

Elmer/Ice advanced Workshop

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Marine ice-sheets and the Grounding line problem

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Grounding Line

✓ The Physics

- Dynamics of ice-sheets
- The transition zone
- Results from grounding line models

✓ Equations to be solved

- The Schoof equation
- Solution of a contact problem

✓ Implementation in Elmer/Ice (Stokes – see SSA presentation also)

- The basal boundary
- How to evaluate the contact?
- Mesh size issue
- Interpolation of the friction

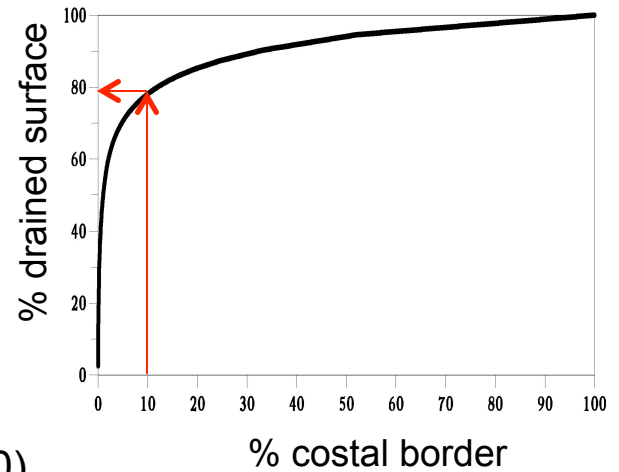
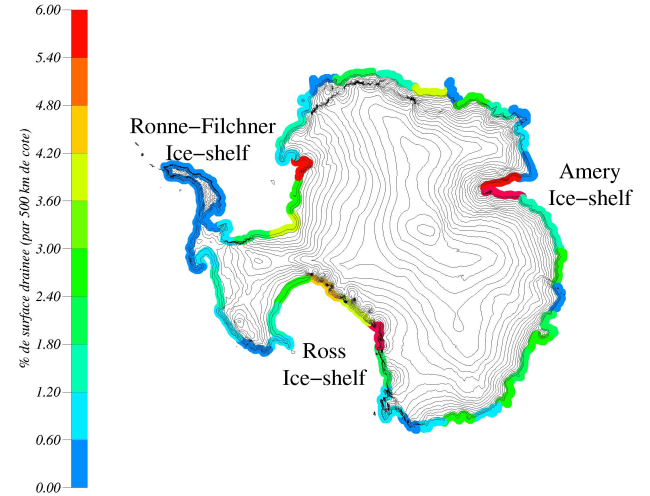
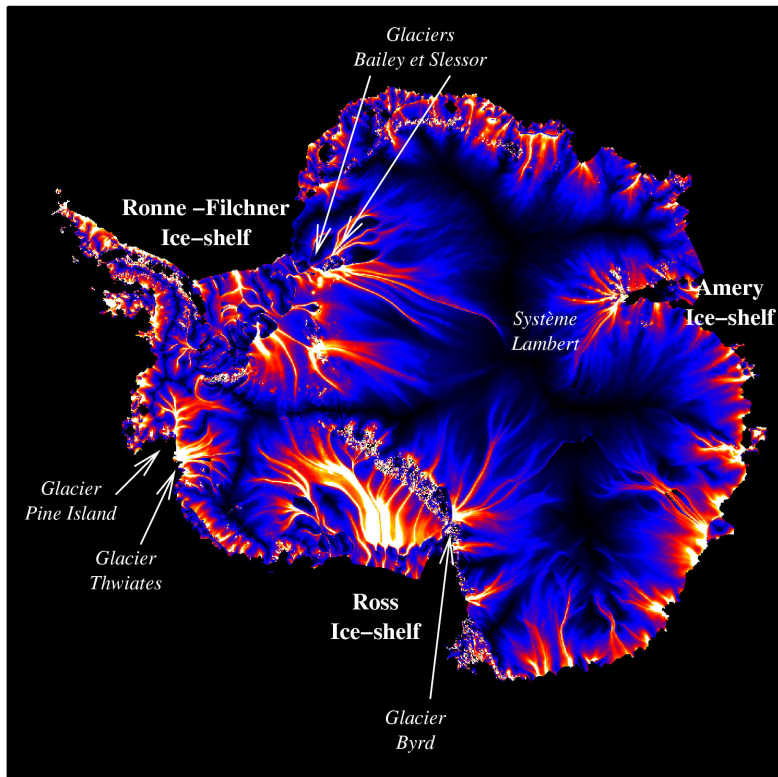
✓ Example

- MISMIP test

Importance of ice-stream



Mass balance velocity

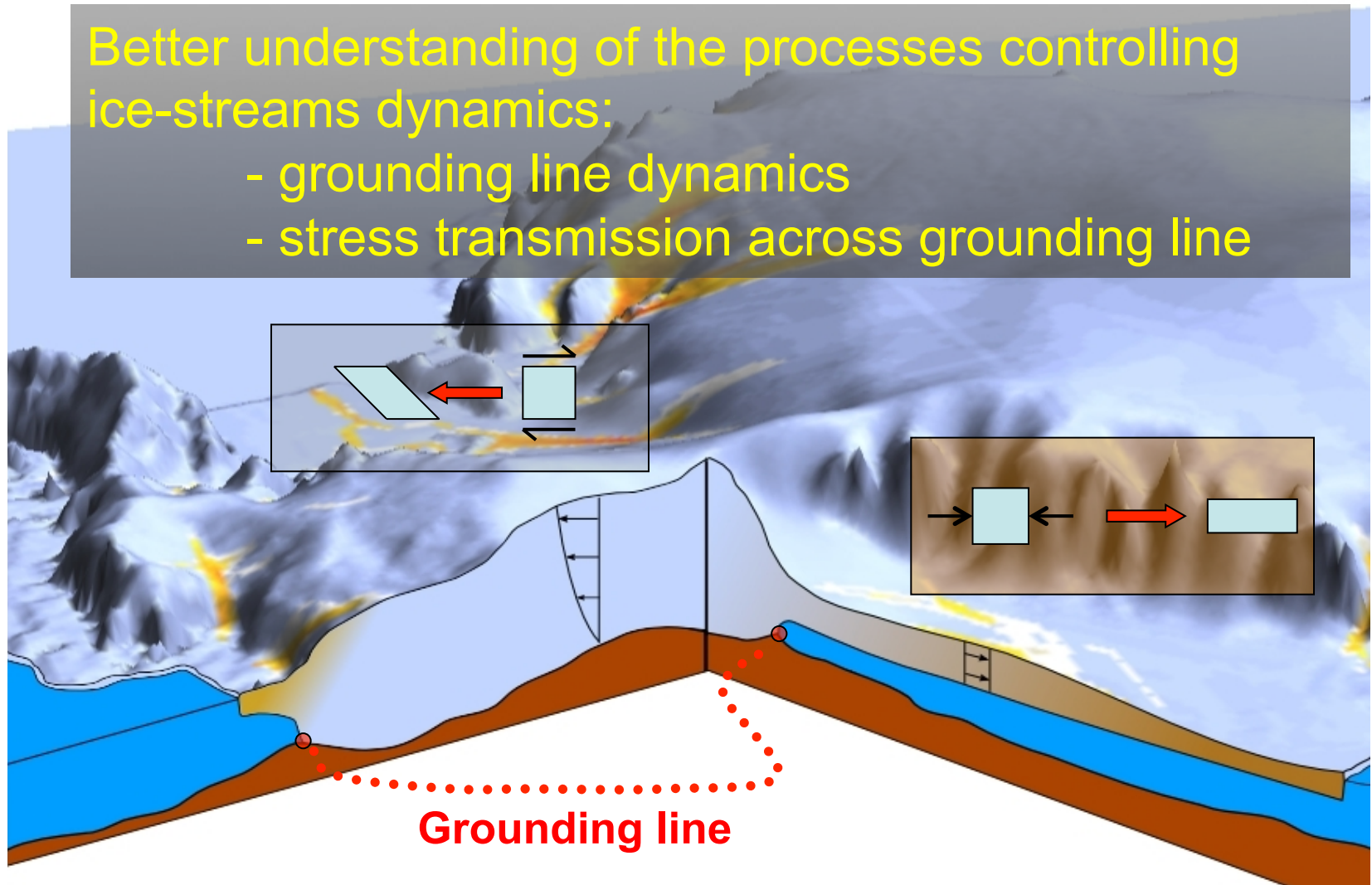


(Testut, 2000)

Transition zone

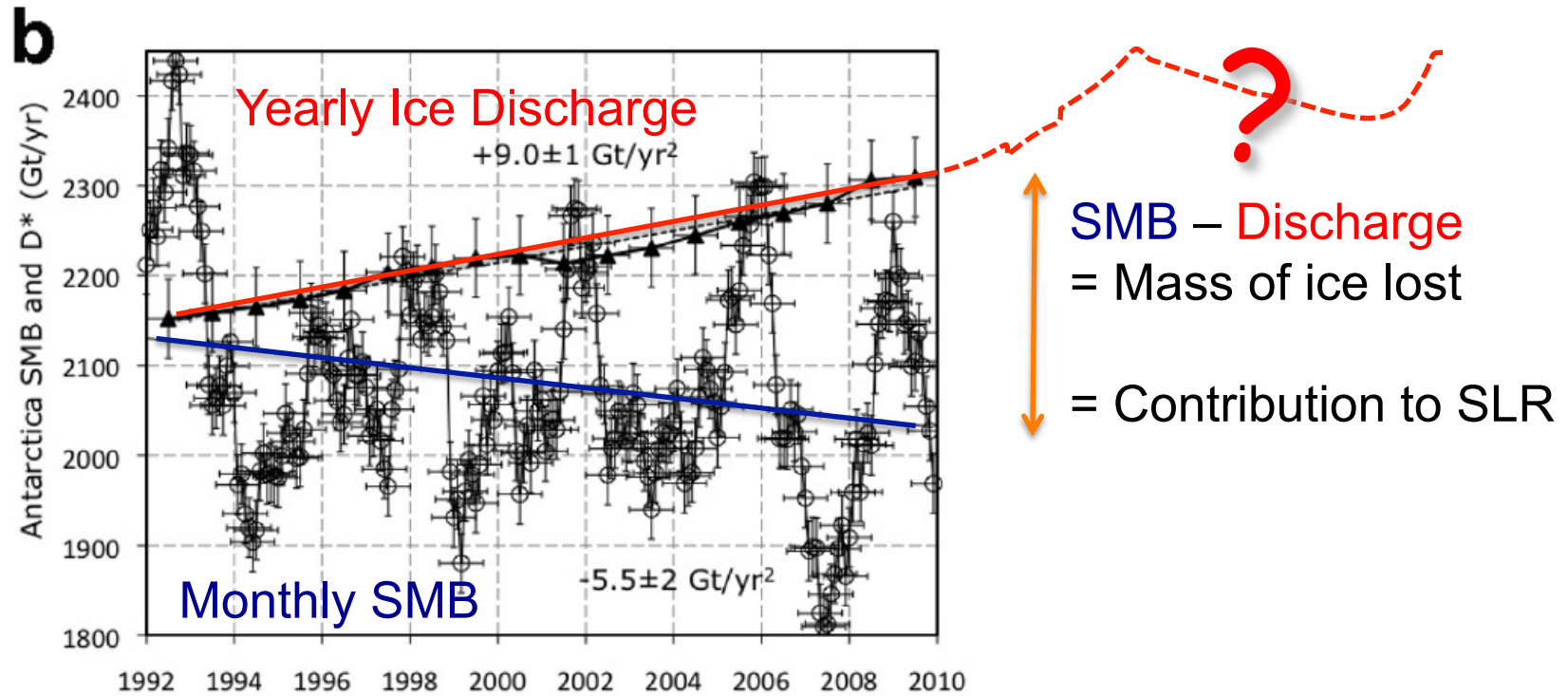
Better understanding of the processes controlling ice-streams dynamics:

- grounding line dynamics
- stress transmission across grounding line



Grounding line

Ice Discharge

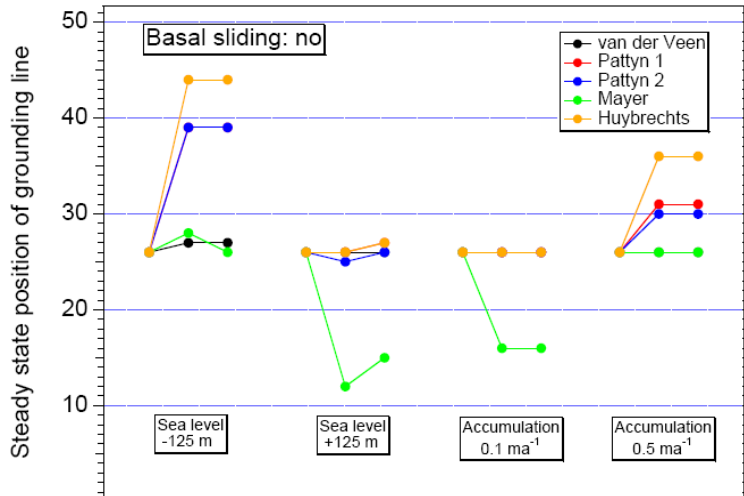


[Rignot et al., 2011]

What will be the future contribution of Ice Discharge for the next centuries?

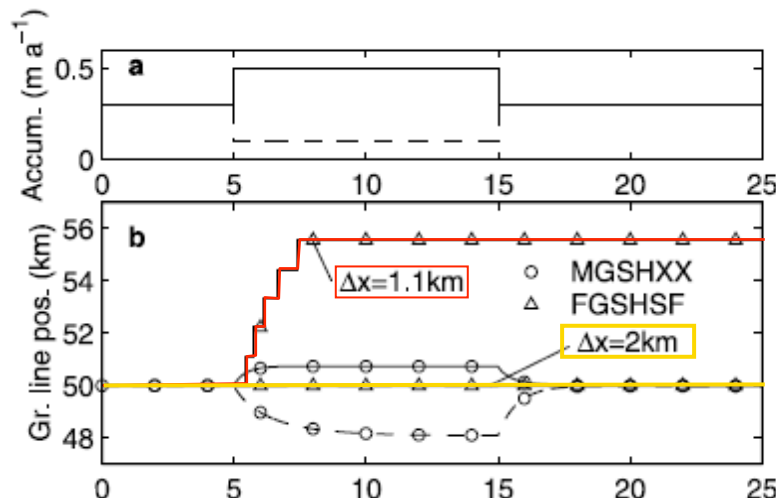
➡ **Need accurate description of the Grounding Line dynamics**

EISMINT Results



- **No consensus** on the results
- **No consensus** on how the GL should be modelled
- It is **unclear** whether these results are indicative of neutral equilibrium

[Huybrechts, 1998]



Influence of the horizontal grid size

[Vieli and Payne, 2005]

Poor ability of the model to capture the GL dynamic until recently

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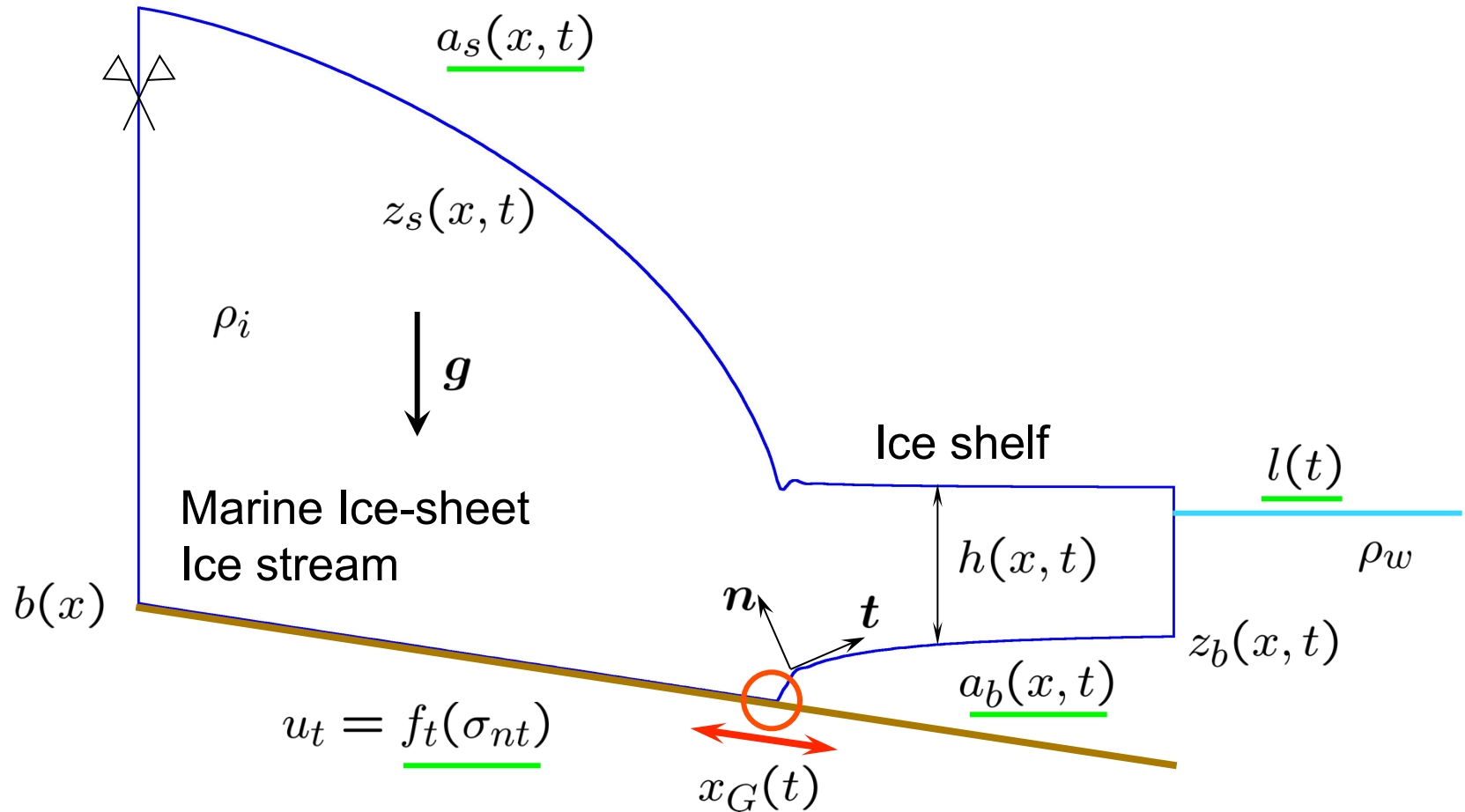
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✓ Example

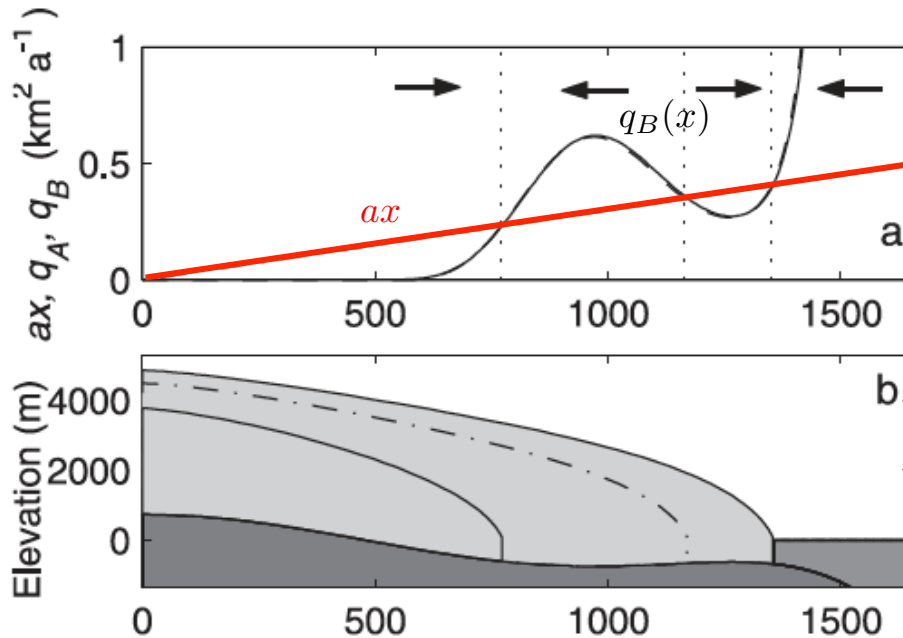
- MISMIP test

Notation / Concept



How the grounding line evolves for different scenarii ?

Schoof's solution (2007) – MISI in 2D

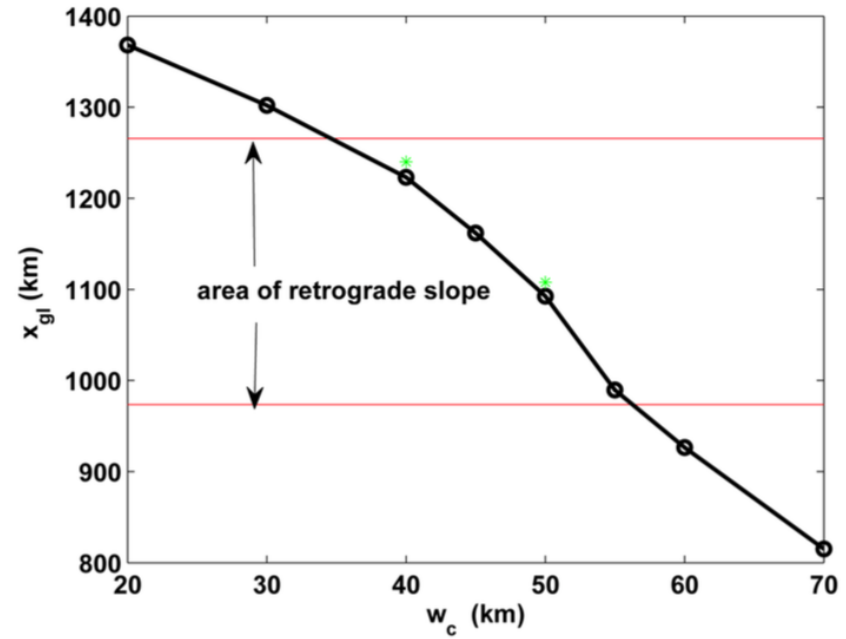
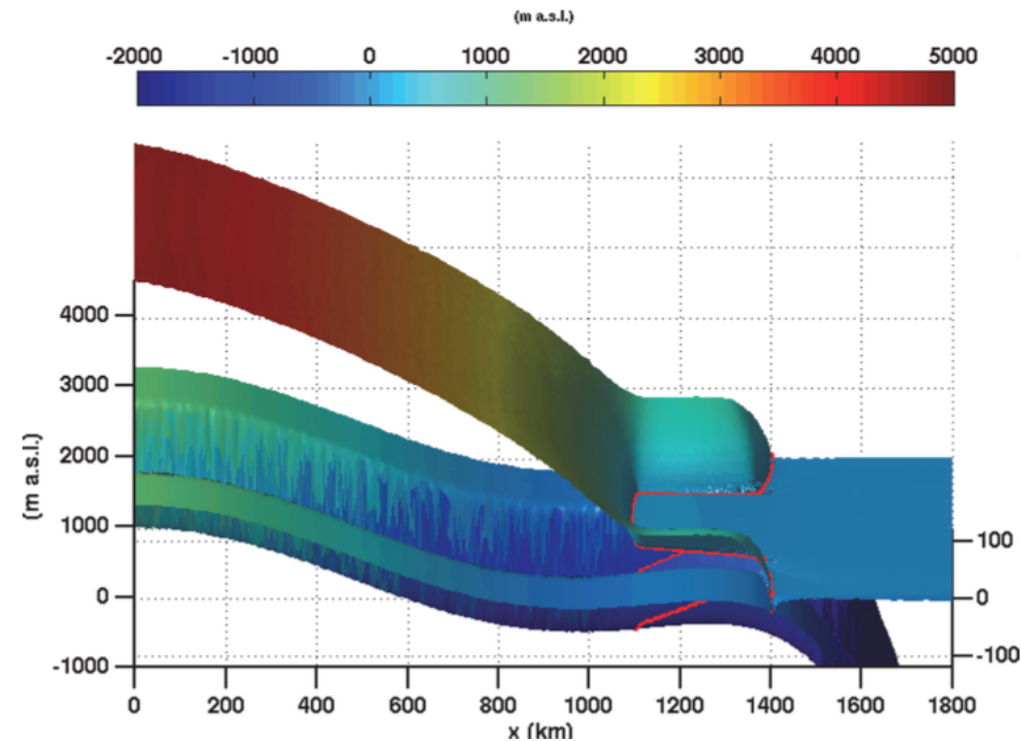


[Schoof, 2007, 2011]

$$ax_g = q_B(x_g) = \left(\frac{\bar{A}(\rho_i g)^{n+1} (1 - \rho_i / \rho_w)^n}{4^n C} \right)^{\frac{1}{m+1}} h(x_g)^{\frac{m+n+3}{m+1}}$$

- Confirms that there is no stable position of the GL on an upsloping bed
- For a given surface mass balance, gives the steady GL position (in 2D)

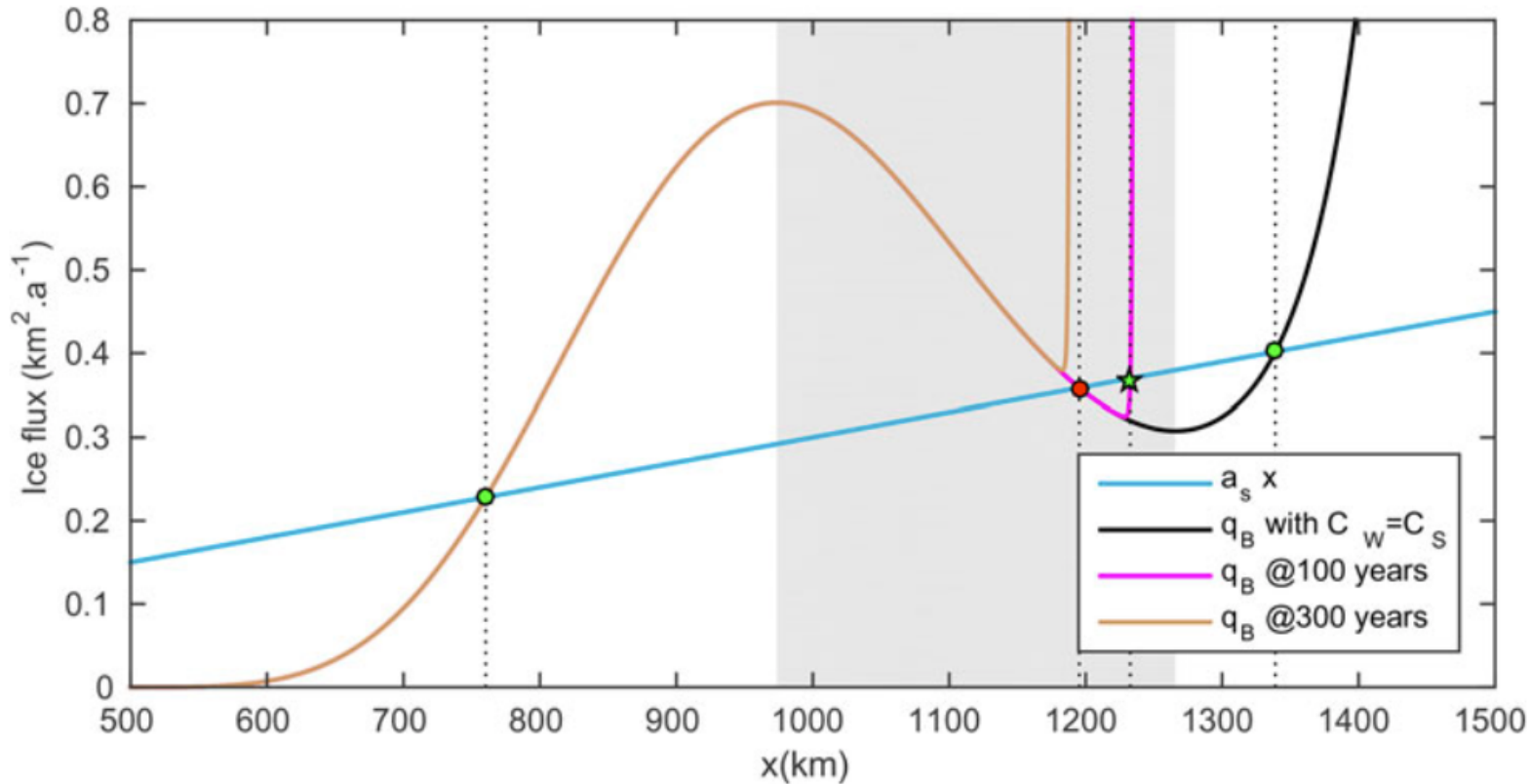
MISI in 3D



[Gudmundsson et al., 2012]

Marine ice sheets are not unconditionally unstable in two horizontal dimensions

MISI in 2D – non uniform friction



[Brondex et al., 2017]

Stable positions can be found in the MISI for non uniform basal friction field

Equations to be solved

<p>Ice Flow</p> $\left\{ \begin{array}{l} \mathbf{D} = A\tau_e^{n-1} \mathbf{S} \\ \operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{g} = 0 \\ \operatorname{div} \mathbf{u} = 0 \end{array} \right.$	<p>Top free surface</p> $\frac{\partial z_s}{\partial t} + u_x \frac{\partial z_s}{\partial x} + u_y \frac{\partial z_s}{\partial y} - u_z = a_s$	<p>Bottom free surface</p> $\left\{ \begin{array}{l} \frac{\partial z_b}{\partial t} + u_x \frac{\partial z_b}{\partial x} + u_y \frac{\partial z_b}{\partial y} - u_z = a_b \\ \text{with } z_b \geq b \end{array} \right.$
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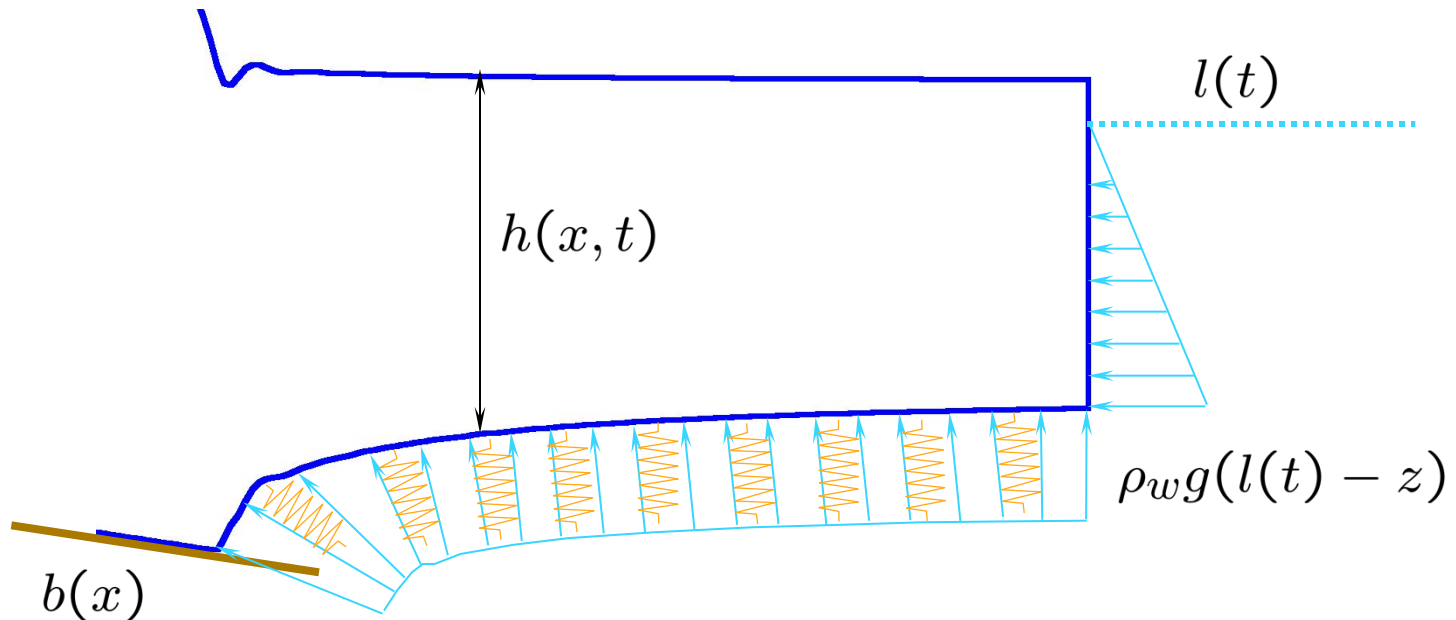
Ice - Bed contact

$$z_b = b \quad \text{and} \quad -\sigma_{nn} > p_w \quad \Rightarrow \quad \begin{array}{l} \mathbf{u} \cdot \mathbf{n} = 0 \\ u_t = f_t(\sigma_{nt}) \end{array}$$

Ice - Sea

$$\left. \begin{array}{l} z_b = b \quad \text{and} \quad -\sigma_{nn} \leq p_w \\ \text{or } z_b > b \end{array} \right\} \Rightarrow \text{Buoyancy condition}$$

Buoyancy BC



BC Stokes: if $z_b(x, t) > b(x)$

$$\sigma_{nn}(x) = \rho_w g (l(t) - z_b(x, t)) \quad \text{and} \quad \sigma_{nt}(x) = 0$$

$$z_b(x, t) = z_b(x, t - dt) + u_n \sqrt{1 + (dz_b/dx)^2} dt$$

$$\Rightarrow \sigma_{nn}(x) = \rho_w g (l(t) - z_b(x, t - dt)) - \rho_w g \sqrt{1 + (dz_b/dx)^2} dt \cdot u_n$$

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✓ Example

- MISMIP test

The basal boundary

Condition applied on the basal boundary depend if

- the ice is in contact with the bedrock
- or the ice is in contact with the sea

The limit between grounded and floating parts (the GL) is unknown and solution of the contact problem

Add a `Mask` variable (only on the basal surface) which tells if grounded, floating or at the GL

`Mask` = 1 if grounded

`Mask` = -1 if floating

`Mask` = 0 if at the GL

The basal boundary

In Elmer, the use of a **conditional Dirichlet** condition allows to deal with this evolving limit.

Example in the SIF:

```
Velocity 1 = Real 0.0  
Velocity 1 Condition = Variable Mask  
Real MATC "tx + 0.5"
```

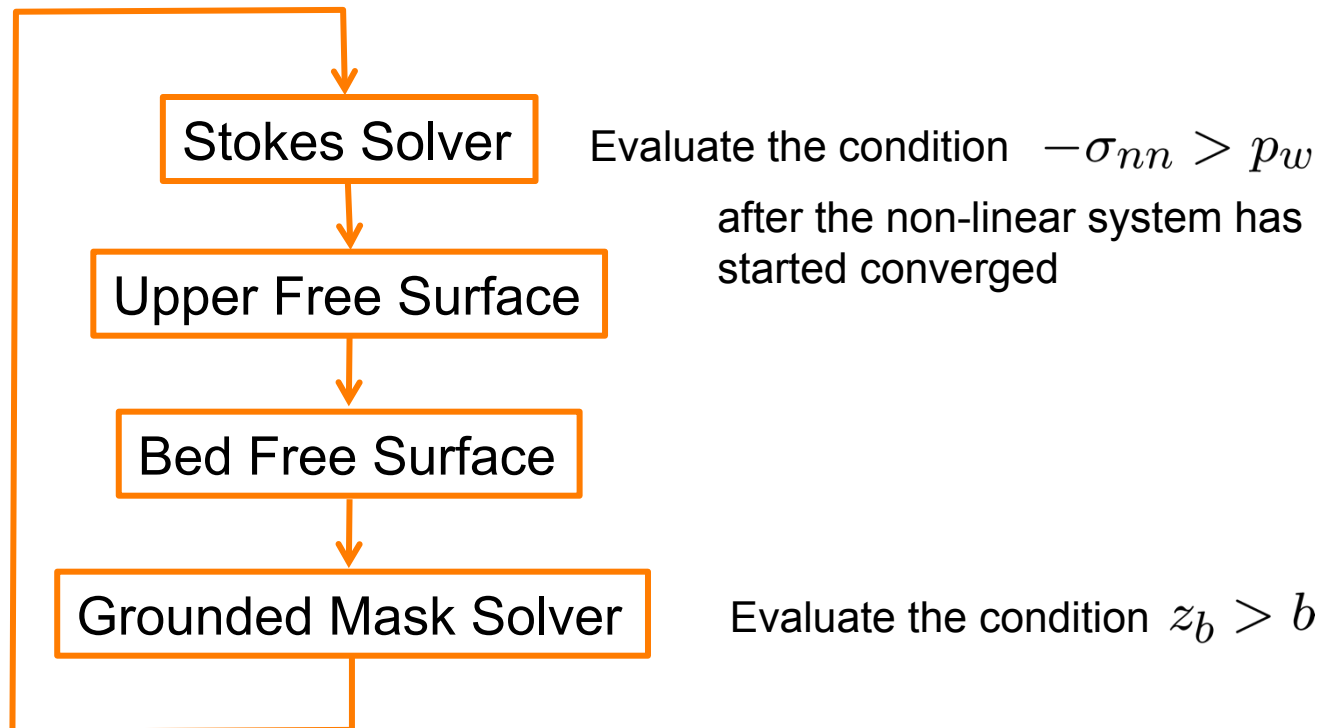
Mask = -1 → the Dirichlet BC is not applied

Mask = 1 or 0 → the Dirichlet BC $u_n = 0$ is applied

The contact problem

$z_b = b$ and $-\sigma_{nn} > p_w$ \Rightarrow Ice - Bed condition

$z_b = b$ and $-\sigma_{nn} \leq p_w$
or $z_b > b$ \Rightarrow Ice - Sea condition



The contact problem

The condition $-\sigma_{nn} > p_w$ is in fact evaluated using nodal force (and not stress)

- the force exerted by the ice on the bed is given by the residual of the Stokes solution

In the Stokes solver

```
Exported Variable 1 = Flow Solution Loads[Stress Vector:2 CEQ Residual:1]  
Calculate Loads = Logical True
```

- the nodal water force is the integrated water pressure with respect to the surface element

add a new solver to integrate the water pressure

Water force

```
SUBROUTINE GetHydrostaticLoads( Model,Solver,dt,TransientSimulation )

VariableValues = 0.0_dp

DO t = 1, Solver % NumberOfActiveElements
  Element => GetActiveElement(t)
  IF (ParEnv % myPe .NE. Element % partIndex) CYCLE
  n = GetElementNOFNodes()

  BC => GetBC( Element )
  pwt(1:n) = -1.0 * ListGetReal(BC, 'External Pressure', n, &
    Element % NodeIndexes , GotIt)
  CALL GetElementNodes( Nodes )
  IP = GaussPoints( Element )
  DO p = 1, IP % n

    stat = ElementInfo( Element, Nodes, IP % U(p), IP % V(p), &
      IP % W(p), detJ, Basis, dBasisdx, ddBasisddx, .FALSE.)
    s = detJ * IP % S(p)

    Normal = NormalVector( Element, Nodes, IP % U(p), IP % V(p), .TRUE.)
    pwi = SUM(pwt(1:n)*Basis(1:n))
    PwVector(1:DIM) = pwi * Normal(1:DIM)

    DO i = 1, n
      Nn = Permutation(Element % NodeIndexes(i))
      DO j = 1, DIM
        VariableValues(DIM*(Nn-1)+j) = VariableValues(DIM*(Nn-1)+j) + PwVector(j) *
s * Basis(i)
      END DO
    END DO
  END DO
  IF ( ParEnv % PES>1 ) CALL ParallelSumVector( Solver % Matrix, VariableValues )
!-----
END SUBROUTINE GetHydrostaticLoads
!-----
```

The bed boundary condition

```
Boundary Condition 1
Target Boundaries = 1
  Body Id = 3

  Normal-Tangential Velocity = Logical True
  Flow Force BC = Logical True
!
! Bedrock conditions
!
Slip Coefficient 2 = Variable Coordinate 1
  Real Procedure "ElmerIceUSF" "SlidCoef_Contact"
  Sliding Law = String "Weertman"
  Weertman Friction Coefficient = Real $C
  Weertman Exponent = Real $(1.0/n)
  Weertman Linear Velocity = Real 1.0

Grounding line Definition = String "Discontinuous"

Velocity 1 = Real 0.0
Velocity 1 Condition = Variable GroundedMask
  Real MATC "tx + 0.5"
!
! Shelf conditions
!
External Pressure = Variable Coordinate 2
  Real Procedure "ElmerIceUSF" "SeaPressure"

Slip Coefficient 1 = Variable Coordinate 2
  Real Procedure "ElmerIceUSF" "SeaSpring"
```

The variable GroundedMask is updated in this User Function SlidCoef_Contact

See note after

Will only apply if the Dirichlet condition Velocity 1 = 0 is not applied

End



The user function SlidCoef_Contact

Test the contact condition:

```
Normal = NormalValues(DIM*(NormalPerm(jj)-1)+1 : DIM*NormalPerm(jj))
Fwater = Hydro(DIM*(HydroPerm(jj)-1)+1 : DIM*HydroPerm(jj))
Fbase = ResidValues((DIM+1)*(ResidPerm(jj)-1)+1 : (DIM+1)*ResidPerm(jj)-1)
```

```
comp = ABS( SUM( Fwater * Normal ) ) - ABS( SUM( Fbase * Normal ) )
```

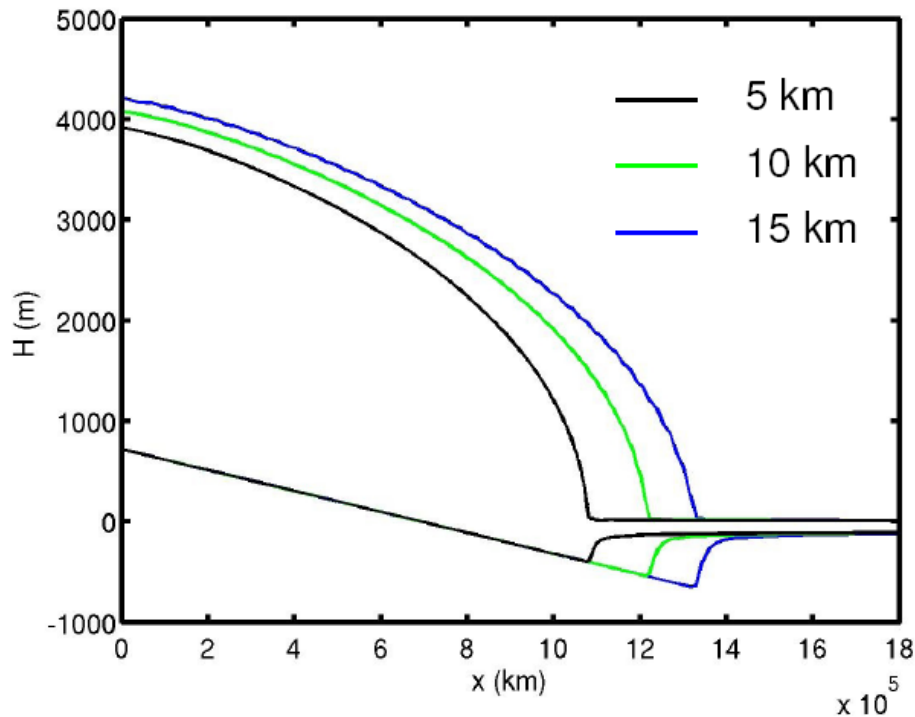
```
IF (comp >= 0.0_dp) GroundedMask(Nn) = -1.0_dp
```

and return the sliding coefficient:

- appropriate if grounded
- 0 if floating

```
cond = GroundedMask(GroundedMaskPerm(nodenumbr))
IF (cond > -0.5_dp) THEN
    Bdrag = Sliding_weertman(Model, nodenumbr, y)
ELSE
    Bdrag = 0.0_dp
END IF
```

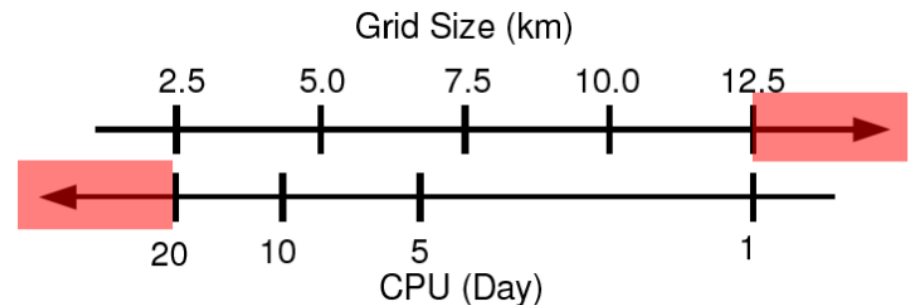
Sensitivity to the grid size



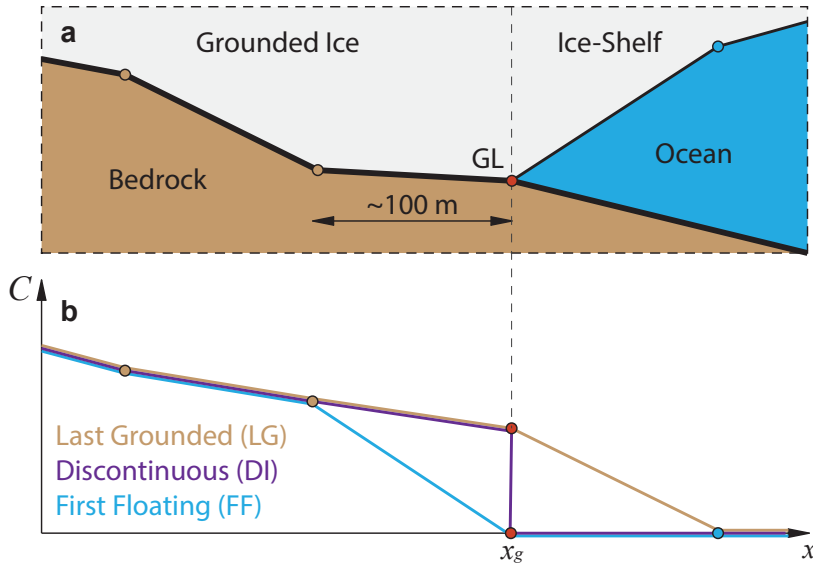
Steady for different horizontal grid sizes

Strong influence of the grid size !

Problem : CPU cost !



Interpolation of the friction

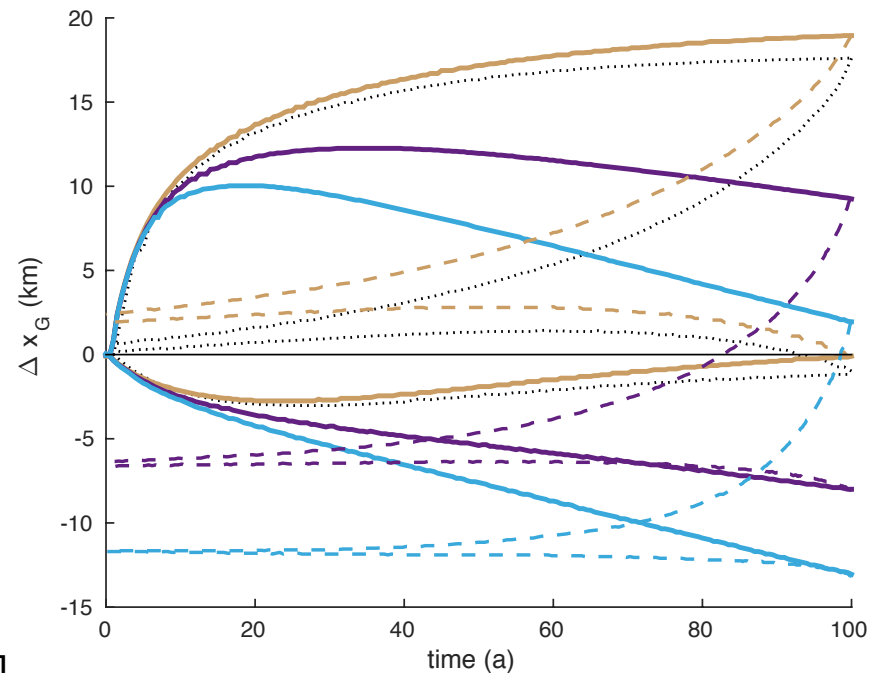


Use Discontinuous!

Or better: use a water pressure dependant friction law (see presentation on friction law)

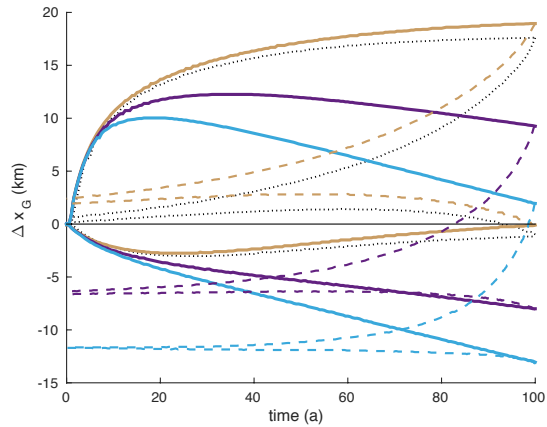
[Gagliardini et al., 2016]

MISMIP3d – $N_y = 20$

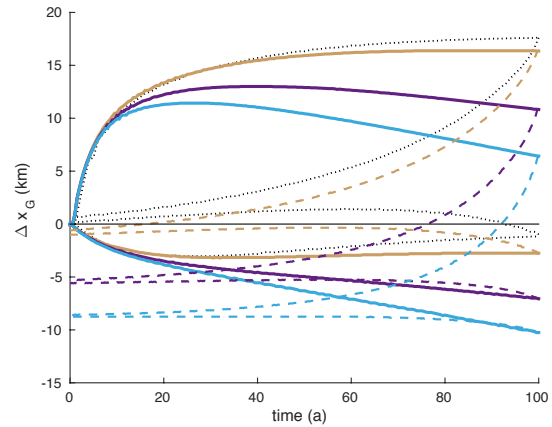


Interpolation of the friction

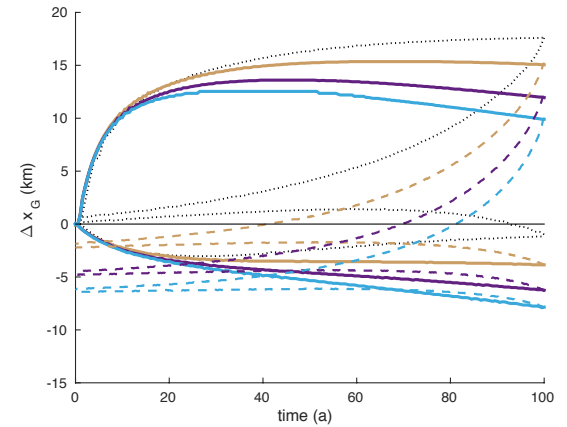
$N_y = 20$



$N_y = 40$

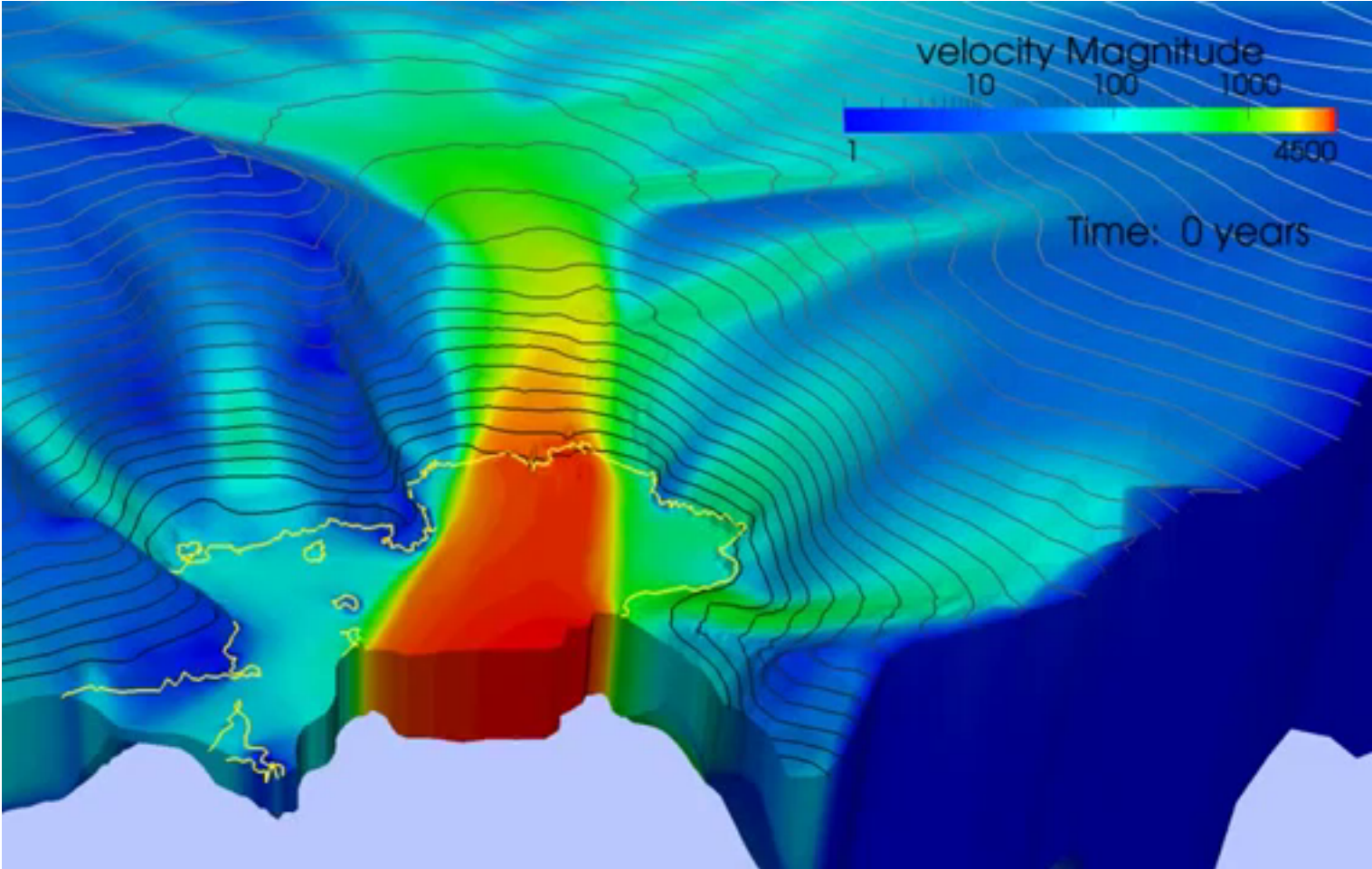


$N_y = 80$



[Gagliardini et al., 2016]

PIG example (Favier et al., 2014)



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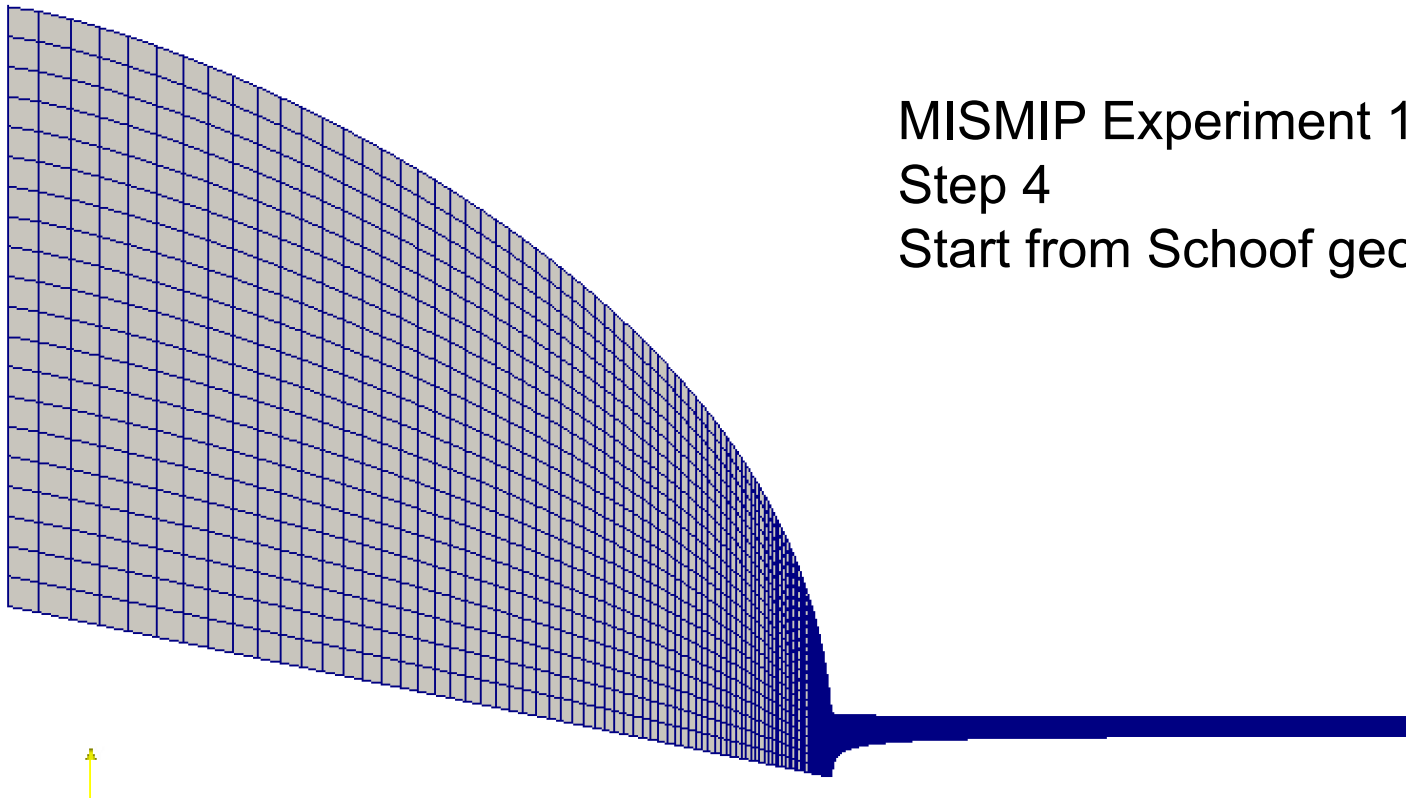
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Example GL_MISMIP



MISMIP Experiment 1a
Step 4
Start from Schoof geometry

<http://elmerice.elmerfem.org/wiki/doku.php?id=problems:groundingline>

[ELMER_TRUNK]/elmerice/Tests/GL_MISMIP

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- ~ 17 papers published so far using the contact problem implemented for the Stokes equations to solve the GL dynamics.