Laboratoire de Glaciologie et Géophysique de l'Environnement

Elmer/Ice advanced Workshop

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Marine ice-sheets and the Grounding line problem

Olivier GAGLIARDINI

LGGE - Grenoble - France

LabEx OSUG@2020

Observatoire des Sciences de l'Univers de Grenoble

Grounding Line

✓ **The Physics**

- Dynamics of ice-sheets
- The transition zone
- Results from grounding line models

✓ **Equations to be solved**

- The Schoof equation
- Solution of a contact problem

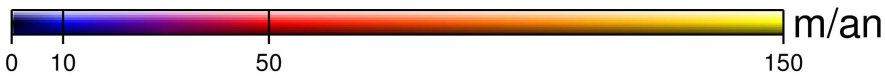
✓ **Implementation in Elmer/Ice**

- The basal boundary
- How to evaluate the contact?
- Mesh size issue
- Interpolation of the friction

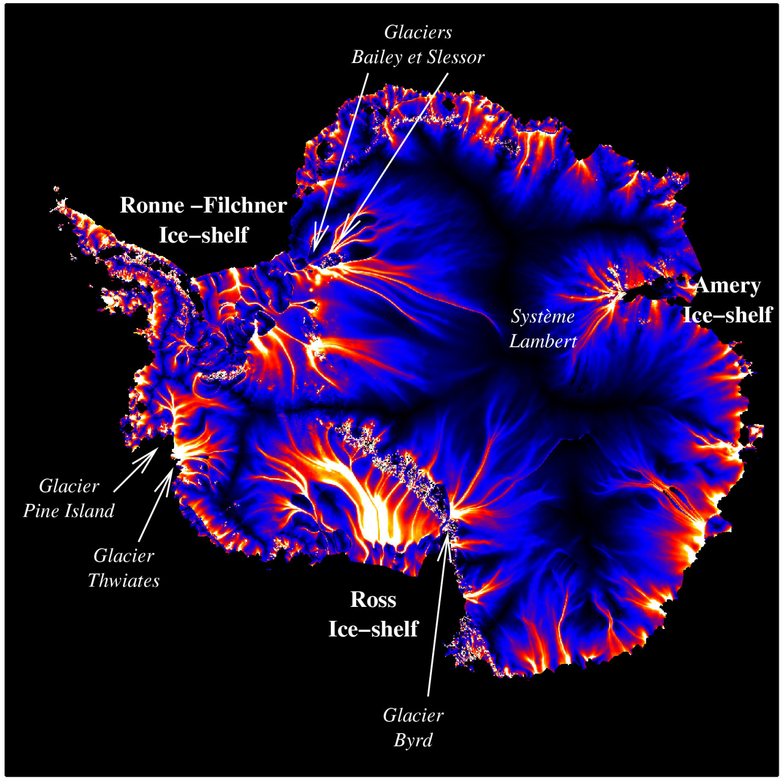
✓ **Example**

- MISMIP test

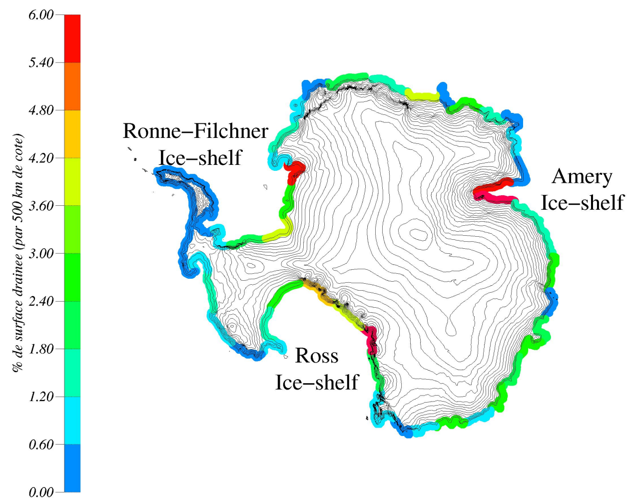
Importance of ice-stream

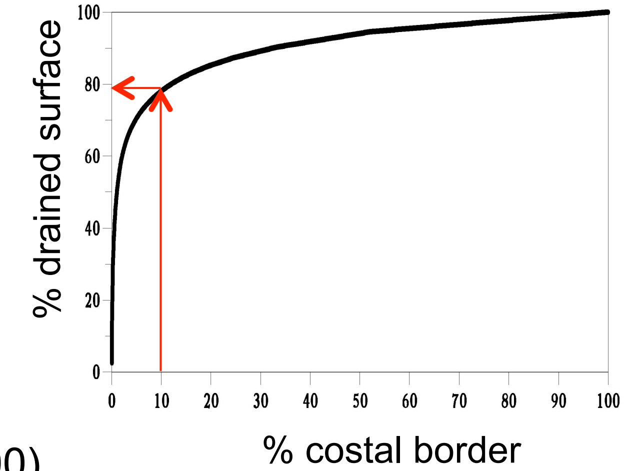


Mass balance velocity



(Testut, 2000)

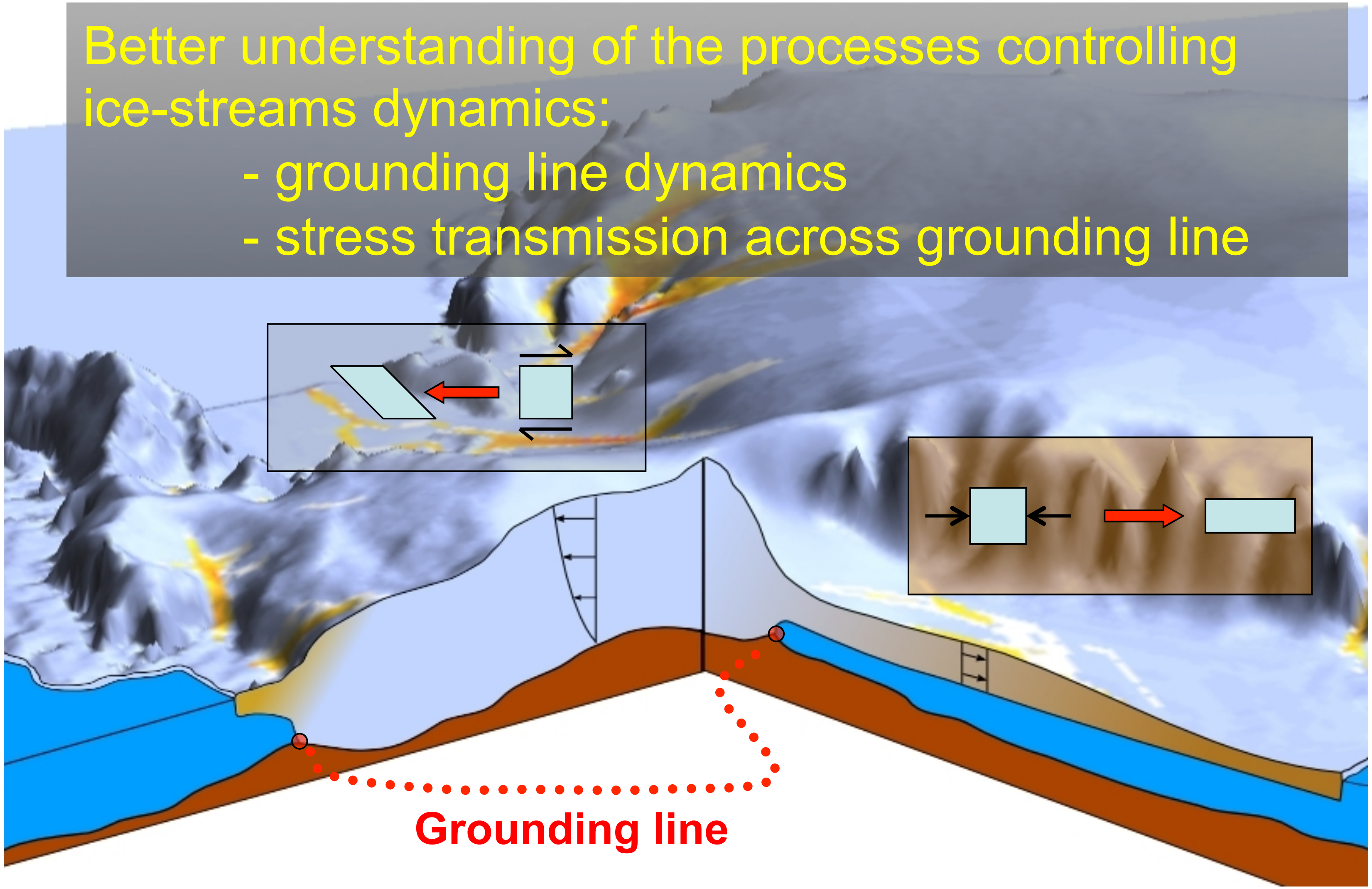




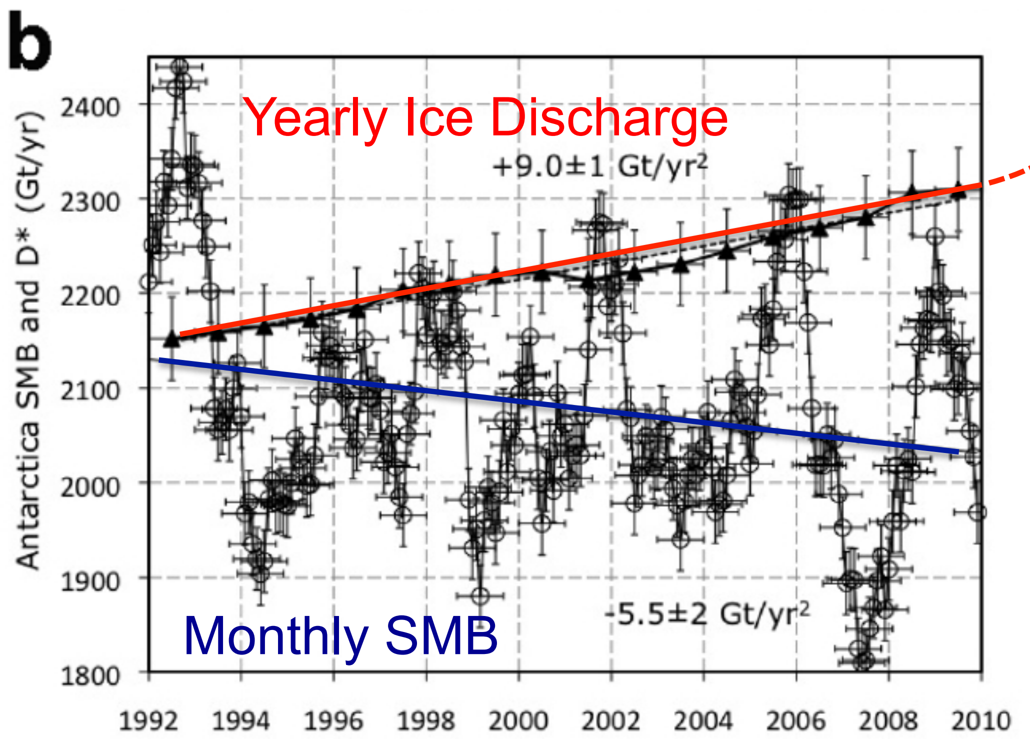
Transition zone

Better understanding of the processes controlling ice-streams dynamics:

- grounding line dynamics
- stress transmission across grounding line



Ice Discharge



[Rignot et al., 2011]

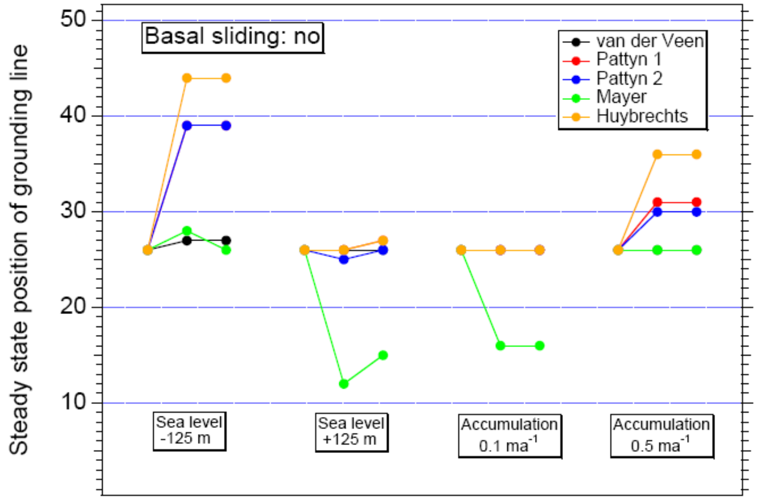
SMB – Discharge
= Mass of ice lost

= Contribution to SLR

What will be the future contribution of Ice Discharge for the next centuries?

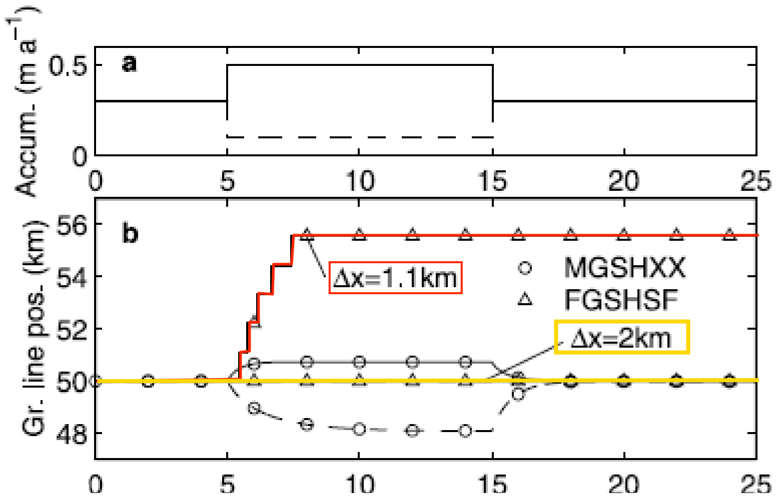
Need accurate description of the Grounding Line dynamics

EISMINT Results



- **No consensus** on the results
- **No consensus** on how the GL should be modelled
- It is **unclear** wheter these results are indicative of neutral equilibrium

[Huybrechts, 1998]



Influnce of the horizontal grid size

[Vieli and Payne, 2005]

Poor ability of the model to capture the GL dynamic until recently

Fourth IPCC report

Dynamical processes related to ice flow not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea level rise. Understanding of these processes is limited and there is no consensus on their magnitude. Current global model studies project that the Antarctic Ice Sheet will remain too cold for widespread surface melting and is expected to gain in mass due to increased snow fall. However, net loss of ice mass could occur if dynamical ice discharge dominates the ice sheet mass balance.

IPCC, Working group I, Summary for policymakers, 2007.

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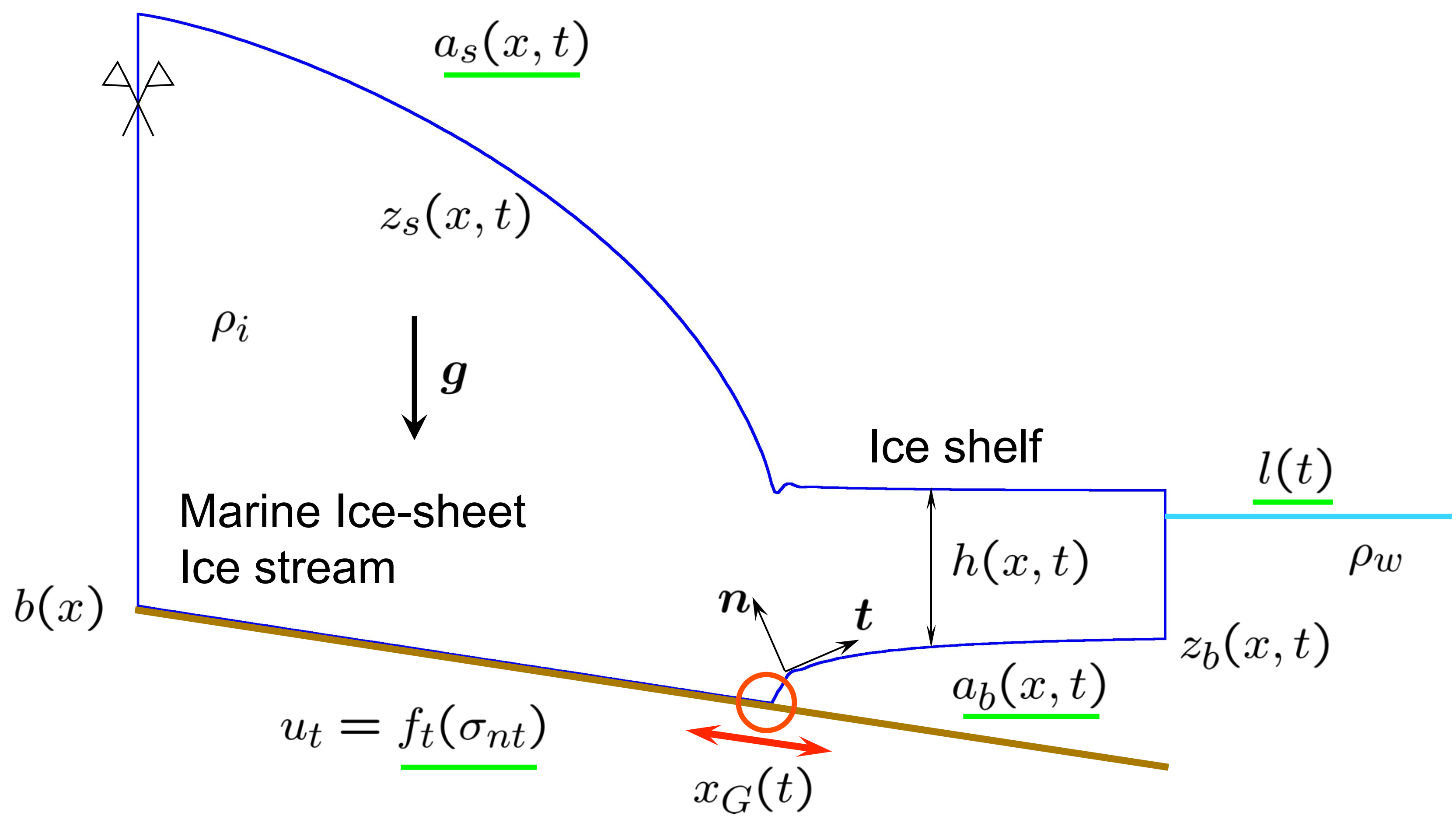
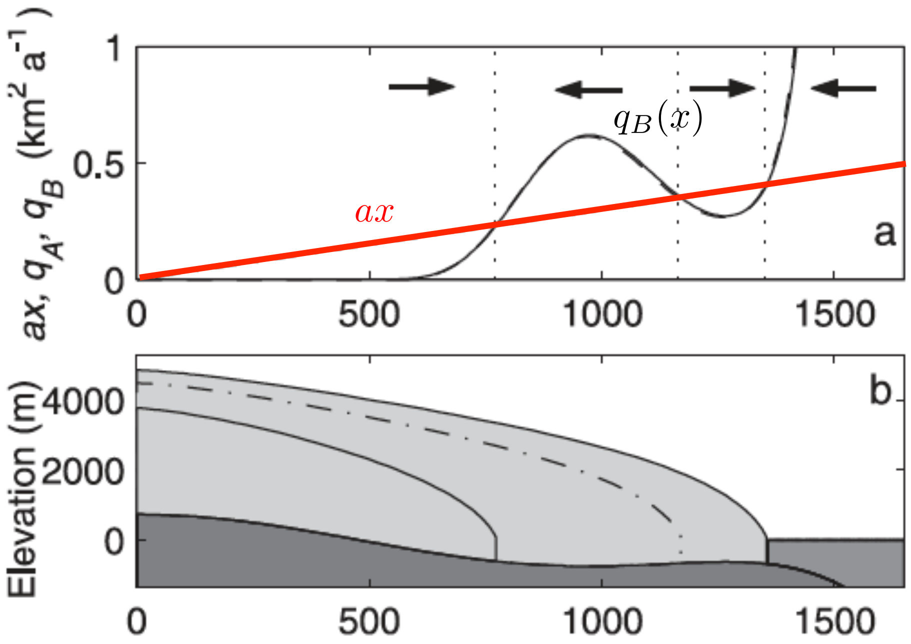
Notation / Concept



How the grounding line evolves for different scenarii ?

Schoof's solution (2007) – MISI in 2D

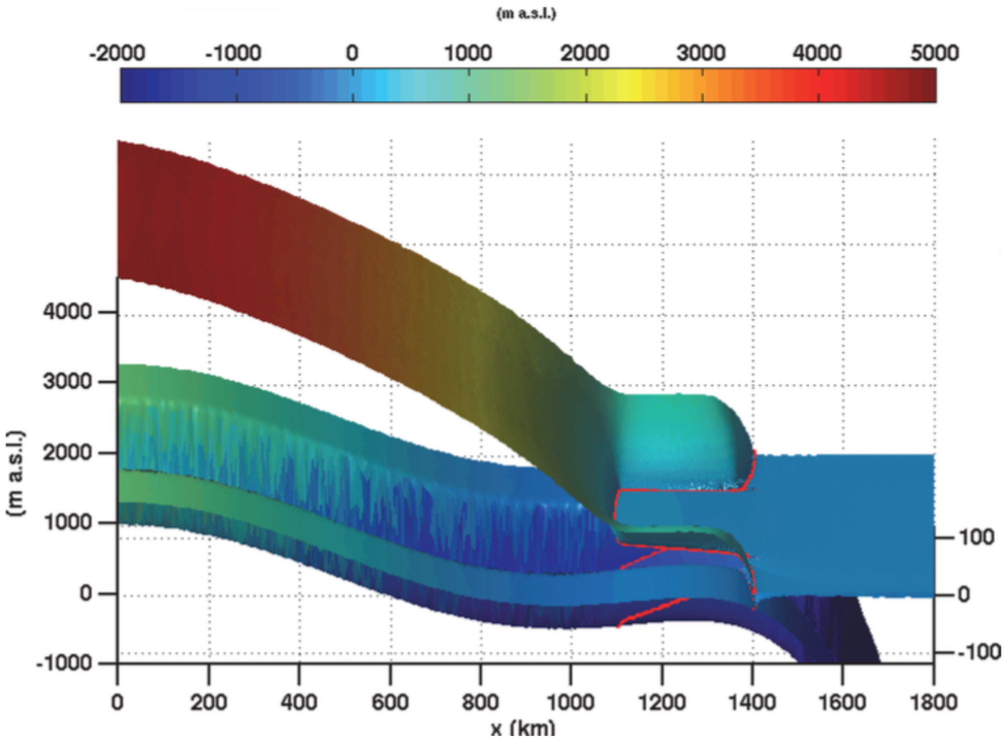


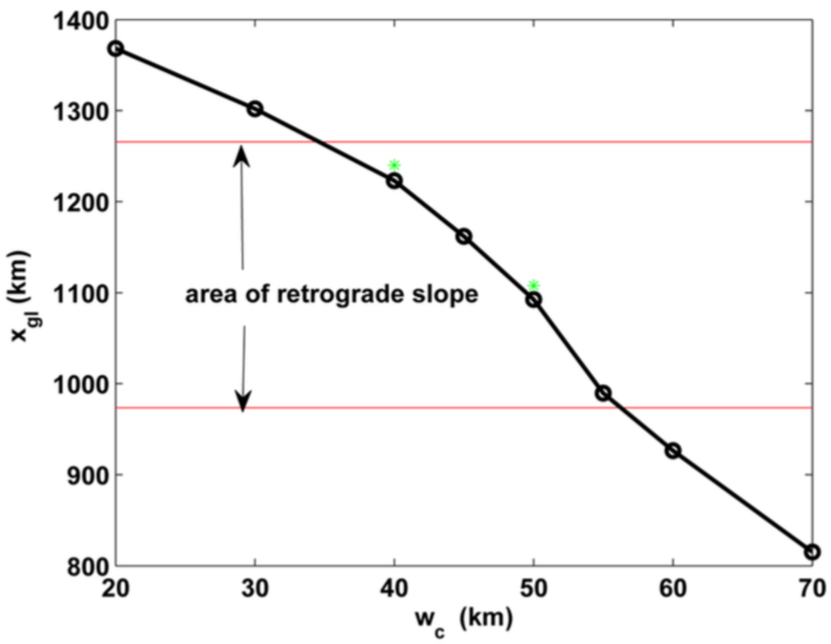
[Schoof, 2007, 2011]

$$ax_g = q_B(x_g) = \left(\frac{\bar{A}(\rho_i g)^{n+1}(1 - \rho_i/\rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}}$$

- Confirms that there is no stable position of the GL on an upsloping bed
- For a given surface mass balance, gives the steady GL position (in 2D)

MISI in 3D





[Gudmundsson et al., 2012]

Marine ice sheets are not unconditionally unstable in two horizontal dimensions

Equations to be solved

Ice Flow

$$\begin{cases} \boldsymbol{D} = A\tau_e^{n-1}\boldsymbol{S} \\ \operatorname{div}\boldsymbol{\sigma} + \rho\boldsymbol{g} = 0 \\ \operatorname{div}\boldsymbol{u} = 0 \end{cases}$$

Top free surface

$$\frac{\partial z_s}{\partial t} + u_x\frac{\partial z_s}{\partial x} + u_y\frac{\partial z_s}{\partial y} - u_z = a_s$$

Bottom free surface

$$\begin{cases} \frac{\partial z_b}{\partial t} + u_x\frac{\partial z_b}{\partial x} + u_y\frac{\partial z_b}{\partial y} - u_z = a_b \\ \text{with} \quad z_b \geq b \end{cases}$$

Ice - Bed contact

$z_b = b$ and $-\sigma_{nn} > p_w$ ⇒ $\boldsymbol{u}.\boldsymbol{n} = 0$, $u_t = f_t(\sigma_{nt})$

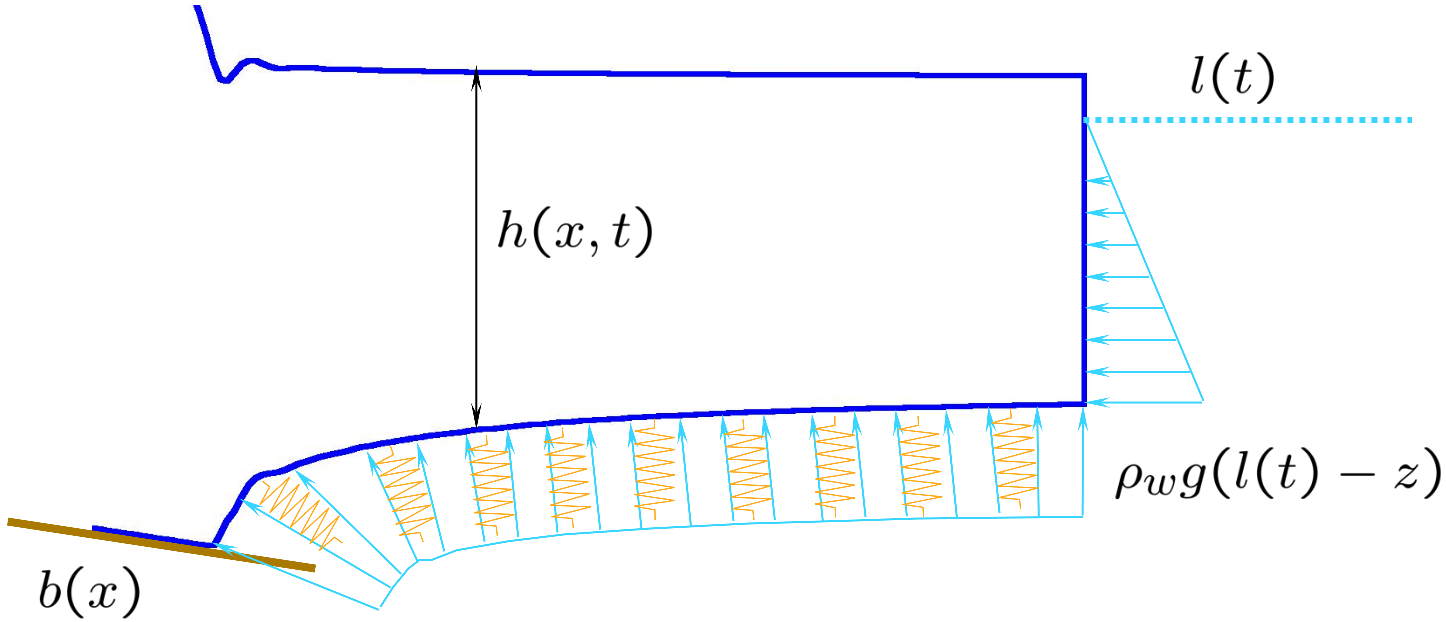
Ice - Sea

$z_b = b$ and $-\sigma_{nn} \leq p_w$

or $z_b > b$

⇒ Buoyancy condition

Buoyancy BC



BC Stokes: if $z_b(x,t) > b(x)$

$$\sigma_{nn}(x) = \rho_w g(l(t) - z_b(x,t)) \quad \text{and} \quad \sigma_{nt}(x) = 0$$

$$z_b(x,t) = z_b(x, t - \mathrm{d}t) + u_n \sqrt{1 + (\mathrm{d}z_b/\,\mathrm{d}x)^2}\,\mathrm{d}t$$

$$\sigma_{nn}(x) = \rho_w g(l(t) - z_b(x, t - \mathrm{d}t)) - \rho_w g \sqrt{1 + (\mathrm{d}z_b/\,\mathrm{d}x)^2}\,\mathrm{d}t.u_n$$

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The basal boundary

Condition applied on the basal boundary depend if
- the ice is in contact with the bedrock
- or the ice is in contact with the sea

The limit between grounded and floating parts (the GL) is unknown and solution of the contact problem

Add a `Mask` variable (only on the basal surface) which tells if grounded, floating or at the GL

Mask = 1 if grounded
Mask = -1 if floating
Mask = 0 if at the GL

The basal boundary

In Elmer, the use of a **conditional Dirichlet** condition allows to deal with this evolving limit.

Example in the SIF:

```
Velocity 1 = Real 0.0
Velocity 1 Condition = Variable Mask
   Real MATC "tx + 0.5"
```

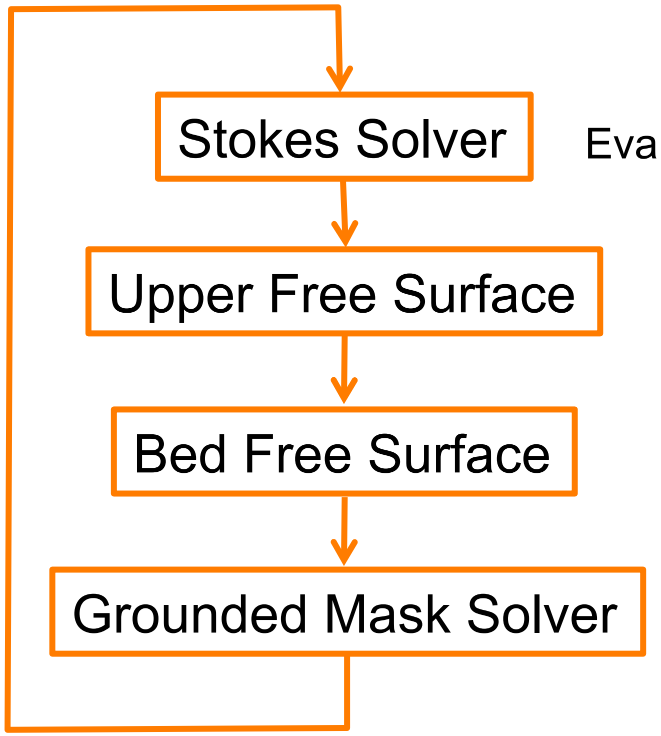
Mask = -1 → the Dirichlet BC is not applied

Mask = 1 or 0 → the Dirichlet BC $u_n = 0$ is applied

The contact problem

$z_b = b$ and $-\sigma_{nn} > p_w$ ⇒ Ice - Bed condition

$z_b = b$ and $-\sigma_{nn} \leq p_w$
or $z_b > b$ ⇒ Ice - Sea condition



Evaluate the condition $-\sigma_{nn} > p_w$ after the non-linear system has started converged

Evaluate the condition $z_b > b$

The contact problem

The condition $-\sigma_{nn} > p_w$ is in fact evaluated using nodal force (and not stress)

- the force exerted by the ice on the bed is given by the residual of the Stoles solution

In the Stokes solver

```
Exported Variable 1 = Flow Solution Loads[Stress Vector:2 CEQ Residual:1]
Calculate Loads = Logical True
```

- the nodal water force is the integrated water pressure with respect to the surface element

add a new solver to integrate the water pressure

Water force

```
SUBROUTINE GetHydrostaticLoads( Model,Solver,dt,TransientSimulation )

VariableValues = 0.0_dp

DO t = 1, Solver % NumberOfActiveElements
    Element => GetActiveElement(t)
    IF (ParEnv % myPe .NE. Element % partIndex) CYCLE
    n = GetElementNOFNodes()

    BC => GetBC( Element )
    pwt(1:n) =  -1.0 * ListGetReal(BC, 'External Pressure', n, &
                    Element % NodeIndexes , GotIt)
    CALL GetElementNodes( Nodes )
    IP = GaussPoints( Element )
    DO p = 1, IP % n

      stat = ElementInfo( Element, Nodes, IP % U(p), IP % V(p), &
      IP % W(p), detJ, Basis, dBasisdx, ddBasisddx, .FALSE.)
      s = detJ * IP % S(p)

      Normal = NormalVector( Element, Nodes, IP % U(p), IP % V(p), .TRUE.)
      pwi = SUM(pwt(1:n)*Basis(1:n))
      PwVector(1:DIM) = pwi * Normal(1:DIM)

      DO i = 1, n
            Nn = Permutation(Element % NodeIndexes(i))
            DO j = 1, DIM
              VariableValues(DIM*(Nn-1)+j) = VariableValues(DIM*(Nn-1)+j) + PwVector(j) *
s * Basis(i)
            END DO
      END DO
    END DO
  END DO
  IF ( ParEnv % PEs>1 ) CALL ParallelSumVector( Solver % Matrix, VariableValues )
!------------------------------------------------------------------------------
END SUBROUTINE GetHydrostaticLoads
!------------------------------------------------------------------------------
```

The bed boundary condition

```
Boundary Condition 1
Target Boundaries = 1
  Body Id = 3

  Normal-Tangential Velocity = Logical True
  Flow Force BC = Logical True
!
! Bedrock conditions
!
  Slip Coefficient 2 = Variable Coordinate 1
    Real Procedure "ElmerIceUSF" "SlidCoef_Contact"
    Sliding Law = String "Weertman"
    Weertman Friction Coefficient = Real $C
    Weertman Exponent = Real $(1.0/n)
    Weertman Linear Velocity = Real 1.0

  Grounding line Definition = String "Discontinuous"

  Velocity 1 = Real 0.0
  Velocity 1 Condition = Variable GroundedMask
    Real MATC "tx + 0.5"
!
! Shelf conditions
!
  External Pressure = Variable Coordinate 2
     Real Procedure "ElmerIceUSF" "SeaPressure"

  Slip Coefficient 1 = Variable Coordinate 2
     Real Procedure "ElmerIceUSF" "SeaSpring"
End
```

The variable `GroundedMask` is updated in this User Function `SlidCoef_Contact`

See note after

Will only apply if the Dirichlet condition `Velocity 1 = 0` is not applied

The user function `SlidCoef_Contact`

Test the contact condition:

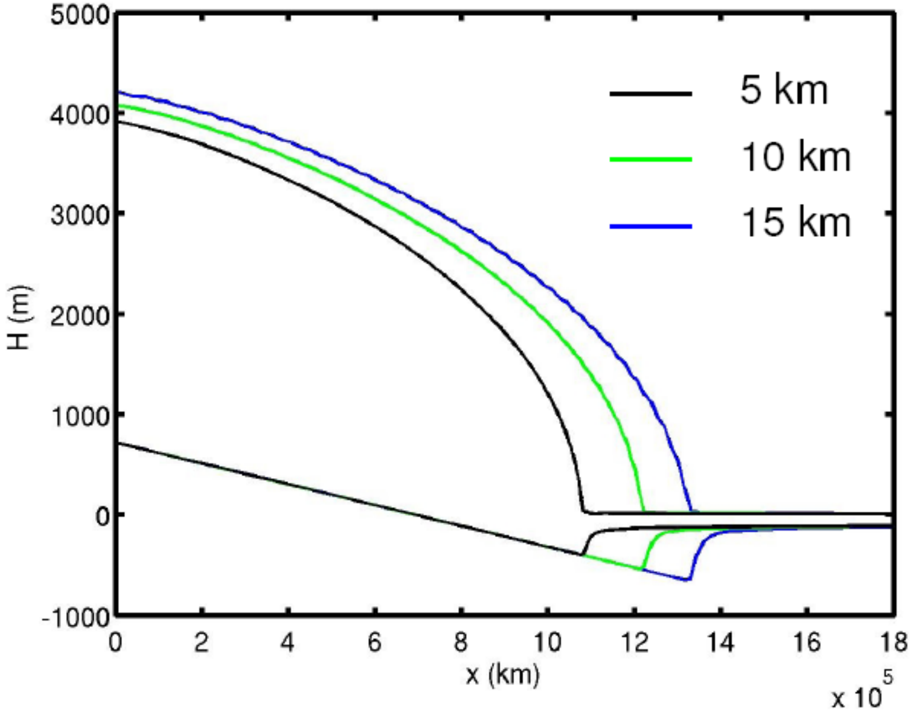
```
Normal = NormalValues(DIM*(NormalPerm(jj)-1)+1 : DIM*NormalPerm(jj))
Fwater = Hydro(DIM*(HydroPerm(jj)-1)+1 : DIM*HydroPerm(jj))
Fbase = ResidValues((DIM+1)*(ResidPerm(jj)-1)+1 : (DIM+1)*ResidPerm(jj)-1)


comp = ABS( SUM( Fwater * Normal ) ) - ABS( SUM( Fbase * Normal ) )

IF (comp >= 0.0_dp)  GroundedMask(Nn) = -1.0_dp
```

and return the sliding coefficient:

- appropriate if grounded
- 0 if floating

```
cond = GroundedMask(GroundedMaskPerm(nodenumber))
IF (cond > -0.5_dp) THEN
          Bdrag = Sliding_weertman(Model, nodenumber, y)
ELSE
          Bdrag = 0.0_dp
END IF
```

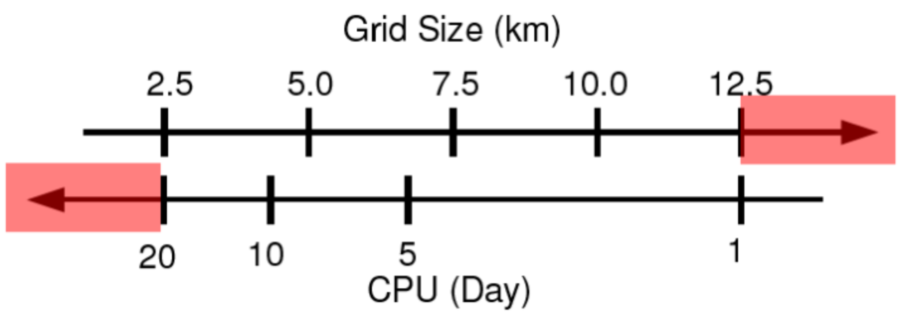
Sensitivity to the grid size



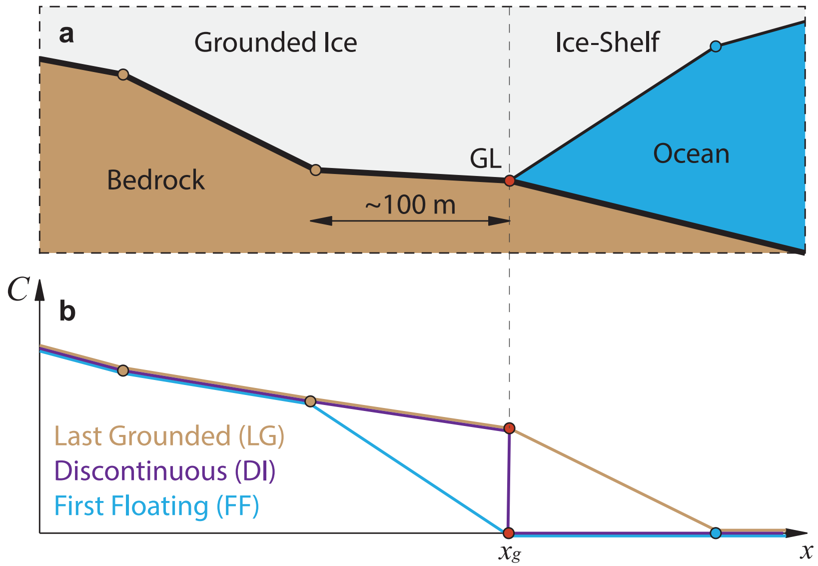
Steady for different horizontal grid sizes

Strong influence of the grid size !

Problem : CPU cost !

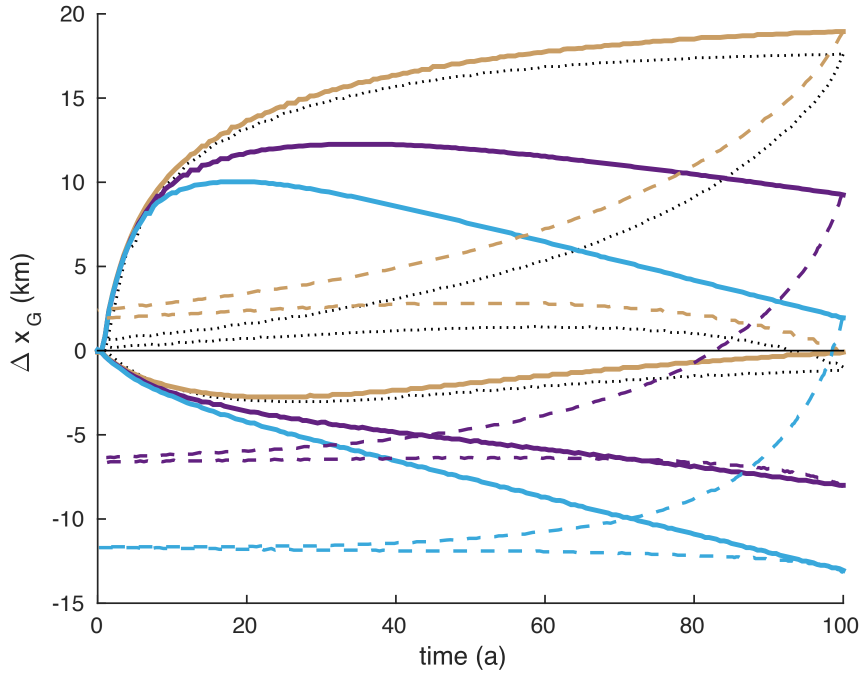


Interpolation of the friction

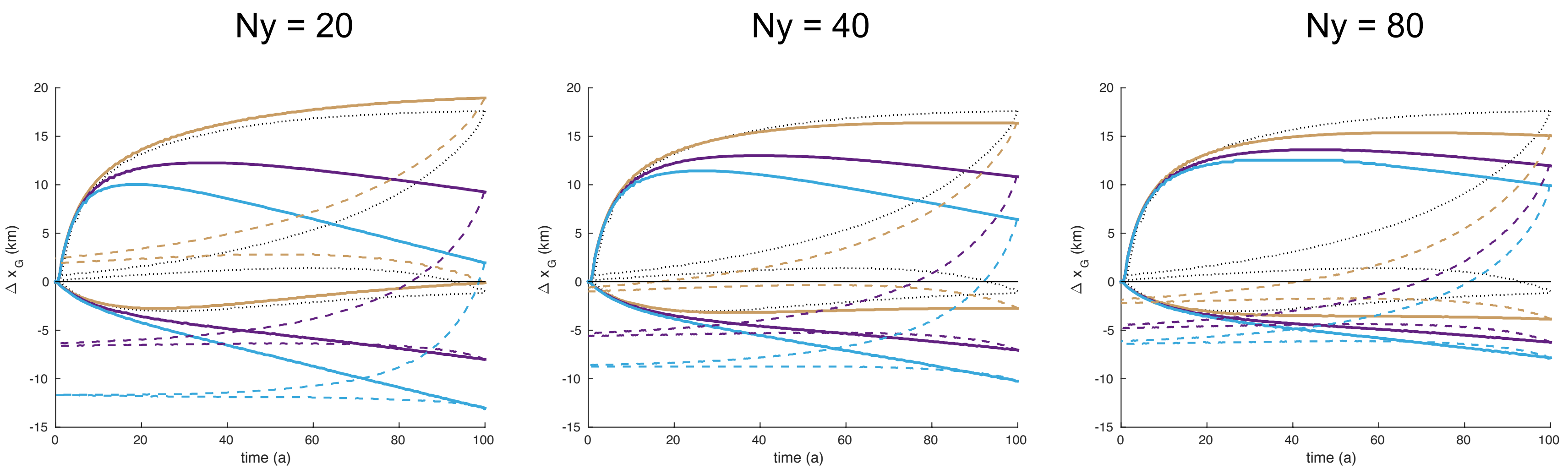


Use Discontinous!

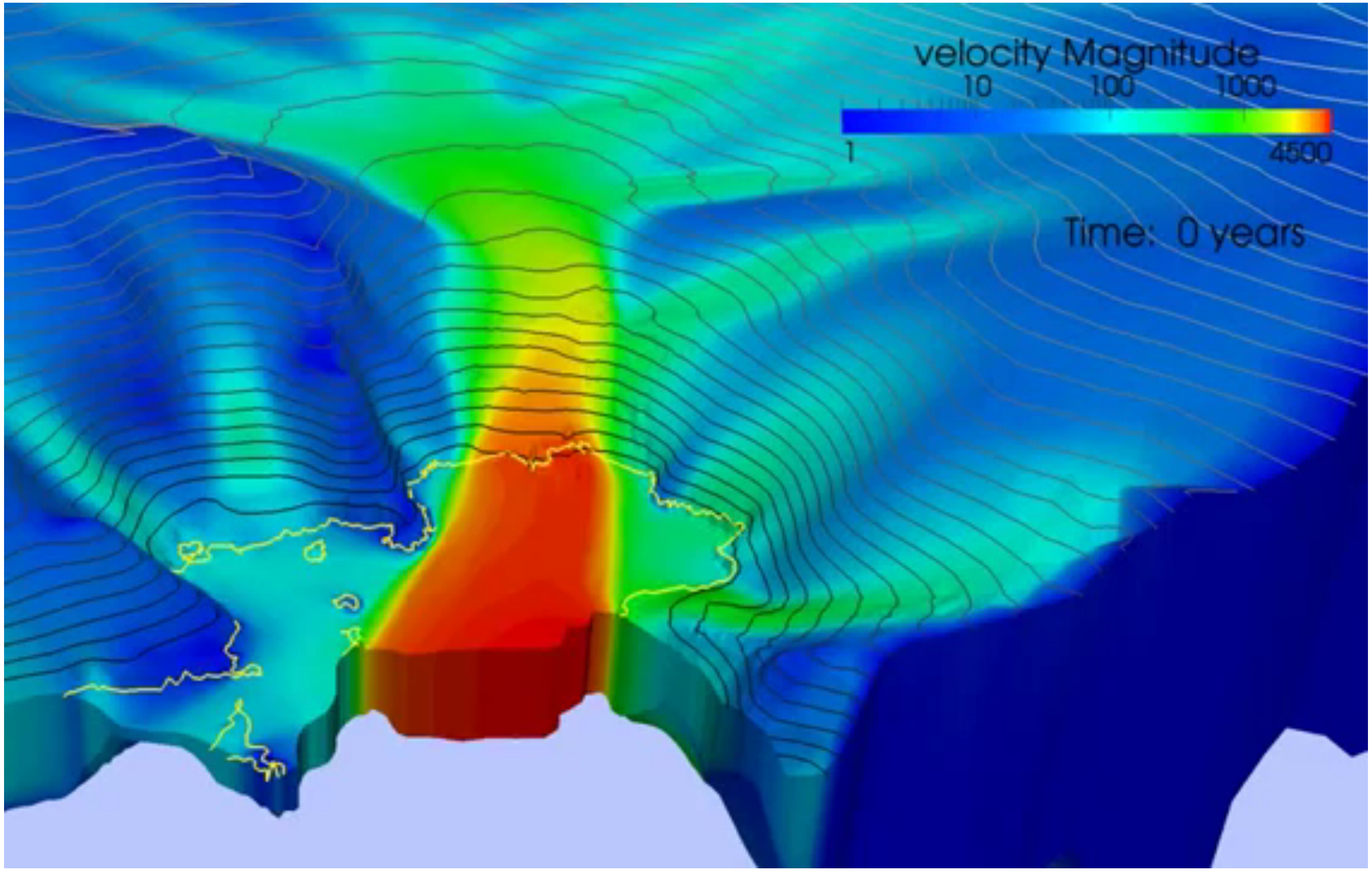
MISMIP3d – Ny = 20



Interpolation of the friction



PIG example (Favier et al., 2014)



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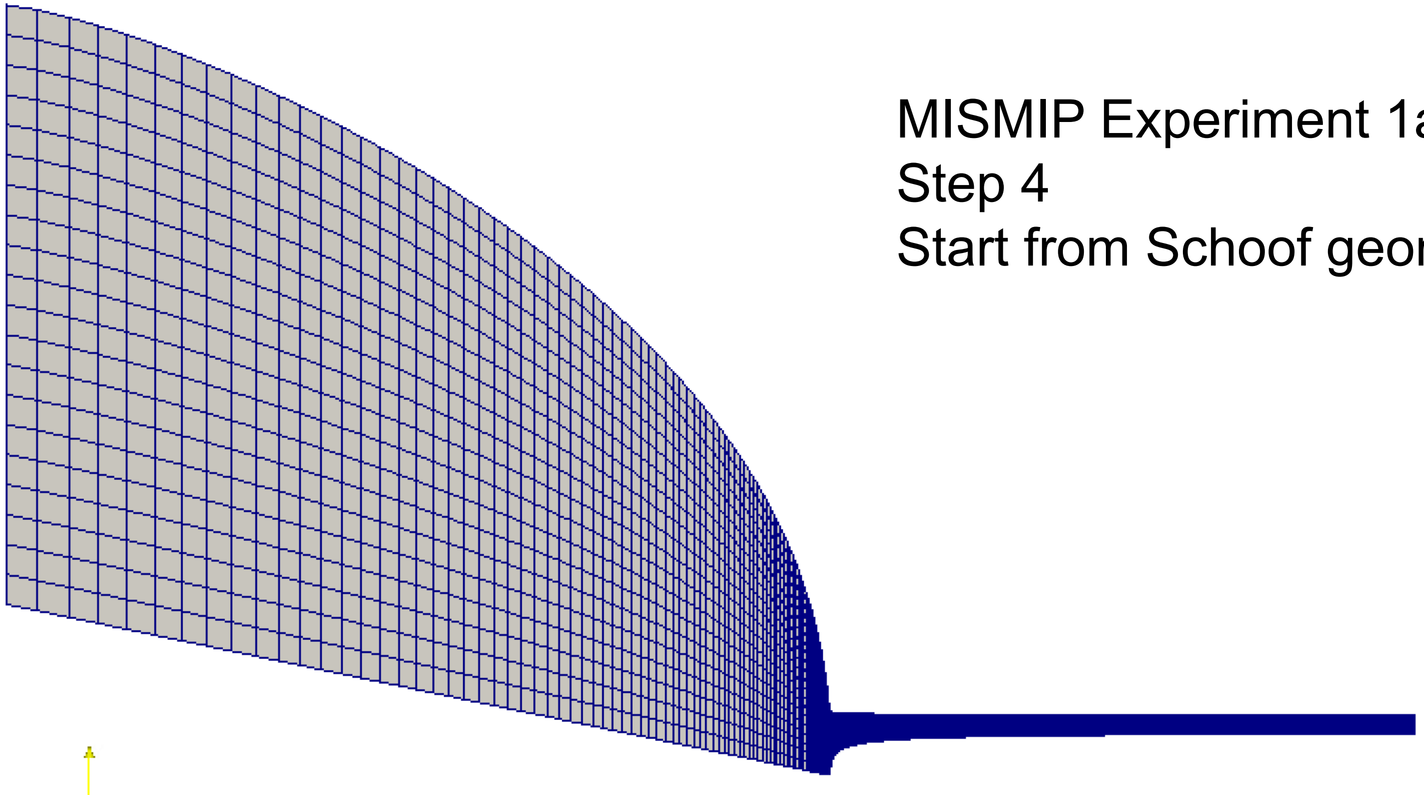
Example GL_MISMIP



MISMIP Experiment 1a
Step 4
Start from Schoof geometry

http://elmerice.elmerfem.org/wiki/doku.php?id=problems:groundingline

[ELMER_TRUNK]/elmerice/Tests/GL_MISMIP

References

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