not Converged = False
   Linear System Residual Output = 1

   Steady State Convergence Tolerance = 1.0e-03

   Relaxation factor = Real 1.0
   Stabilization Method = Bubbles
   End
```

Step 4 – Free surface Solver

Body Force 2:

```
Body Force 2 ! The Cartesian components
  Zs Accumulation Flux 1 = Real 0.0e0
  Zs Accumulation Flux 2 = Real 0.0e0
End
```

Equation 2:

```
Equation 2
  Active Solvers(1) = 2
  Flow Solution Name = String “Flow Solution”
  Convection = String Computed
End
```

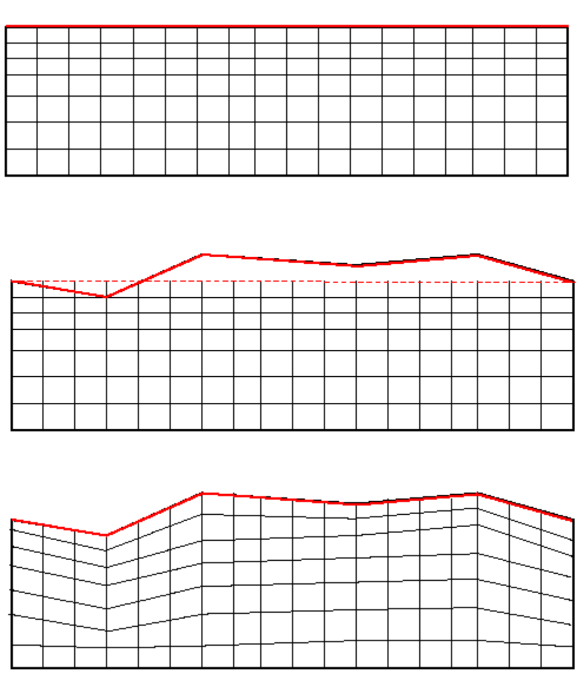
Initial Condition 2: give $z_s(x, 0)$

```
Initial Condition 2
  Zs = Variable Coordinate 1
     Real MATC “-tx*tan(Slope)”
End
```

Step 4 – StructuredMeshMapper

- The `Top Surface` variable is now equal to the variable `Zs`

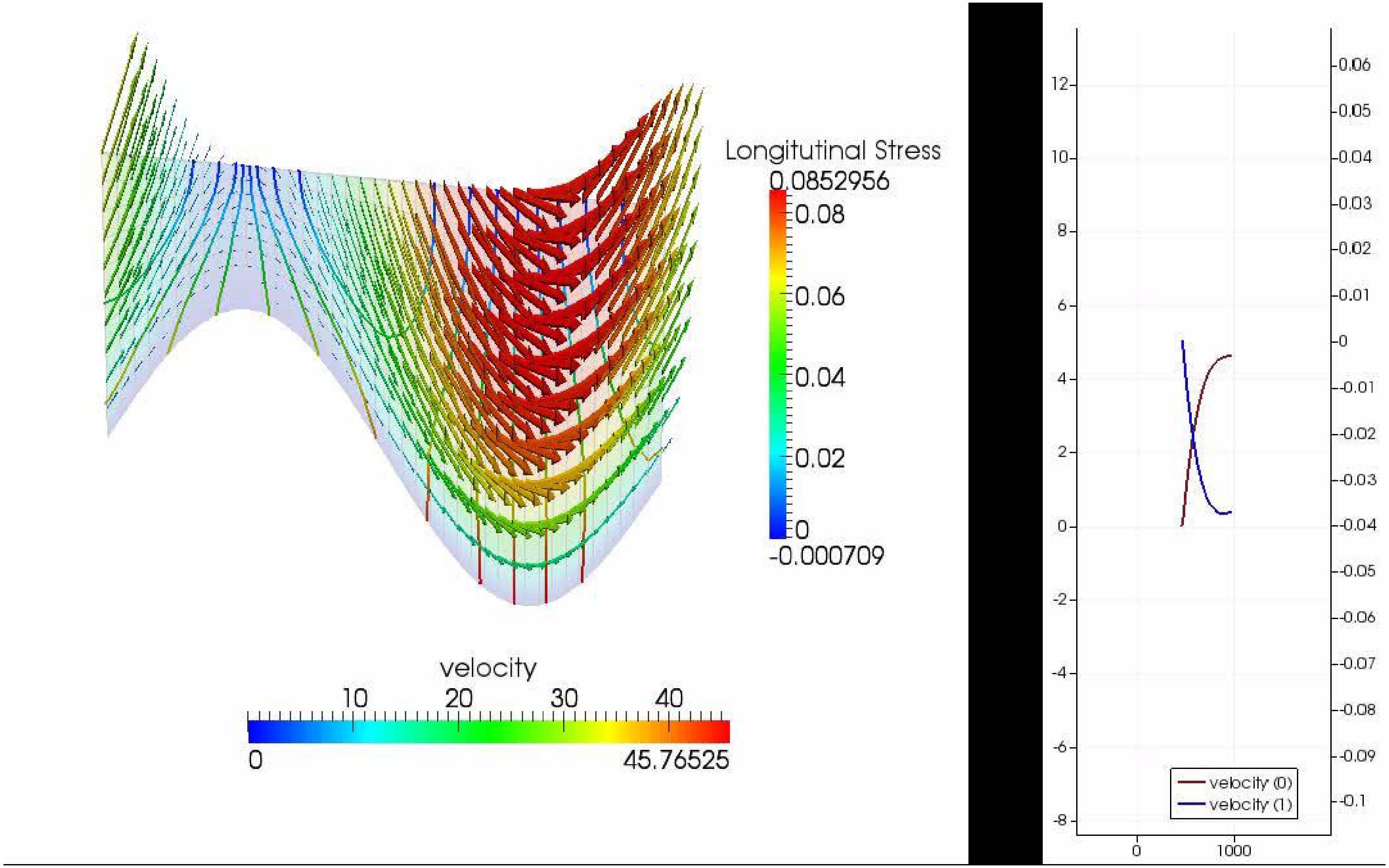
```
Solver 1
  Equation = "MapCoordinate"
  Procedure = "StructuredMeshMapper" "StructuredMeshMapper"
  Active Coordinate = Integer 2

  Mesh Velocity Variable = String  "dSdt"
  Mesh Update Variable = String  "dS"
  Mesh Velocity First Zero = Logical True

  Top Surface Variable = String "Zs"
  Dot Product Tolerance = Real 1.0e-3
End
```



-And delete in the top surface BC the definition of `Top Surface`.

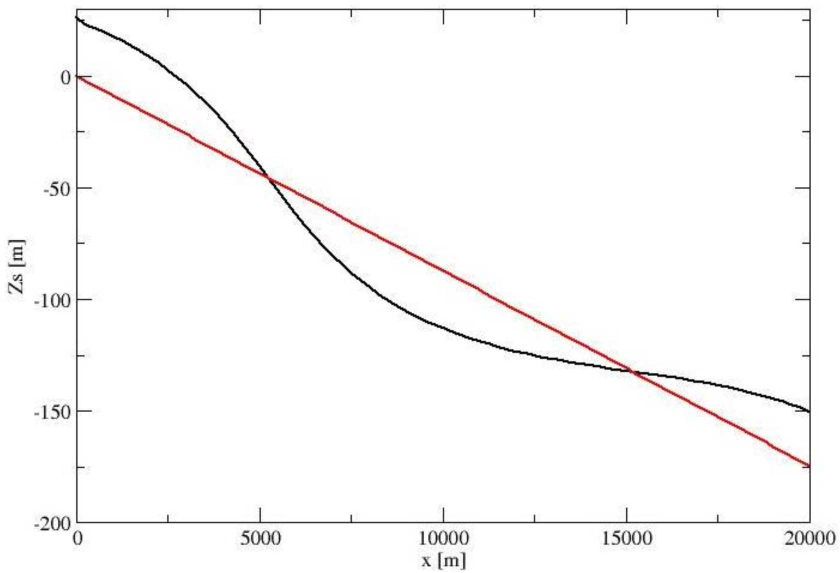
Step 4 – Results !

Animation made with ParaView

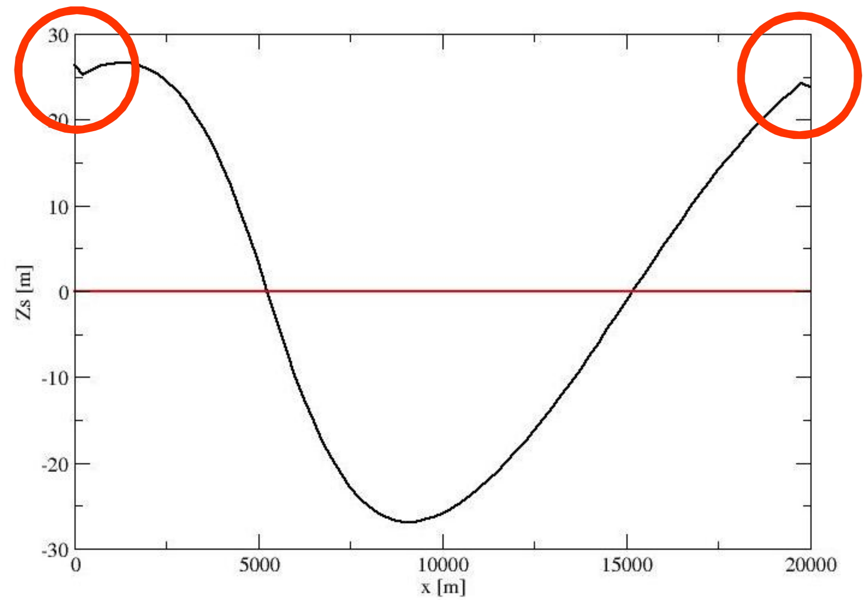


Step 4 – Results !

Comparison of the initial and steady surface of the prognostic run



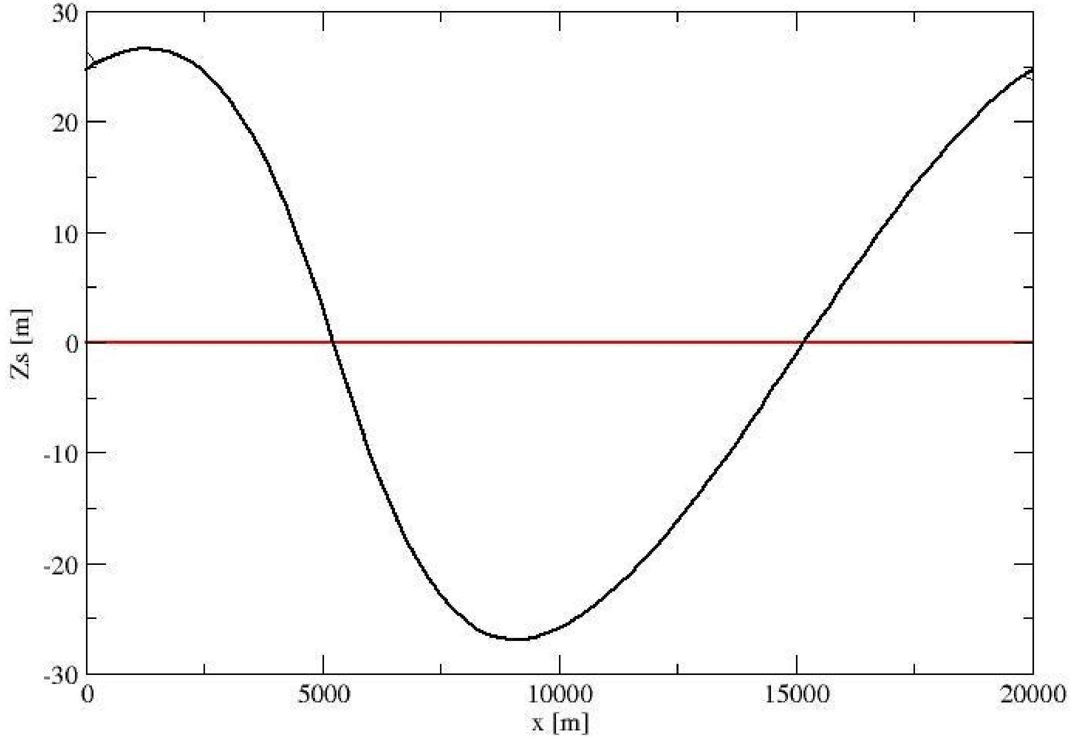
Not periodic



Step 4 – better results !

Turn the mesh so that zs = 0 (turn the gravity vector also !)
Force zs to be periodic

See Step4_hori



Step 5 – Move to prognostic D020

Merge Step 3 and Step 4 and it should work !