

Elmer/Ice advanced Workshop

22-24 November 2017

Basal Conditions (Friction laws & Hydrology)

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LabEx OSUG 2020



Basal Conditions

✓ The Physics

- Sliding at the base of glacier
- The role of basal water
- Different drainage systems

✓ Friction laws and Hydrology

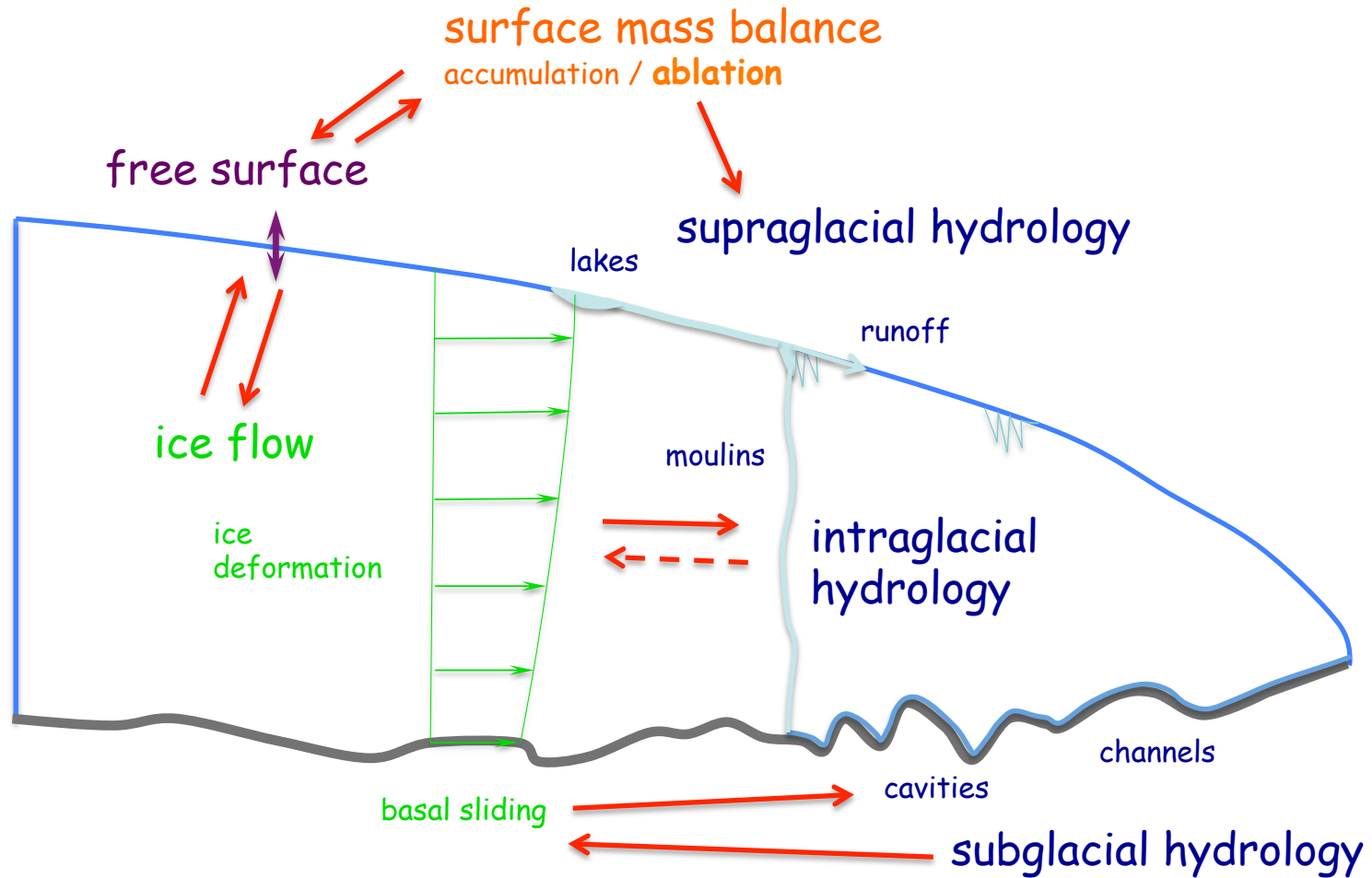
- Linear friction law
- Weertman type friction law
- Water-pressure dependant friction laws
- Double continuum hydrology model
- GlaDS model

✓ Implementation in Elmer/Ice

- Various friction laws

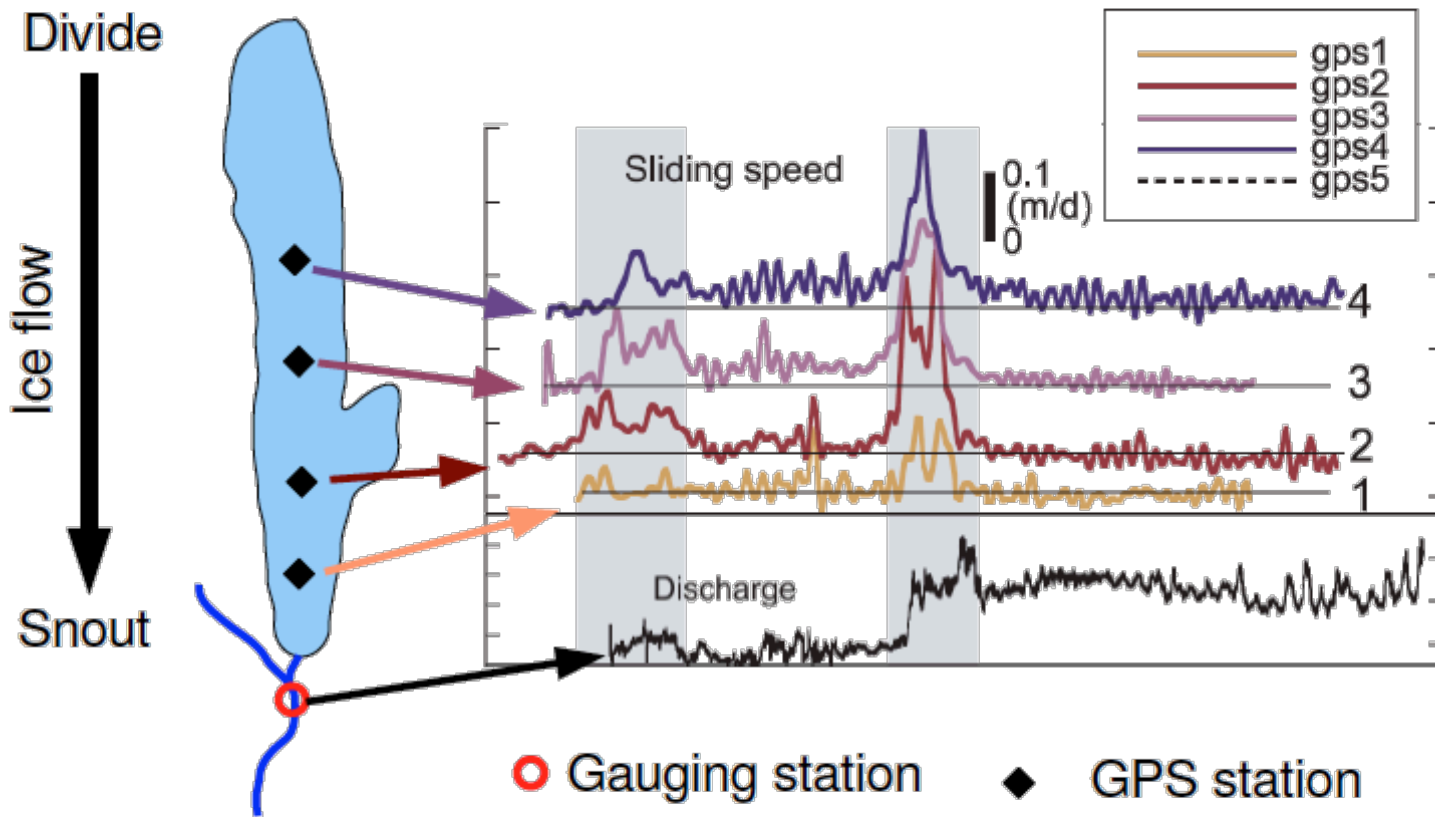
✓ Examples

Coupling water / friction and more...



Relationship between velocity and water

Velocity and discharge measurements on Bench glacier (Alaska)

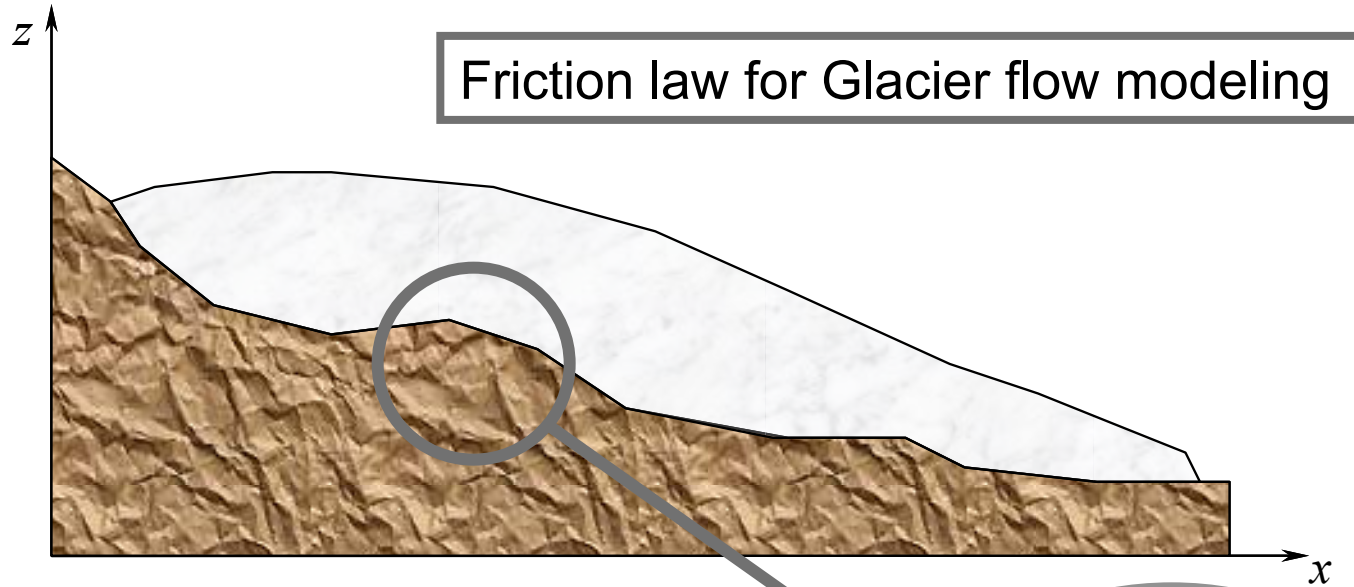


@deFleurian

Figure adapted from [Anderson et al., 2004]

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Scale of interest



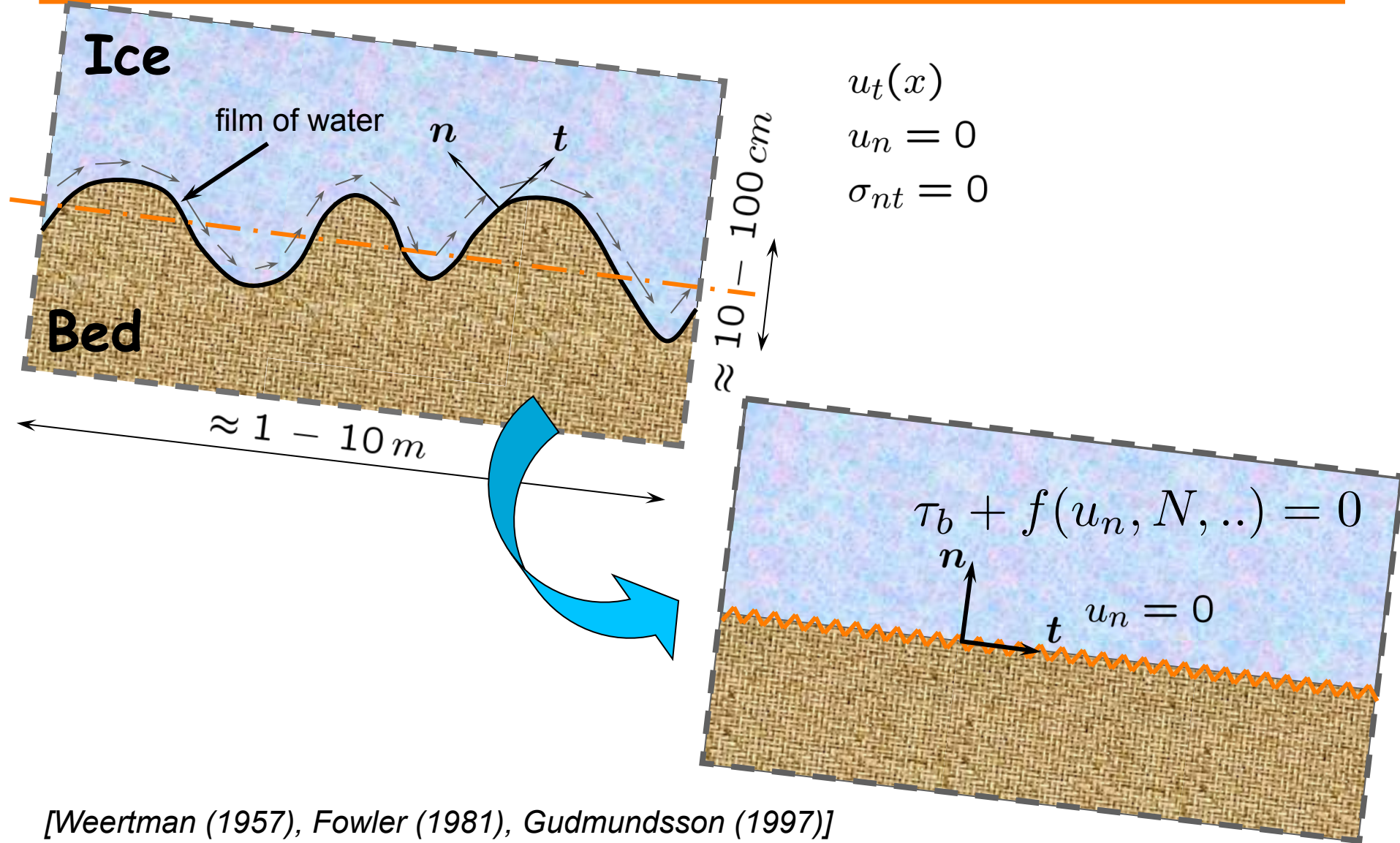
Bedrock roughness

Friction law

$$\tau_b + f(u_n, N, ..) = 0 \quad ?$$

$$N = p_i - p_w (= -\sigma_{nn} - p_w)$$

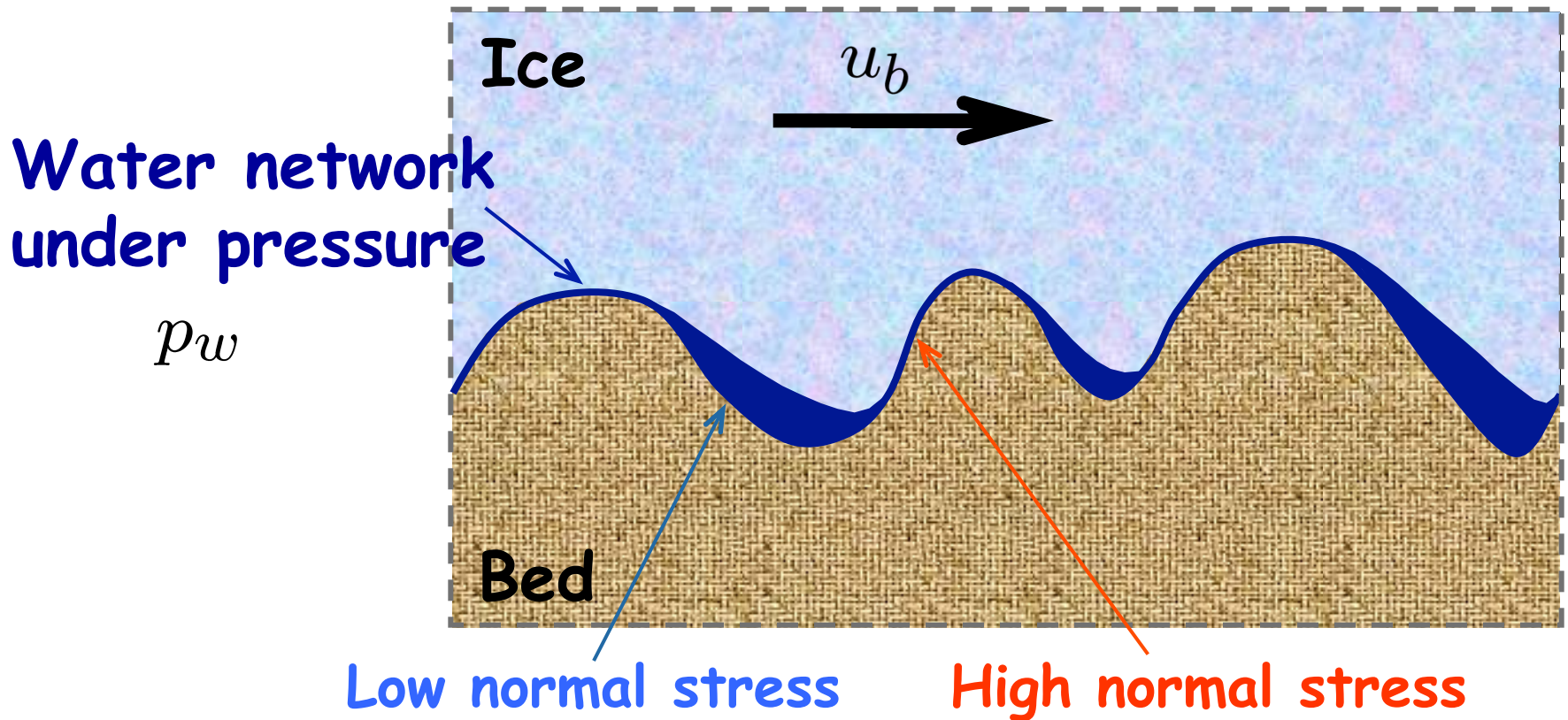
Concept of friction law



[Weertman (1957), Fowler (1981), Gudmundsson (1997)]

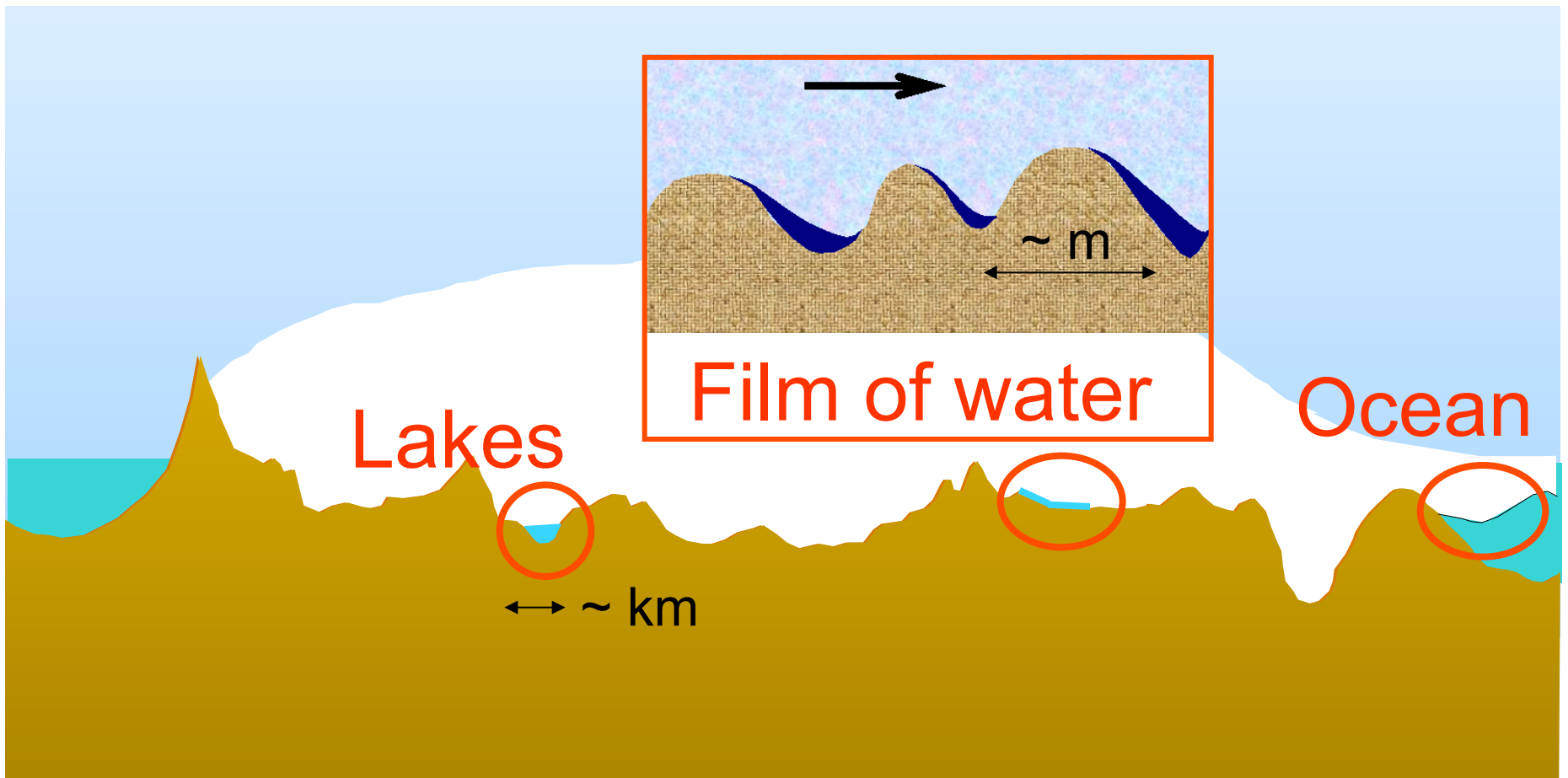
How water enhances glacier sliding

If water pressure and/or velocity increase



➡ How does it affect the form of the sliding law ?

Water at the base of glaciers

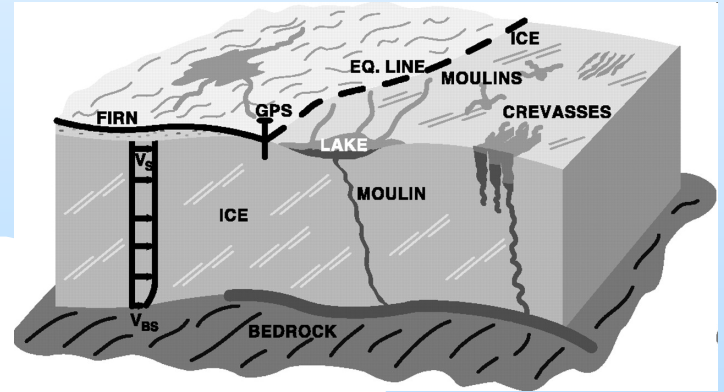


Effective pressure: $N = -\sigma_{nn} - p_w$

Why is there (liquid) water?

negative surface temperature

positive surface temperature



Deformational heat

Increase of T

melting point

friction heat

geothermal heat flux

Two types of drainage systems

▶ Inefficient drainage systems

low conductivities
high water pressure
distributed systems }

Two types of drainage systems

► Inefficient drainage systems

low conductivities
high water pressure
distributed systems

► Sediment layer

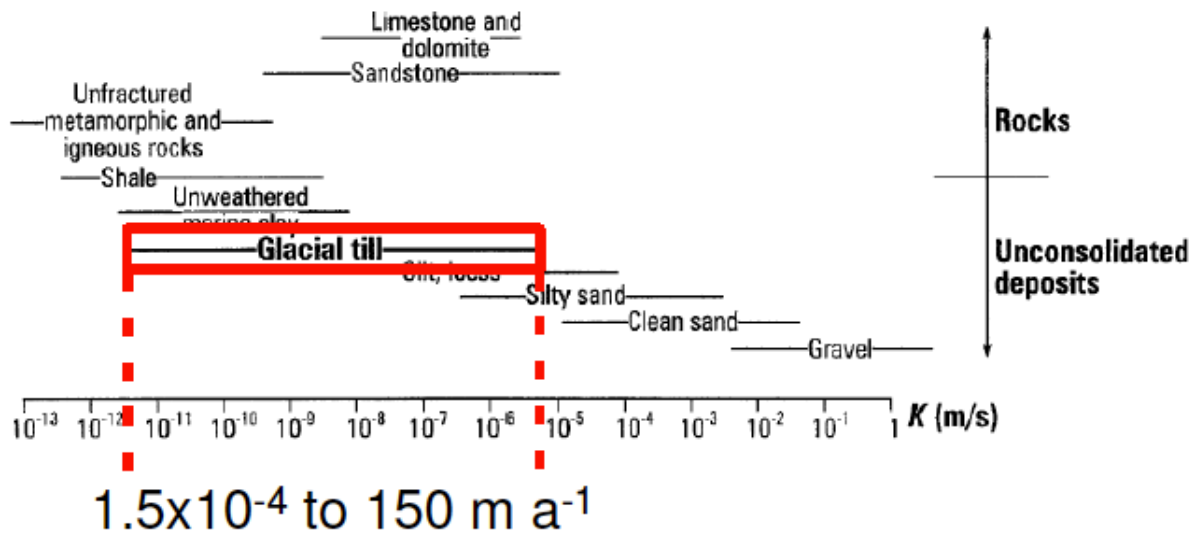


Figure from [Freeze and Cherry, 1979]

Two types of drainage systems

▶ Inefficient drainage systems

low conductivities
high water pressure
distributed systems

- ▶ Sediment layer
- ▶ Linked cavities

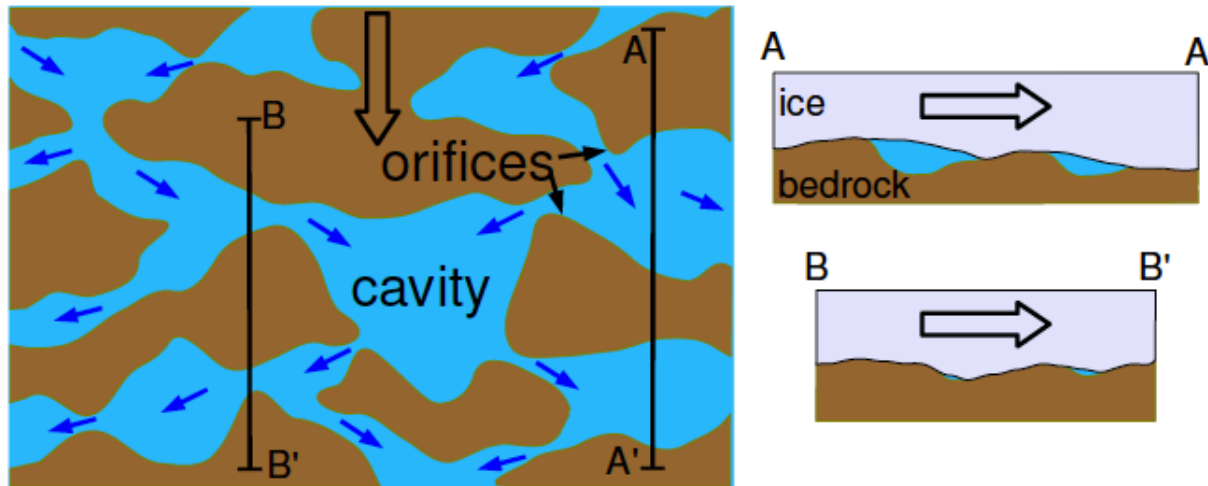


Figure from [Kamb, 1987]

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Two types of drainage systems

▶ Inefficient drainage systems

low conductivities
high water pressure
distributed systems

- ▶ Sediment layer
- ▶ Linked cavities
- ▶ Water film

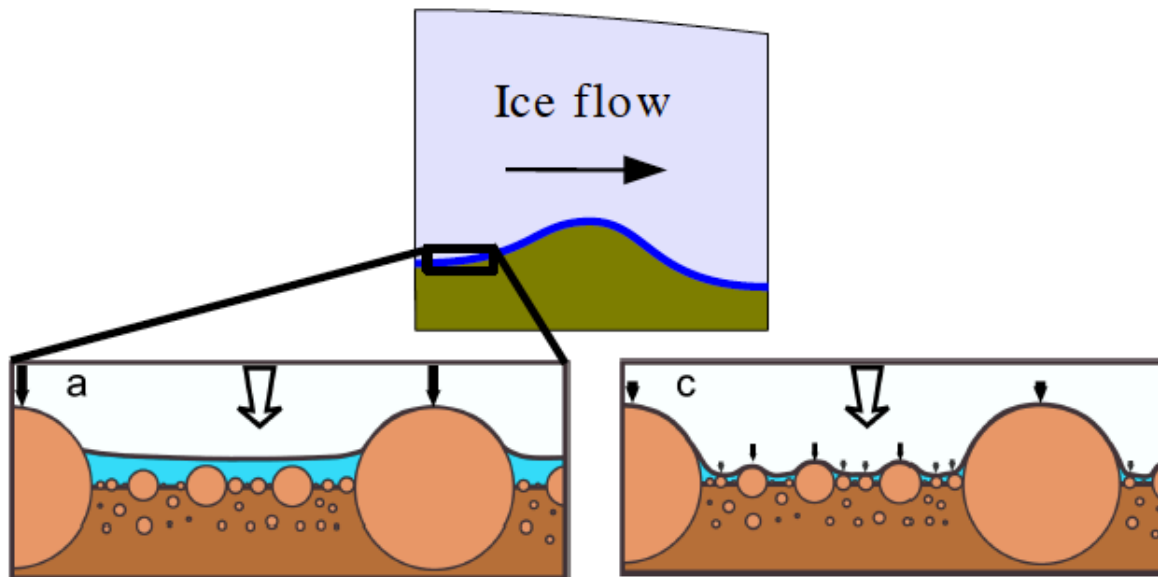


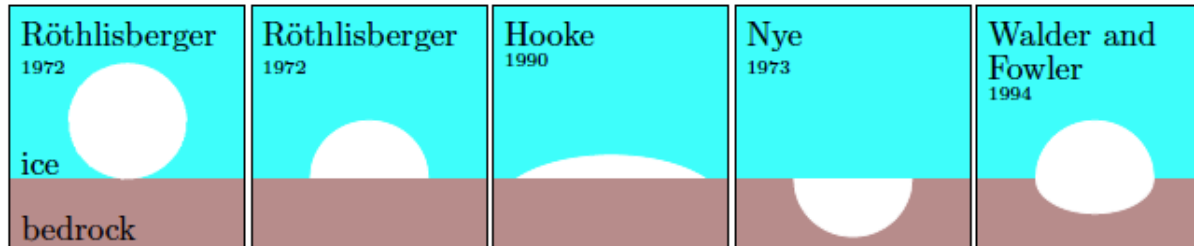
Figure from [Creys and Schoof, 2009]

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Two types of drainage systems

- ▶ Inefficient drainage systems
- ▶ Efficient drainage systems

high conductivities
low water pressure
localized systems } ▶ Channels

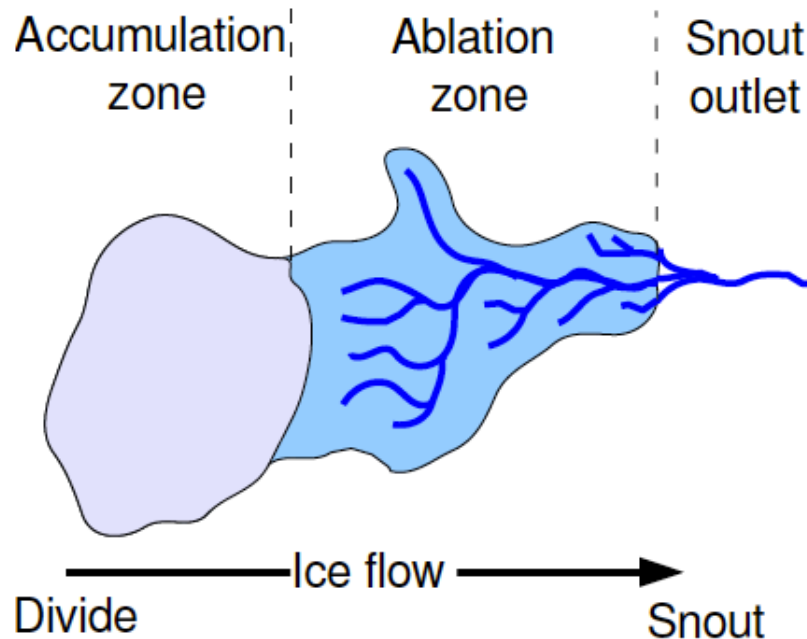


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Two types of drainage systems

- ▶ Inefficient drainage systems
- ▶ **Efficient drainage systems**

high conductivities
low water pressure
localized systems } ▶ Channels



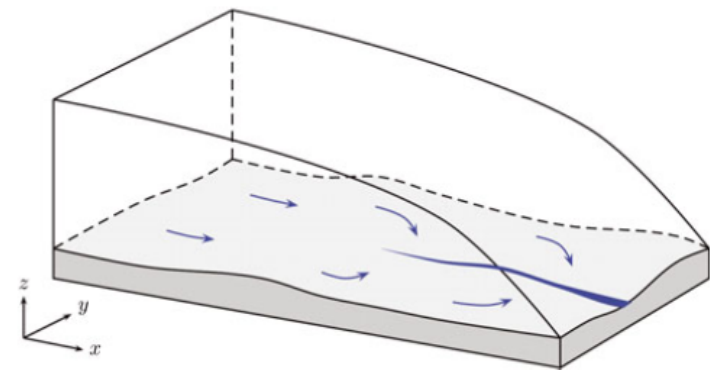
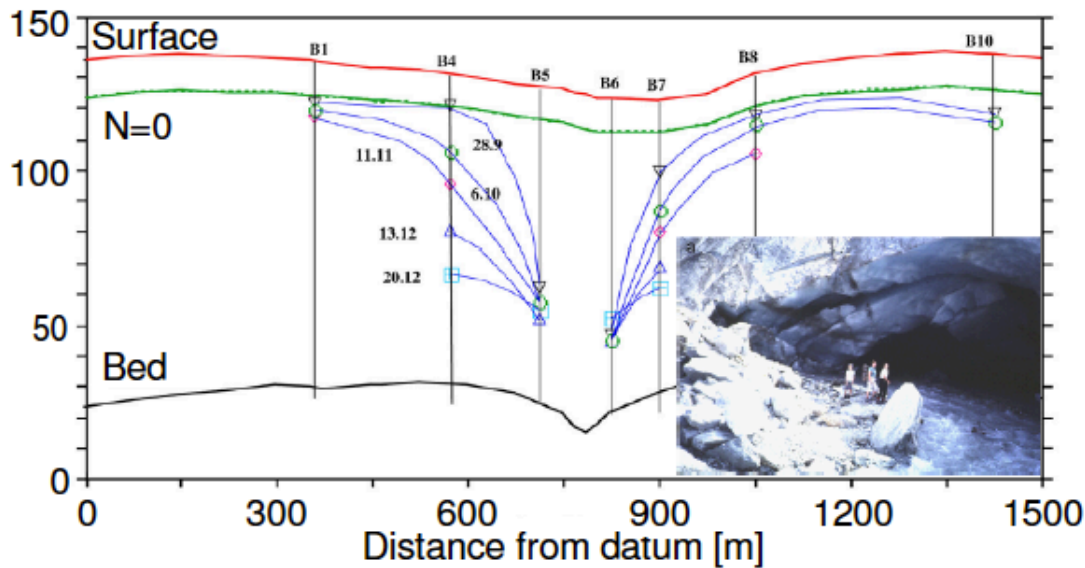
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Two tightly-related systems

The link between inefficient and efficient systems is observable in the field

- ▶ As a spatial variation

Water load observed across Breidamerkujökull during Autumn Winter transition



From [Hewitt, 2011]

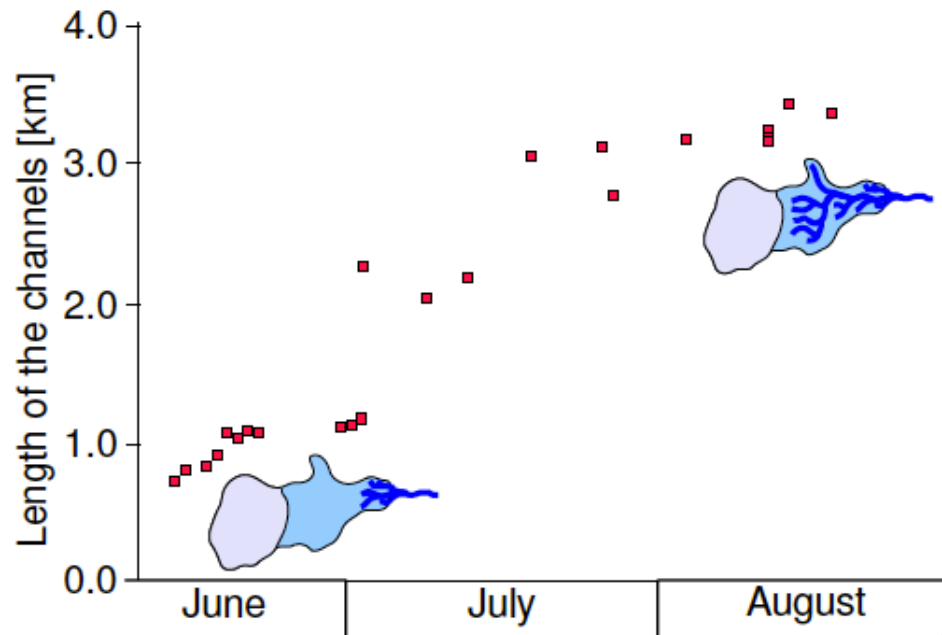
Figure adapted from [Boulton et al., 2007]

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Two tightly-related systems

The link between inefficient and efficient systems is observable in the field

- ▶ As a spatial variation
- ▶ As a temporal evolution



Channel length evolution during summer on Arolla glacier (Switzerland)

Figure adapted from [Nienow et al., 1998]

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✓ Implementation in Elmer/Ice

- Various friction laws

✓ Examples

Friction laws

A friction law is a relation that gives the basal shear stress as a function of the sliding velocity and other variables (effective pressure, ...):

$$\left\{ \begin{array}{l} \tau_b = \mathbf{t} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \\ u_b = \mathbf{u} \cdot \mathbf{t} \\ \sigma_{nn} = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \\ N = -\sigma_{nn} - p_w \end{array} \right.$$

$$\tau_b + f(u_b, N, \dots) = 0$$

Linear friction laws:

$$\tau_b + \beta u_b = 0 \quad \beta \text{ Drag factor or friction parameter}$$

$$u_b + C\tau_b = 0 \quad C \text{ Sliding parameter}$$

Weertman type friction law (non-linear):

$$\tau_b + (u_b/A_s)^{1/n} = 0$$

A_s Sliding parameter
 n Glen's flow law exponent

Friction laws – water pressure dependant

The friction should depend on the water pressure $N = -\sigma_{nn} - p_w$

Raymond and Harrison, 1987, Bindschadler (1983), Budd et al. (1984) :

$$u_b + k\tau_b^p N^{-q} = 0 \quad \text{e.g. } p = n = 3, q = 1$$

Iken's bound, 1981:

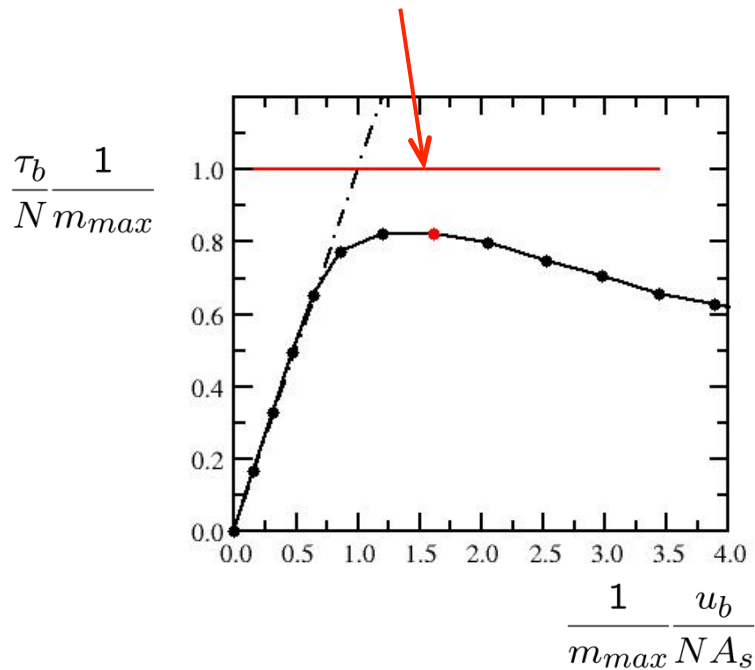
$$\tau_b/N < m_{\max} \quad m_{\max} \text{ the maximum up-slope of the bed}$$

not fulfilled by the previous law

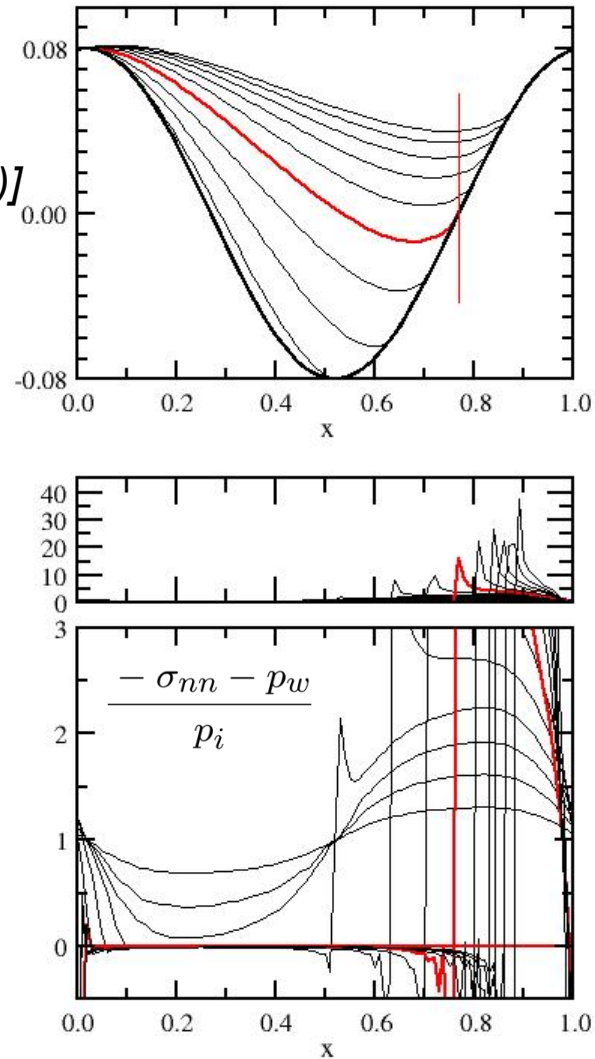
Illustration of Iken's bound

Iken's bound : $\frac{\tau_b}{N} \leq m_{max}$

[Iken (1981), Fowler (1986), Schoof (2005)]



[Gagliardini et al., 2007]



Coulomb-type friction law

Schoof (2005), Gagliardini et al., 2007:

$$\frac{\tau_b}{N} + C \left(\frac{\chi}{1 + \alpha \chi^m} \right)^{1/n} = 0$$

where

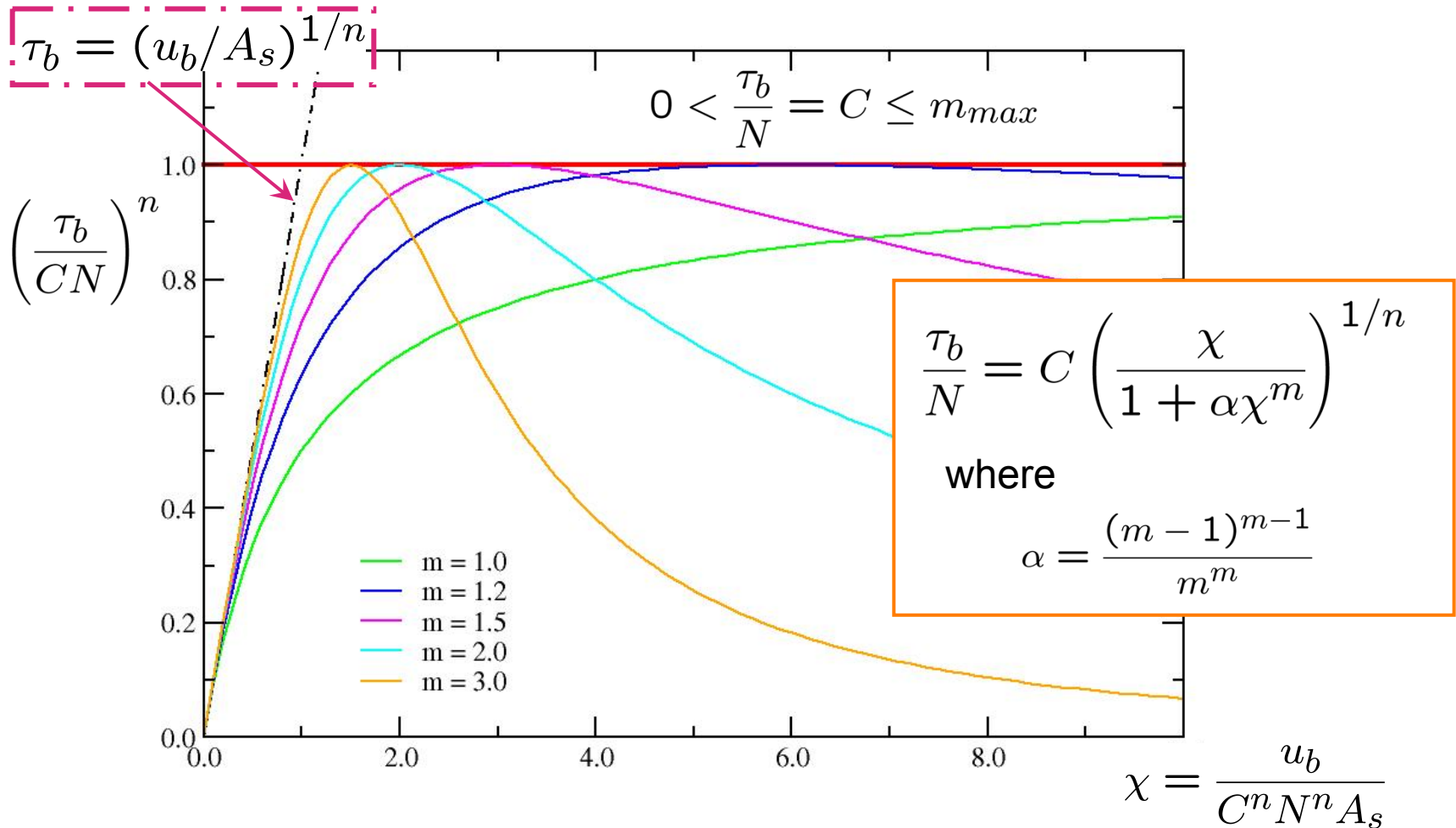
$$\begin{cases} \chi = \frac{u_b}{C^n N^n A_s} \\ \alpha = \frac{(m-1)^{m-1}}{m^m} \end{cases}$$

Fulfills the Iken's bound: $0 < \frac{\tau_b}{N} \leq C \leq m_{max}$

3 parameters:

$$\begin{cases} A_s & [mMPa^{-n}a^{-1}] & \text{Sliding parameter in absence of cavitation} \\ C \leq m_{max} & & \text{Maximum value of } \tau_b/N \\ m \geq 1 & & \text{Post-peak exponent} \end{cases}$$

Coulomb-type friction law



[Schoof (2005), Gagliardini et al., 2007]

Tsai Coulomb law

$$\tau_b + \min((u_b/A_s)^{1/n}; fN) = 0$$

[Tsai et al., 2015]

Fulfills the Iken's bound: $0 < \frac{\tau_b}{N} \leq f$

2 parameters:

$$\begin{cases} A_s & [mMPa^{-n}a^{-1}] \\ f & \end{cases} \begin{array}{l} \text{Sliding parameter in absence of cavitation} \\ \text{friction coefficient} \end{array}$$

Comparison

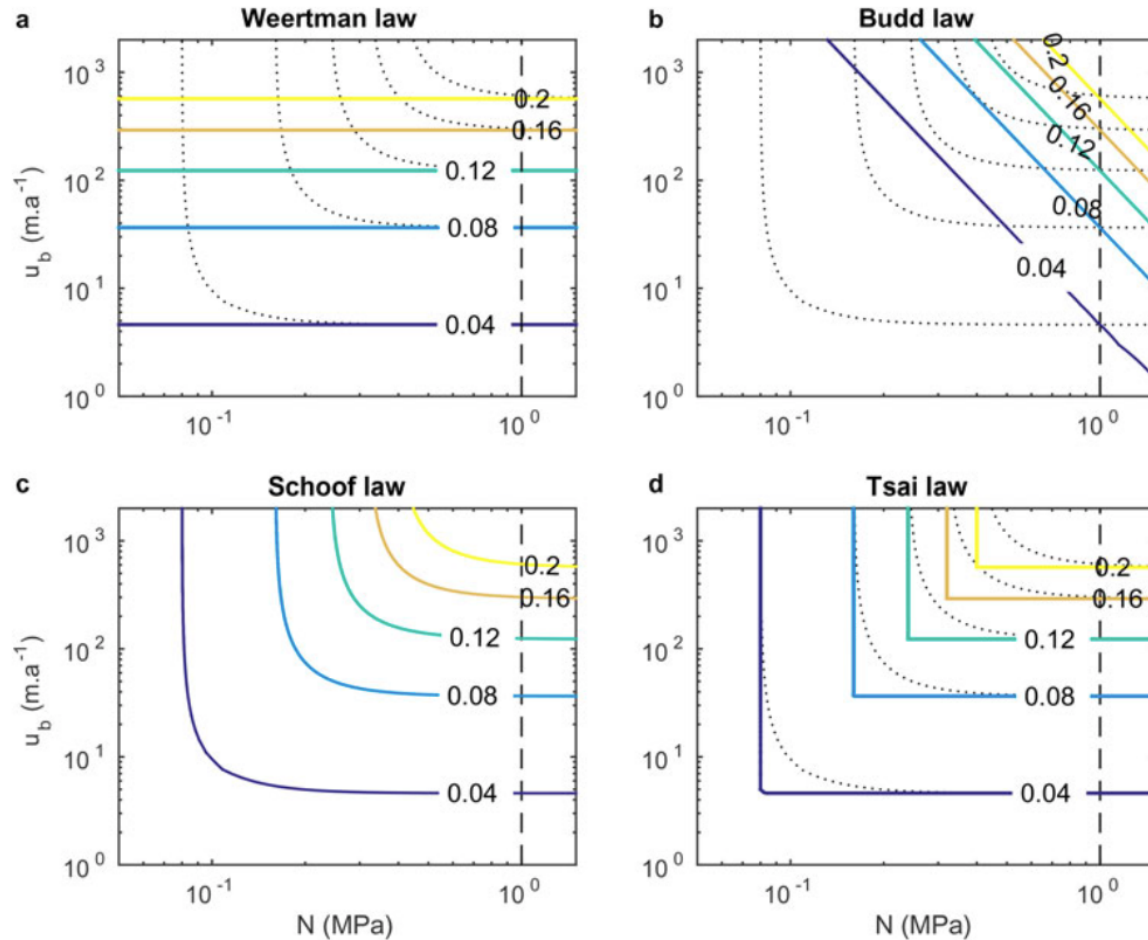
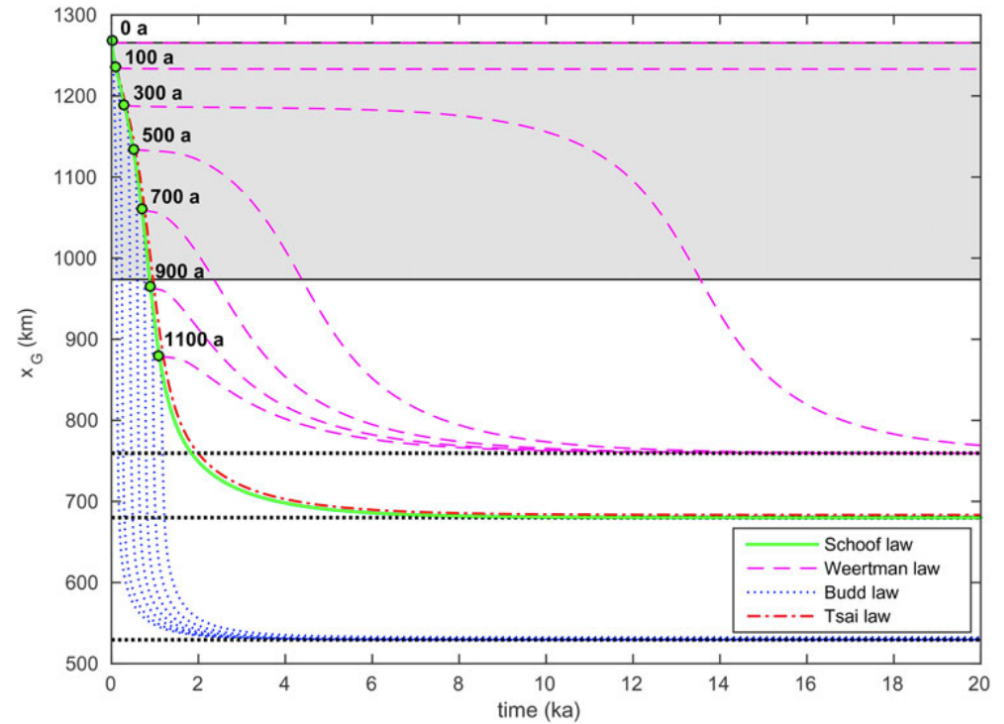
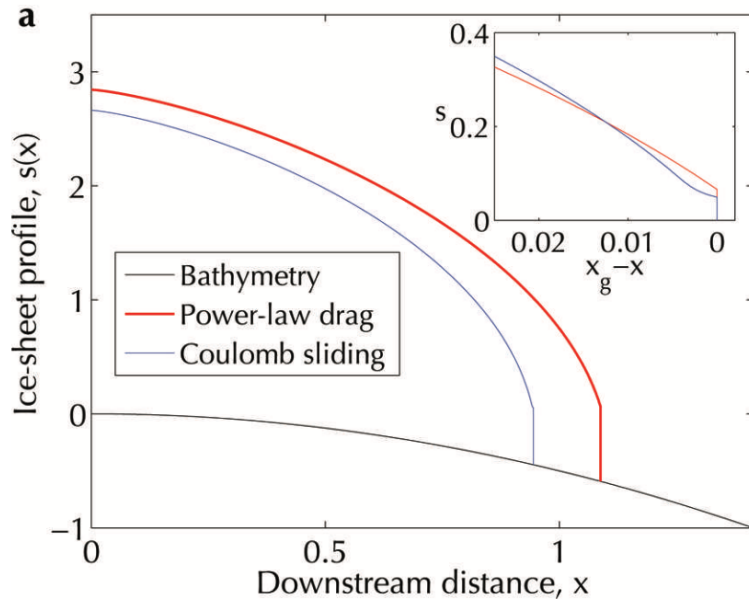


Fig. 1. Iso-values of τ_b ranging from 0.04 to 0.2 MPa given in the basal velocity-effective pressure log-log plane with: (a) Weertman, (b) Budd, (c) Schoof and (d) Tsai laws (Eqns (1)–(4)), for $m = 1/3$, $q = 1$, $C_W = C_B = C_S = 7.624 \times 10^6$ S.I. and $f = C_{\max} = 0.5$. Dotted black lines reported on each plot are the iso-values of τ_b given with the Schoof law. The vertical black dotted line corresponds to $N = 1$ MPa.

[Brondex et al., 2017]

Influence of Friction law on GL



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✓ Examples

Two approaches in Elmer/Ice

Double continuum approach

- Implemented by Basile de Fleurian
- in the distribution
- <http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:hydrologydc>

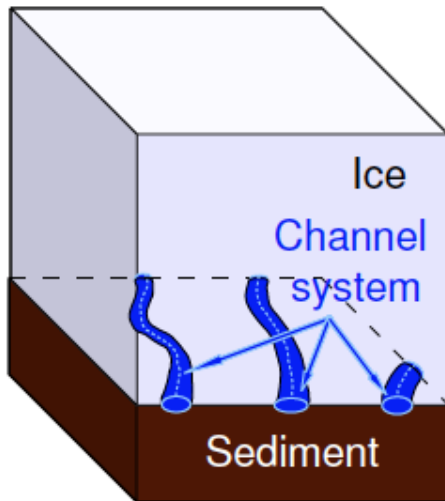
Cavity sheet and discrete channels

- Model developed by Mauro Werder (Werder et al., 2013)
- Implemented in Elmer by O. Gagliardini
- In the distribution
- <http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:glads>

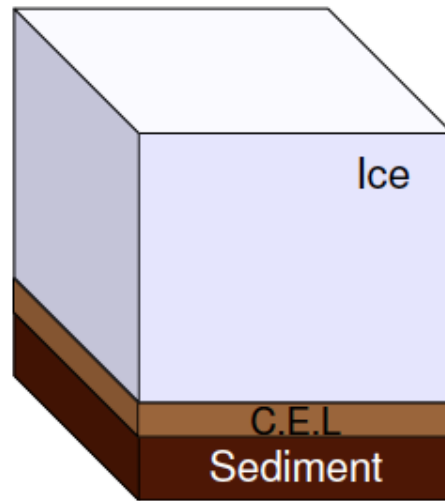
The double continuum approach

Karstified hydrology methods developed while facing difficulties to model conduit drainage [Teutsch and Sauter, 1991]

2 systems
≠ characteristics

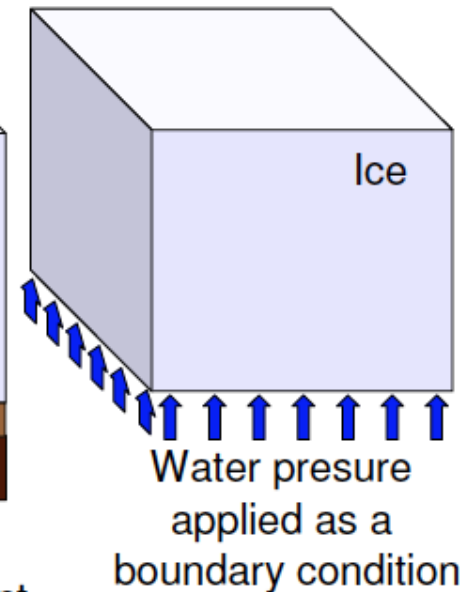


Similar media
≠ conductivities



C.E.L : Channel
Equivalent
Layer

Vertically
integrated



@deFleurian

Computation of the water load

Vertically-integrated computation of the water load h_w

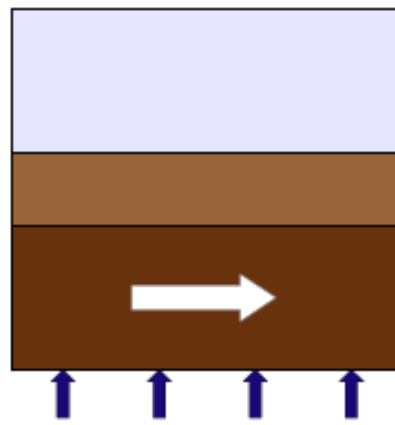
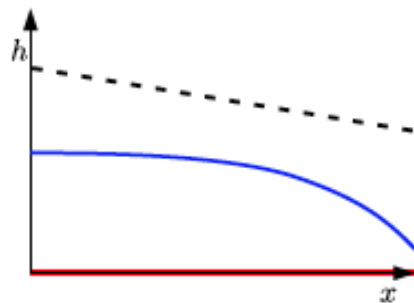
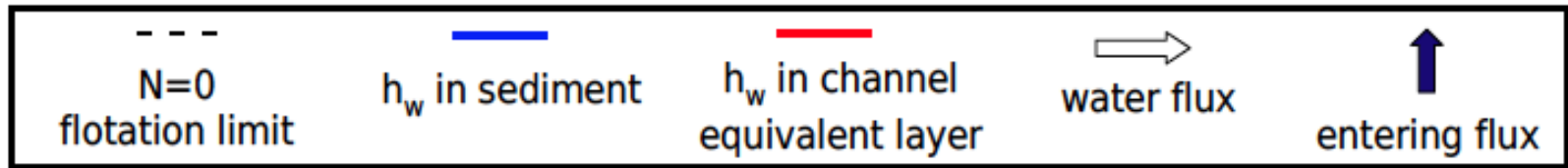
$$\operatorname{div} [\mathbf{T} \operatorname{grad} h_w] = S \frac{\partial h_w}{\partial t} + qe$$

Relies on the transmissivity \mathbf{T} and storage coefficient S of the aquifer

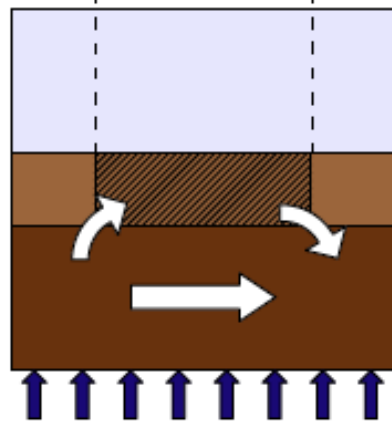
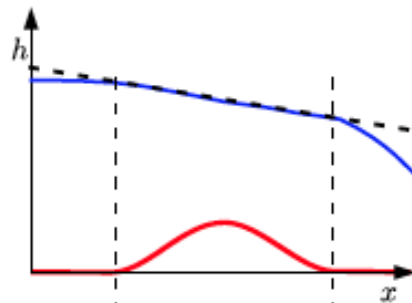
$$\mathbf{T} = \mathbf{K}e; S = \rho_w g e \omega \left[\beta_w + \frac{\alpha}{\omega} \right]$$

ρ_w	Water density	q	Sink/Source term
e	Layer thickness	ω	porosity
\mathbf{K}	Sediment conductivity	α	Porous media compressibility
β_w	Water compressibility		

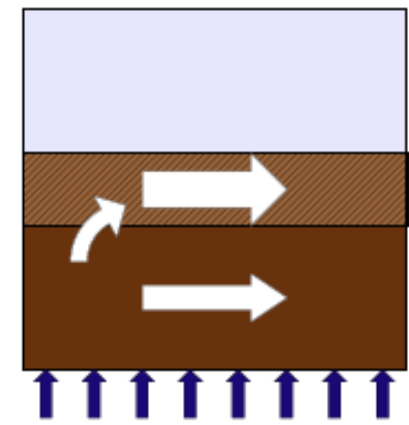
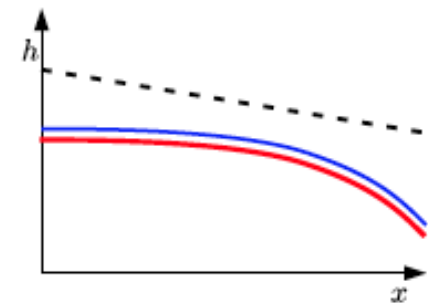
3 states of the Channel Equivalent Layer



closed CEL



active CEL



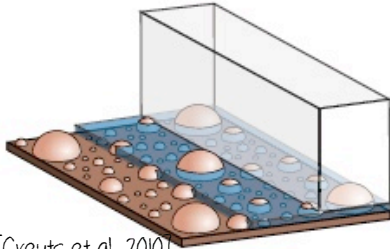
efficient CEL

GlaDS model

(Werder et al., 2013)

Two components system, 3 variables: Φ, h, S

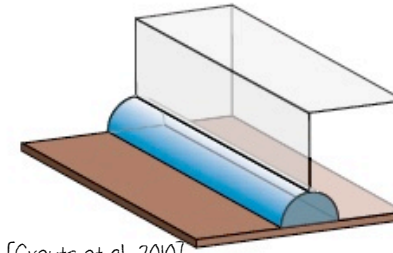
Network of cavities



[Creutz et al., 2010]

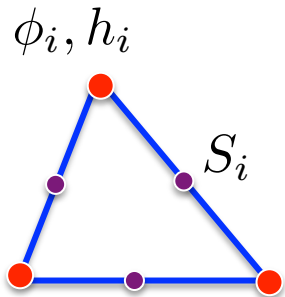
Cavity thickness h
(nodal variable)

Channels



[Creutz et al., 2010]

Channel cross-sectional area : S
(edge variable)



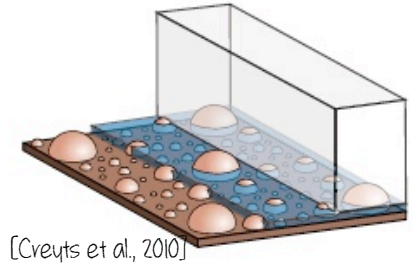
$$\begin{aligned} & \sum_i \int_{\Omega_i} \left[\theta \frac{e_v}{\rho_w g} \frac{\partial \phi}{\partial t} - \nabla \theta \cdot \mathbf{q} + \theta (w - m_b - v) \right] d\Omega \\ & + \sum_j \int_{\Gamma_j} \left[-\frac{\partial \theta}{\partial s} Q + \theta \left(\frac{\Xi - \Pi}{L} \left(\frac{1}{\rho_i} - \frac{1}{\rho_w} \right) - v_c \right) \right] d\Gamma \\ & + \int_{\partial \Omega_N} \theta q_N d\Gamma - \sum_k \theta \left(-\frac{A_m^k}{\rho_w g} \frac{\partial \Phi}{\partial t} + Q_s^k \right) = 0, \end{aligned}$$

GlaDS model, cavities

(Werder et al., 2013)

Discharge (Darcy-Weisbach law) :

$$\mathbf{q} = -kh^\alpha |\text{grad } \phi|^{\beta-2} \text{grad } \phi$$



Cavity thickness evolution :

$$\frac{\partial h}{\partial t} = w(h) - v(h, \phi)$$

with

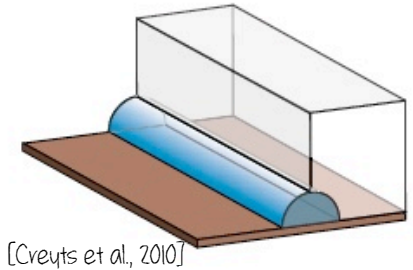
$$\begin{cases} v(h, \phi) = \tilde{A}h|N|^{n-1}N & \text{creep, closing (opening)} \\ w(h) = \max(0; \frac{u_b}{l_r}(h_r - h)) & \text{opening term} \end{cases}$$

GlaDS model, Channels

(Werder et al., 2013)

Discharge (Darcy-Weisbach law) :

$$Q = -k_c S^{\alpha_c} \left| \frac{\partial \phi}{\partial s} \right|^{\beta_c - 2} \frac{\partial \phi}{\partial s}$$



Channel cross-sectional area evolution :

$$\frac{\partial S}{\partial t} = \frac{\Xi(S, \phi) - \Pi(S, \phi)}{\rho_i L} - v_c(S, \phi)$$

with

$$\left\{ \begin{array}{ll} v_c(S, \phi) = \tilde{A}_c S |N|^{n-1} N & \text{Creep, closing (opening)} \\ \Xi(\phi) = \left| Q \frac{\partial \phi}{\partial s} \right| + \left| l_c q_c \frac{\partial \phi}{\partial s} \right| & \text{Energy dissipated} \\ \Pi(S, \phi) = -c_t c_w \rho_w (Q + f l_c q_c) \frac{\partial \phi - \phi_m}{\partial s} & \text{Sensible heat change} \end{array} \right.$$

Comparison

	Double Continuum	GlaDS
Cavity only	=	=
Channels	continuous	discrete
Coupling	$N \rightarrow u$	$N \leftrightarrow u$
Channels closing	No	Yes

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✓ Examples

Friction laws in Elmer/Ice

Friction law in Elmer:

$$C_i u_i = \sigma_{ij} n_j \text{ with } i = 1, 2, 3$$

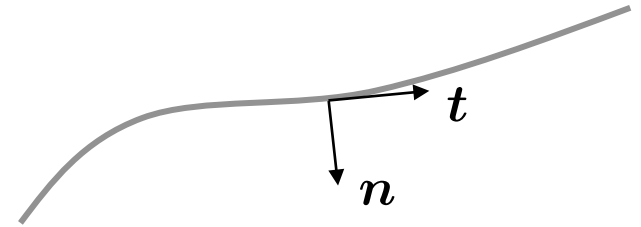
where \mathbf{n} is the surface normal vector

In Normal-Tangential coordinate : $\mathbf{n} = (1, 0, 0)$

and

$$\left\{ \begin{array}{l} C_n u_n = \sigma_{nn} \\ C_{t_1} u_{t_1} = \sigma_{nt_1} \\ C_{t_2} u_{t_2} = \sigma_{nt_2} \end{array} \right\}$$

Friction law applied through the two Slip Coefficients 2 and 3



```
! Bedrock BC
Boundary Condition 1
  Target Boundaries = 1

  Flow Force BC = Logical True
  Normal-Tangential Velocity = Logical True

  Velocity 1 = Real 0.0e0
  Slip Coefficient 2 = Real 0.1
  Slip Coefficient 3 = Real 0.1
End
```

Friction laws in Elmer/Ice

Linear friction laws:

$$\tau_b = \beta u_b$$

```
$beta = 0.1
```

```
Slip Coefficient 2 = Real $ beta
```

```
Slip Coefficient 3 = Real $ beta
```

Non-Linear friction laws:

Need a User Function to evaluate the Slip Coefficient

Rewrite the friction law in the form $\tau_b = C_t(u_b)u_b$

where $C_t(u_b)$ is the Slip Coefficient estimated through a user function

Weertman:
$$C_t(u_b) = u_b^{(1-n)/n} / A_s^{1-n}$$

Schoof, 2005
Gagliardini et al., 2007

$$C_t(u_b) = CN \left(\frac{\chi u_b^{-n}}{1 + \chi^m} \right)^{1/n}$$

with
$$\chi = \frac{u_b}{C^n N^n A_s}$$

Friction laws in Elmer/ice

Problem when $u_b \rightarrow 0$

The law is linearized for small velocity:

$$\begin{cases} C_t(u_b) = C_t(u_b) & \text{for } u_b > u_{t0} \\ C_t(u_b) = C_t(u_{t0}) & \text{for } u_b \leq u_{t0} \end{cases}$$

Example of a call (File USF_Sliding.f90):

```
Normal-Tangential Velocity = Logical True
Flow Force BC = Logical True
```

```
!! Water pressure given through the Stokes 'External Pressure' parameter
!! (Negative = Compressive)
External Pressure = Equals Water Pressure
```

```
Velocity 1 = Real 0.0
Slip Coefficient 2 = Variable Coordinate 1
Real Procedure "ElmerIceUSF" "Friction_Coulomb"
```

```
!! PARAMETERS NEEDED FOR THE BASAL SLIDING LAW
Friction Law Sliding Coefficient = Real $As
Friction Law Post-Peak Exponent = Real $m
Friction Law Maximum Value = Real $C
Friction Law PowerLaw Exponent = Real $n
Friction Law Linear Velocity = Real $ut0
```

GlaDS solvers

Three solvers:

- GlaDSCoupledSolver: main solver, all 3 variables ϕ, h, S are solved in a coupled way
- GlaDSsheetThickDummy: just here to declare h (to save previous values)
- GlaDSchannelOut: just here to export in VTU format edge type variables (not accounted for by ResultOutput solver).

Coupling with SSA using user functions HorizontalVelo and OverburdenPressure.

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Examples

Friction :

Weertman

- Tests/GL_MISMIP, Tests/Contact, Tests/Friction_Weertman.
- <http://elmerice.elmerfem.org/wiki/doku.php?id=userfunctions:weertman>

Coulomb :

- Tests : Tests/Friction_Coulomb and Tests/Friction_Coulomb_Pw
- <http://elmerice.elmerfem.org/wiki/doku.php?id=userfunctions:coulomb>

Hydrology :

Double continuum approach

- Tests/Hydro_SedOnly and Tests/Hydro_Coupled
- <http://elmerice.elmerfem.org/wiki/doku.php?id=solvers:hydrologydc>

Cavity sheet and discrete channels

- Tests/GlaDS and Tests/GlaDS_SSA

References

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